

Using the Knothe Time Function to Predict Surface Subsidence: A Case Study from Ha Long City, Vietnam

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Abstract: This article investigates the applicability of the Knothe Time Function (KTF) in forecasting surface subsidence caused by underground mining activities, using the Thong Nhat coal mine in Ha Long City, Vietnam, as a case study. The study derives two key parameters, η_k and c , from seven initial monitoring stages along line T and employs them to predict subsidence in the subsequent eighth and ninth stages. Results indicate a strong correlation between predicted and measured values, with maximum deviation limited to 61 mm (7.5% of actual subsidence). The Root Mean Squared Errors (RMSE) for these stages are 0.017 and 0.024 meters, respectively, underscoring the reliability of the model. This research highlights KTF's potential for effective dynamic subsidence trough prediction and risk reduction in mining operations.

Keywords: Knothe Time Function, surface subsidence, mining deformation, predictive modeling, coal mine monitoring

1. Introduction

Subsidence is an inevitable consequence of underground mining, bringing about changes to surface landforms, ground and surface water, and damage to constructions on surface as well as underground mines. Many serious incidents have been recorded due to underground mining in Ha Long city, Quang Ninh basin, Vietnam (Long N.Q, Ahhmad, Cuong, & Canh, 2018). For example, a fan station at 142 m level was damaged required costly repairs at the Mao Khe coal mine in the year 2000; 110 Kv electricity line destroyed, and cracks in house's walls around Mong Duong coal mine, etc. The prediction of the mining subsidence is an important task of the mine surveyors, it enables to repair the mining damages efficiently and has a positive impact on the economic results of mining. Thus, the preliminary aim of mine surveyors is to estimate the impact of underground mining on the surface of mines. They started to measure the subsidence of points on the mine surface in order to be able to control the subsidence process and to reduce the damages caused by the underground excavation activity (Kratzsch Helmut, 2012).

Many methods for subsidence prediction have been created, each method has its own advantages and disadvantages as well as conditions for individual applications, but the same point in these methods is that they are only used to predict the subsidence of points in the main section at the centre of trough in the case of critical excavation. In which, empirical prediction methods have high reliability because they are built based on a large number of field measurements. Subsidence profile functions are based on a curve fitting procedure that uses a mathematical function to match the measured subsidence profile. When this mathematical function is established by use of actual field data then it can be used for the future prediction of surface subsidence in the mining area.

In Quang Ninh coal basin of Vietnam, there are some monitoring lines constructed in Ha Lam, Nam Mau, Mao Khe, Thong Nhat, and Mong Duong (Truc, 2012). It is noted that the data of subsidence observation is not collected in the

case of critical excavaton, so subsidence curves in the main section of trough do not represent completely the influence of exploited activities on the surface. In order to determine the total subsidence curve, ongoing observation is required.

This study aims to evaluate the effectiveness of the Knothe Time Function in predicting surface subsidence due to underground coal mining, with specific application to the Thong Nhat coal mine in Ha Long City, Vietnam. By accurately forecasting surface subsidence, this study supports safer mine planning, structural integrity of surface infrastructure, and minimizes environmental disruptions caused by underground mining.

2. Determining the parameters of KTF

Ones describe surface point subsidence by an KTF as follows (Long et al., 2017):

$$\eta_{ti} = \eta(t_i) = \eta_k(1 - e^{-ct_i}) \quad (1)$$

Where, η_k is total value of subsidence at t time, c is time coefficient, t_i is time of ith stage monitoring from the first stage. It can be seen that when $t = \infty$ then $\eta_{\infty} = \eta_k = \eta_{max}$. It is clear given concrete values for η_k and c we can predict η_{ti} for any surface point.

The values of η_k and c can be determined based on observed point subsidence. If the number of observation stages is greater than or equal 3, the parameters η_k and c can be determined by the least square principle. The error equation of observations can be written as follows:

$$V_i = \eta_k(1 - e^{-ct_i}) - \eta_{ti} \quad (2)$$

If the subsidence values are observed with the same accuracy, the parameters η_k and c are solved with the condition of least square $[VV] = \min$. The approximate value of η_k (η_0) is assigned by subsidence value of the last

monitoring stage, and the approximate value of c (c_0) is assigned by 0.1.

