NGUYEN TAT THANH UNIVERSITY

ĐẠI HỌC ĐỒNG THÁP
DONG THAP UNIVERSITY

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Some experience in teaching Probability

Vu Tien Viet - People's security Academy
Pham Tuan Cuong - University of Mining and Geology

Abstract: *I would like to introduce some experience in teaching Probability.*

Key word: Teaching, Probability

1 Introduction

(See [1, 2] and [3]) In textbooks faculty and textbooks on Probability we already know determination the classical meaning of Probability is: with random testing of course there is a sample space Ω with a finite number limit n possible elements (outcome - primary event). The probability of appearing is equal, then the probability of event A (subset of Ω) with quantity m elements ($0 \leq m \leq n$) are calculated according to the formula:

$$P(A) = \frac{|A|}{|\Omega|} = \frac{m}{n}$$

This definition is given when the subject new probability was born in the 17th century when two French scientist Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665, who is a lawyer) exchanged letters with each other to discuss about the possibilities in games of chance.

Teaching this subject is still dry dry, difficult to understand, difficult to apply. Other eyes, clear in this definition, the two basic requirements are: feasible number the energy n of the space Ω must be finite, possibility of primary events occurring (outcome) must be equal.

Many pupils and students have asked: when does not satisfy either or both of those requirements, then the above definition cannot be used, so how to solve?

To answer, we have consulted the literature specialized data, find the answer and that's it the reason for this research article.

2 Theoretical research

2.1 Neoclassical definition

(See [4]) When elements (outcome - variables elementary fixed) of the sample space Ω is not possible equal probability of occurrence or number of parts the elements of Ω are infinite (but countable), then they we need to use a different definition (which we temporarily call it *neoclassical*) as follows:

Suppose the sample space Ω consists of a number finite (or countable) elements (outcome - primary event), but the possibility of occurrence of they are not the same. For example, toss one the coin is unbalanced and not of the same quality.

We assign each element $\omega_i \in \Omega, (i = 1, 2, ...)$ a *weight*, denoted by $p(\omega_i)$ and call it *probability* of ω_i. We assume that:

a) $p(\omega_i) \geq 0, \ \forall i = 1, 2, ...$

b) $p(\omega_1) + p(\omega_2) + \cdots = 1.$

From the probabilities $p(\omega_i)$ of the ω_i, for each event A of the sample space Ω, we define the probability of A to be the number $P(A)$ is determined by the formula

$$P(A) = \sum_{\omega_i \in A} p(\omega_i)$$

Comment. If the sample space Ω consists of only a finite number of elements $\omega_1, \omega_2, ..., \omega_n$ and their probability is the same, then by assignment

$$p(\omega_1) = p(\omega_2) = \cdots = p(\omega_n) = \frac{1}{n},$$

the probability of the event A is

$$P(A) = \frac{|A|}{|\Omega|} = \frac{m}{n}$$

Thus, the neoclassical definition is completely consistent with the considered classical definition.

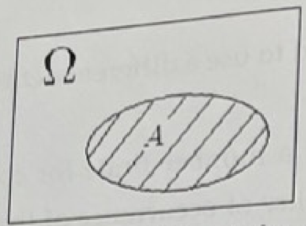
With the neoclassical definition, the properties of probability remains the same value as the field meets the classical definition.

2.2 Geometric definition

(See [4, 5] and [6]) In the case when the number of elements in the sample space Ω is infinite (uncountable), people use the definition in geometric form to determine probability. When then the sample space Ω is represented as one geometric domain, and event A is a subdomain of Ω and the probability of A is given by the formula:

$$P(A) = \frac{\text{mes}(A)}{\text{mes}(\Omega)}$$

where mes(.) indicates the measure (measure) of the corresponding geometric domain.



$$P(A) = \frac{\text{mes}(A)}{\text{mes}(\Omega)}$$

2.3 Definition by statistical frequency

(See [4, 5] and [6]) For the present phenomenon that occurs many times, people can use it statistical frequency to determine the probability of events event that occurs. It is a stable value of frequency:

$$P(A) = \frac{\#(A)}{\#(\Omega)}$$

where $\#(\Omega)$ is the total number of possible cases survey and $\#(A)$ is the number of cases the survey satisfies the condition for occurrence of A.