The equation of error (2) expanded by linear transformation is expressed as (Chinh, 1997):

$$V_i = A_i d\eta + B_i dc - L_i \quad (3)$$

where,

$$\begin{aligned} A_i &= (1 - e^{-c_0 t_i}) \\ B_i &= \eta_0 \cdot t_i e^{-c_0 t_i} \\ L_i &= \eta_{t_i} - \eta_0 (1 - e^{-c_0 t_i}) \end{aligned} \quad (4)$$

The error equation (3) can be expressed by matrix form as below:

$$V = A \cdot X - L \quad (5)$$

where,

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \dots \\ V_n \end{bmatrix} \quad A = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \\ \dots & \dots \\ A_n & B_n \end{bmatrix} \quad X = \begin{bmatrix} d\eta \\ dc \end{bmatrix} \quad L = \begin{bmatrix} L_1 \\ L_2 \\ \dots \\ L_n \end{bmatrix} \quad (6)$$

The system of normal equations as follow:

$$A^T A X = A^T L \quad (7)$$

Solving the system of normal equations obtained vector X , the best probability values of η and c are calculated as below:

$$\begin{aligned} \eta &= \eta_0 + d\eta \\ c &= c_0 + dc \end{aligned} \quad (8)$$

The parameters η_k and c show the rule of subsidence process, the subsidence of points at the time $t_k = t_n + \Delta t$ can be predicted (t_n is the time at the last monitoring stage). Subsidence value at t_k is computed as follow:

$$\eta_{t_k} = \eta_k (1 - e^{-c t_k}) \quad (9)$$

3. Experiment Data

The Experimental data are collected on Lo Tri leveling network composed of 5 observation lines on Thong Nhat coal mine. Lines A, B and C are established in the deep direction and lines T and P in the strike direction. Each line includes 20 to 40 benchmarks with interval distance of about 10 to 40m. 8 measurement stages were carried out on the network using Ni 030 instrument with levelling staff. The period of two successive stages is approximately 3 months. Precision of measured data is satisfied with Vietnam National Specifications on Mine Surveying (Long, 2010). All observation data, that belong to route T presented in Table 1, were used to calculate values of the two parameters η_k and c .

Table 1: The data of subsidence measurement (unit mm)

Point ID	Distance	Period of time (month)								
		0	3	6	9	12	15	18	21	24
T1	0	0	0	-17	-25	-32	-45	-53	-66	-70
T2	34	0	-5	-23	-28	-38	-51	-69	-73	-80
T3	62	0	-7	-25	-29	-39	-50	-73	-85	-91
T4	105	0	-11	-27	-39	-48	-53	-78	-95	-120
T5	133	0	-17	-59	-78	-77	-87	-115	-155	-175
T6	154	0	-31	-70	-102	-117	-135	-165	-210	-230
T7	177	0	-45	-85	-159	-183	-199	-231	-287	-307
T8	190	0	-54	-90	-150	-194	-212	-249	-310	-350
T9	206	0	-86	-157	-286	-356	-396	-431	-527	-585
T10	232	0	-134	-217	-445	-501	-555	-597	-705	-815
T11	250	0	-197	-351	-588	-753	-791	-832	-980	-1046
T12	263	0	-215	-362	-629	-844	-956	-1015	-1142	-1206
T13	279	0	-183	-335	-651	-893	-1062	-1154	-1301	-1365
T14	291	0	-189	-289	-596	-810	-990	-1108	-1286	-1339
T15	307	0	-201	-289	-495	-757	-934	-1102	-1262	-1311
T16	341	0	-254	-352	-551	-795	-956	-1139	-1269	-1326
T17	350	0	-220	-350	-493	-688	-888	-1071	-1179	-1268
T18	373	0	-197	-274	-441	-627	-798	-934	-1081	-1116
T19	390	0	-93	-175	-302	-471	-643	-776	-887	-923
T20	401	0	-61	-113	-204	-370	-454	-522	-594	-639
T21	418	0	-38	-86	-142	-185	-243	-306	-366	-398
T22	438	0	-42	-50	-80	-117	-159	-221	-287	-311
T23	458	0	-11	-18	-31	-40	-85	-113	-157	-170
T24	473	0	-3	-28	-41	-62	-84	-117	-148	-155
T25	492	0	0	-9	-12	-14	-19	-30	-36	-45
T26	536	0	0	-3	-6	-7	-8	-11	-14	-19