Mathematical basis for using definition equal to statistical frequency is the Law of Large Numbers and Limit Theorems that we do not have the conditions to talk about come here.

3 Apply and experiment

Example 1. (See [4]. Illustration for work the sample space is infinite but infinite element).

Toss a coin until the first heads (N) appear then stop. The sample space is

$$\Omega = \{N, SN, SSN, ..., SS...SN, ...\}.$$

Since the number of elements of the sample space is infinite (countable), we will use the neoclassical definition of probability. Assign$P(N) = \frac{1}{2}$ for an event that is only tossed once to be successful. Assign $P(SN) = \frac{1}{4}$ to the event that takes a second toss to succeed. Assign $P(SSN) = \frac{1}{8}$ to the event that must be thrown a third time to succeed. Similarly we assign to $\omega_n \in \Omega$ the probability is

$$p(\omega_n) = P(\underbrace{SS...S}_{n-1}N) = \frac{1}{2^n}, \; n = 1, 2, ...$$

We have

$$\sum_{\omega_n \in \Omega} p(\omega_n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1,$$

infer that if you keep tossing the coin in a row, it is certain (with probability 1) that there will be heads (N) and also that it is impossible (with probability 0) that only fully tails (S).

Let A be the event that the test succeeds on even-ordered coin tosses, i.e. $A = \{\omega_2, \omega_4, \omega_6, \omega_8, \ldots\}$, so there

$$P(A) = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \frac{1}{3}.$$

It also follows from this that the probability of $\overline{A}$, which is the event that the test succeeds on odd-ordered coin tosses, will be equal to $\dfrac{2}{3}$.

This problem can be described as a problem of a family having children until they have a son (or daughter), then stop.

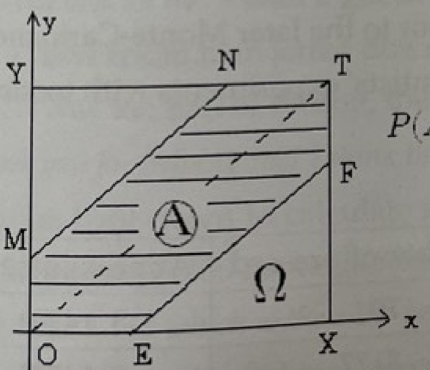
Example 2. (See [4, 5] and [6]. Illustrations the sample space is infinite, but it is not countable elements).

Two people X and Y make an appointment to meet somewhere between 12 o'clock and 1 o'clock. Each person's arrival time at the rendezvous point is random. The sample space is

$$\Omega = \{\omega : \omega = (x, y),\ 0 \leq x \leq 60,\ 0 \leq y \leq 60\},$$

where x is the time of appearance of the first person and y is the time of appearance of the second person (in minutes).

For the event $A = \{(x, y) : |x - y| \leq 20\}$, then A is the event that the person who arrives at the rendezvous point first will wait for the other 20 minutes and then leave if the other person does not appear.



$$P(A) = \frac{\text{mes}(OEFTNM)}{\text{mes}(OXTY)}$$

$$= \ldots ? = \frac{5}{9}$$

On the Oxy coordinate system $OXTY$ rectangle with points

$X(60, 0), T(60, 60), Y(0.60)$ represents the sample space Ω.

Each point $(x, y) \in \Omega$ represents the arrival time of person X is x and the arrival time of person Y is y. We have

$$A = \{(x, y) \in \Omega \mid |x - y| \leq 20\} = \{(x, y) \in \Omega \mid x - 20 \leq y \leq x + 20\}$$

The polygon $OEFTNM$ with the lines $EF: y = x - 20$ and $MN: y = x + 20$ represents the event A.

Example 3. (See [5] and [6]. Illustrations for definitions probability by frequency statistics).

A well-known famous problem using geometrical probability definitions is the *Buffon's Needle* problem.

Count George-Luis Leclerc de Buffon (1707-1788) was a great natural scientist who studied plants, animals, the earth, natural history,... In his youth he was particularly interested in Mathematics. . In 1733 he submitted to the Paris Academy of Sciences a work on the game franc-careau (a popular betting game at the time: people toss a coin into a square and bet each other on the position of the player). where it will be located).

In this work, differential and integral calculus were introduced by Buffon to calculate probability (as defined by geometry). The needle problem proposed by Buffon is as follows:

On a large sheet of paper draw parallel lines at regular intervals a distance greater than the length of a needle. Toss the needle at random onto the paper. There are two possibilities, either the needle is on one of the lines, or the needle is between the two lines. With a geometric probability definition (using calculus) Buffon calculates the probability that the needle will overlap one of the lines as $p = 2/\pi$. On the other hand, if toss the needle n times and m times the needle lies on the line (over the line), the frequency is $\overline{p} = m/n$. From the point of view of defining probability by statistical frequency, we can consider $p \approx \overline{p}$. From this it can be deduced that the approximation $\pi \approx 2n/m$ can be calculated. It is from here that the term "*drop the needle to find* π".

Buffon's needle toss method was the precursor to the later Monte-Carlo method.

Here are some scientists' results of some scientists' experiments with tossing a needle

Experimenter	Number of tosses	Number of pressed	Approximate value π
Buffon, 1777	1106	704	$3,14204$
Wolf, 1850	5000	3177	$3,14762$
Smith, 1855	3204	2030	$3,15665$
De Morgan, 1860	600	382	$3,14136$
Fox, 1864	1030	653	$3,15467$
Lazzerini, 1901	3408	2169	$3,14246$
Reina, 1925	2520	1604	$3,14214$

Note. The Count of Buffon (1707-1788) or rather the Earl of Buffon was a French savant, whose real name was Georges Louis Leclerc. He was born on September 7, 1707 in Monbard, Côte d'Or, France and died on April 16, 1788 in Paris. Buffon is best known as a naturalist, but in his youth he had a passion for mathematics, although at the time he was pursuing his father's ambition to become a lawyer. From 1739, Buffon became the curator of the Royal Botanical Garden (Jardin du Roi) and he worked there for the rest of his life. Buffon's most famous work is Histoire Naturelle (Natural History, 1749 - 1785) which presents all the problems of nature from humans, animals, plants to minerals.

Buffon (1707-1788)


Bá tước Buffon (1707-1788)

Tên thật Georges Louis Leclerc

One night in 1777, in a luxurious building in the French capital Paris, a group of people were huddled around a large piece of paper with parallel lines drawn at regular intervals, and strangely, they were taking turns throwing the needles on the paper according to the wishes of the host.

"Earl, 33 cut and 77 no" – said a guest.

Nearby, a 70-year-old man jotted down numbers on a piece of paper he was holding in his hand. It was the Earl of Buffon, the master of that day.

"Oh, thank you for helping me! I think this is enough for us!" said the Count.

Then Buffon bent down to calculate something on the paper, and after a while he continued: *"There were 1106 throws in total and 704 of them intersected the lines. So we have π which is close to 3.142 which is the ratio between 2212 and 704."*

When Boffon finished speaking, everyone present was bewildered, looking at each other, those eyes were full of doubt but there were also eyes full of excitement. It took a long time, and then, as if unable to contain their curiosity, they both turned to Buffon, waiting for an explanation from him. At that moment, the guests all recognized in that person a pride reigning in his soul. The old earl scanned the cozy room with his bright eyes, then began to say:

During the years 1989-1999 worldwide there were an average of 18 million flights per year, 24 fatal plane crashes and 750 deaths in plane crashes. During the same period in France, an average of 8,000 people died in car accidents each year out of a population of 60 million.