4. Discussion of Results

Measured data subsidence of 7 first stages (from 0 month to 18th month) of all points belong to monitoring line T were

used to build subsidence models over time of points. Using the algorithm introduced in section 2 to determine the parameters η_k and c for all 26 points. Applying the calculated parameters to Equation (1) and recalculate the

subsidence of all points in the 7 first stages. The accuracy of the prediction model is evaluated using RMS of each point. The obtained values of parameters η_k and c with its Root Mean Square Error (RMSE) for each point in line T are set in Table 2.

Table 2: Results of KTF parameters computation

Point ID	η_k	c	RMS (mm)	Point ID	η_k	c	RMS (mm)
T1	-170,112	0,026	2	T14	-1835,903	0,064	70
T2	-154,074	0,036	4	T15	-2091,093	0,049	75
T3	75,981	-0,050	22	T16	-1829,486	0,063	92
T4	596,753	-0,008	8	T17	-1903,559	0,053	84
T5	-355,841	0,030	17	T18	-1723,568	0,052	73
T6	-351,788	0,047	17	T19	-2026,631	0,031	42
T7	-370,240	0,076	22	T20	-1167,714	0,039	29
T8	-512,498	0,050	24	T21	-1056,621	0,023	16
T9	-692,918	0,077	39	T22	746,417	-0,017	18
T10	-876,402	0,094	63	T23	104,982	-0,048	10
T11	-1076,981	0,124	80	T24	929,670	-0,008	6
T12	-1330,386	0,106	79	T25	47,414	-0,032	2
T13	-1663,839	0,083	67	T26	25,070	-0,026	1

The accuracy of prediction model is assessed by Root Mean Square Error (http://www.math-interactive.com/fit_curve_to_data/root_mean_squared_error.htm):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\eta_i - \eta_i^p)^2} \quad (10)$$

where,

η_i^p is subsidence value of point i from KTF;

η_i is subsidence experimental value of point i .

The RMSE between the values computed using KTF parameters and those of experiment range between 1 mm and 92.8 mm. It is obvious that the above mentioned RMSE varies among the cases, and some filters should be done in advance so that the data represent the general subsidence trend, and outliers are removed.

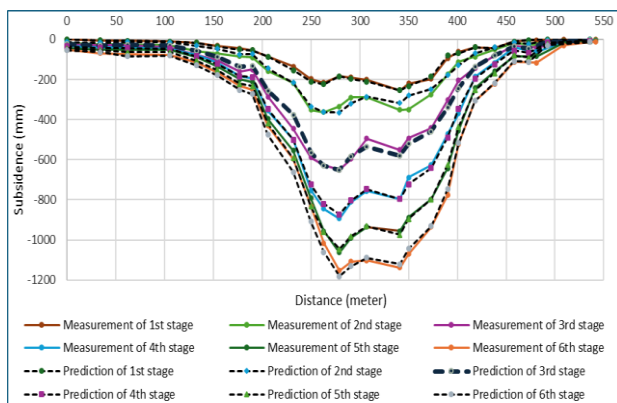


Figure 1: Subsidence scheme of measured and predicted values from stage 2 to 7

Apply parameters η_k and c to equation (1) in order to predict the subsidence of 26 above points at 21th month (8th stage) and 24th month (9th stage). The results of predicted subsidence are shown in table 4. Comparing them to the measured data, the largest and smallest difference are 61 mm at point No10 and 0 mm at point No2 and point No25, equivalent to 7.5% and 0% of measured subsidence value. The point measured subsidence and its corresponding predicted value of trough by KTF are shown in figure 2 and figure 3. The Root Mean Squared Error (RMSE) of the difference between measurement and predicted values of 8th and 9th stages are 0.017 meter and 0.024 meter respectively (table 4). The correlation between measured and predicted values of 26 points in the last two stages, 8th and 9th, is illustrated in figure 4 and figure 5 that present linear relationship with correlation coefficients are 0.997 and 0.998 respectively. Correlation coefficients are computed using following equation:

$$r = \frac{\sum_{i=1}^n (\eta_i - \bar{\eta})(\eta_i^p - \bar{\eta}^p)}{\sqrt{\sum_{i=1}^n (\eta_i - \bar{\eta})^2 * \sum_{i=1}^n (\eta_i^p - \bar{\eta}^p)^2}} \quad (11)$$

where,

r is correlation coefficient;

η_i and η_i^p are measured and predicted values;

$\bar{\eta}$ and $\bar{\eta}^p$ represent correspondent medium values.

Table 4: Results of subsidence prediction

Point ID	21 months		Difference (mm)	24 months		Difference (mm)
	Mea	Pre		Mea	Pre	
T1	-66	-63	-3	-70	-71	1
T2	-73	-73	0	-80	-81	1
T3	-85	-111	26	-91	-101	10
T4	-95	-97	2	-120	-115	-5
T5	-155	-148	-7	-175	-166	-9
T6	-210	-202	-8	-230	-222	-8
T7	-287	-276	-11	-307	-295	-12
T8	-310	-306	-4	-350	-335	-15
T9	-527	-521	-6	-585	-557	-28
T10	-705	-715	10	-815	-754	-61
T11	-980	-961	-19	-1046	-997	-49
T12	-1142	-1134	-8	-1206	-1188	-18
T13	-1301	-1288	-13	-1365	-1370	5
T14	-1286	-1254	-32	-1339	-1356	17
T15	-1262	-1227	-35	-1311	-1345	34
T16	-1269	-1245	-24	-1326	-1346	20
T17	-1179	-1170	-9	-1268	-1278	10
T18	-1081	-1045	-36	-1116	-1142	26
T19	-887	-859	-28	-923	-962	39
T20	-594	-589	-5	-639	-653	14
T21	-366	-355	-11	-398	-402	4
T22	-287	-268	-19	-311	-322	11
T23	-157	-142	-15	-170	-180	10
T24	-148	-137	-11	-155	-162	7
T25	-36	-36	0	-45	-45	0
T26	-14	-15	1	-19	-18	-1
RMSE			17			24

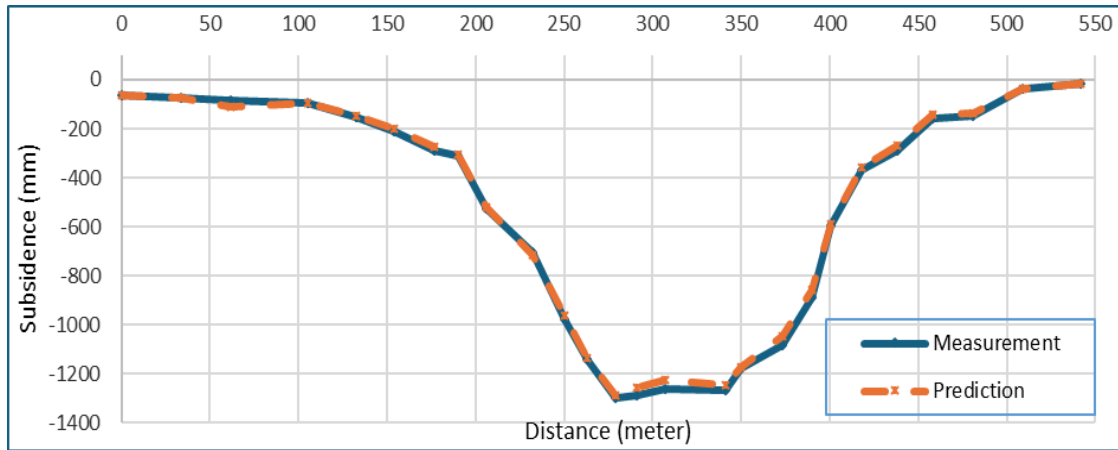


Figure 2: Subsidence scheme of measured and predicted values in 8th stages (month 21)

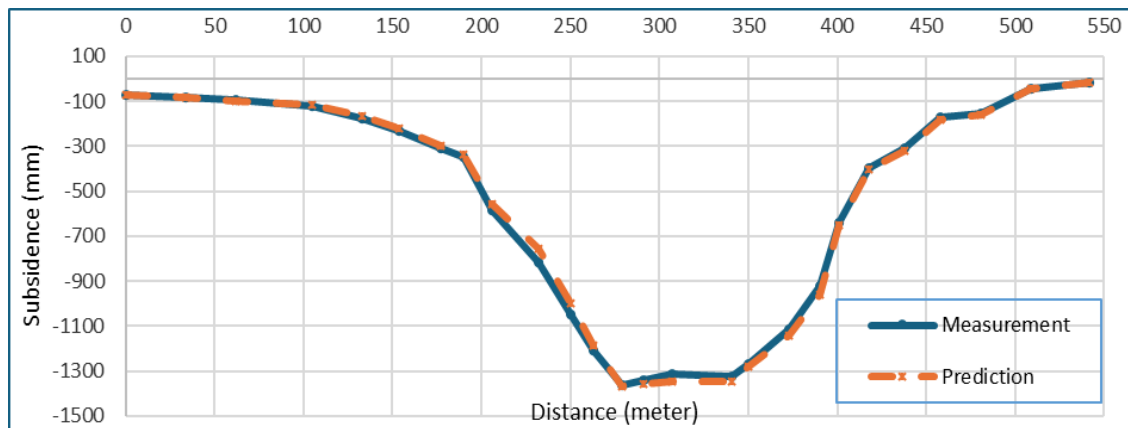


Figure 3: Subsidence scheme of measured and predicted values in 9th stages (month 24)

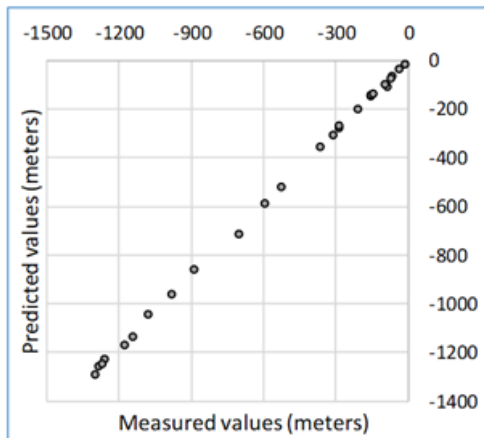


Figure 4: Correlation between the measured and predicted values in 8th stages

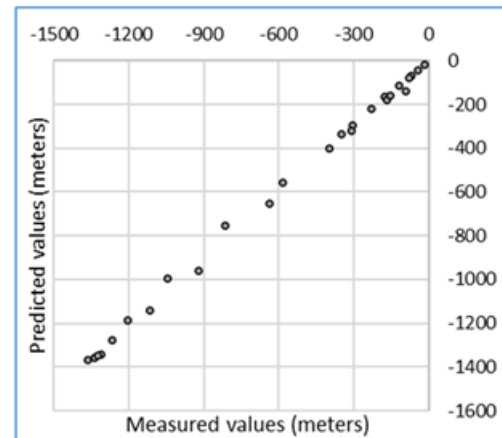


Figure 5: Correlation between the measured and predicted values in 9th stages

5. Conclusions

The study demonstrates that the Knothe Time Function is a promising tool for predicting dynamic surface subsidence caused by underground coal mining. The close agreement between predicted and observed values validates the function's applicability at the Thong Nhat coal mine. This approach can guide proactive surface risk management and improve construction safety in mining areas. Future research should consider broader geological conditions and longer-term data to assess KTF's robustness across diverse settings.

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