From these figures we can calculate: The probability that a French person dies in a car accident in a year is $8,000/6,000,000 = 0.0133\%$. The probability to take a flight to have a fatal accident is $24/18,000,000 = 0.000133\%$, which is only $1/100$ of the probability of being killed in a car accident in a year. If a person takes 20 flights in a year, the probability of dying in a plane crash is about $20 \times 0.000133\% = 0.00266\%$, which is only 1/5 of the probability of being killed in a plane crash. died in

Example 5. (See [4-6]. Illustration for work define probability using frequency statistics).

Mr. Gregor Mendel (1822-1884) was an Austrian monk interested in the study of biology. He planted various varieties of beans in the monastery's garden and took detailed notes on their genetic and crossbreeding properties. In 1866 G. Mendel published a paper on the phenomena he observed along with a theory to explain those phenomena.

One of the observations was about color: When yellow peas were crossed with green peas (first generation), the hybrids (second generation) all produced yellow peas, but continued to cross pea plants. If the second generation gold seeds are together, then the third generation probability of green beans is $\frac{1}{4}$. This $\frac{1}{4}$ figure is due to Mendel's statistics that the percentage of green beans in the third generation is approximately equal to $\frac{1}{4}$.

From there, Mendel developed a genetic theory to explain this phenomenon: The color of a pea is determined by a gene that has two parts. The first generation yellow pea plants have the purebred gene "YY" and the green pea plants have the purebred gene "yy" (the names "Y" and "y" here are symbolic). When crossing each other, half the genes of one plant merge with half of the genes of the other to form the genes of the offspring. The second generation plants all have the "Yy" gene and the color of the peas is also yellow. By the third generation as a result of crossing the second generation plants together, there are four possible genetic occurrences: "YY", "Yy", "yY", "yy" ("Yy" genes) and "yY" are the same, but written so to distinguish the "Y" from the first or the second of the two hybrids). Theoretically, the above 4 possibilities can be considered to have equal probability. So the probability that the third generation tree has the "yy" gene (green seed) is $\frac{1}{4}$. For many years after its publication, Mendel's work was of no interest to other scientists, but today Mendel is considered the father of genetics.

Example 6. (See [4-6]. Illustration for work define probability using frequency statistics.)

The frequency of appearing T (tail, tails=S) when tossing a coin (coin) many times is given in the table below:

Experimenter	Number of tosses	Number of occurrences of tails	Frequency
Buffon	4040	2048	$0,5080$
Pearson	12000	6010	$0,5016$
Pearson	24000	12012	$0,5005$

So we can conclude that the probability of a tail when tossing a coin is $\frac{1}{2}$. It follows that the probability of coming up heads when tossing a coin is also $\frac{1}{2}$.

In the past, when computers and computers had not yet developed, people had to do very elaborate experiments to find a stable value of the frequency with a large enough number of observations. Today computers make it easier to calculate Probability & Statistics problems, when the right data and the right model are in place. However, the computer itself does not know which model is reasonable. It's the computer user's problem. It is necessary to understand the nature of Probability & Statistical concepts and models before they can be used.

Example 7. (See [4]). A boy know 2 girls (let's call them A and B), emotional level same. Every time a boy want to visit a friend some girl or boy go to the bus stop and meet the bus heading to Ms. A's house, get on the car to Ms. A's house. If you see a car going to Ms. B's house, get in the car to Ms. B's house. Those two vehicles come every 30 minutes there is one trip, the moment of complete appearance cool for about 30 minutes. According to speculation if you think normally, after a long time, no time to visit the 2 girlfriends of the right guy almost the same. But after 3 years of recording, the boy saw that the number of times he visited Ms. A increased 2 times the number of visits to Ms. B. Can explain how does this work from a Probability perspective?

From the point of view of possible Geometric Probability the explanation is as follows: consider the sample space Ω as time interval of 30 minutes, with measure $mes(\Omega) = 1$. Collection of all bus departure times to the direction of Ms. A's house in 30 minutes $mes(A) = \frac{2}{3}$. Collection of all output times the bus is currently going to Ms. B's house in the direction of 30 that minute has a measure of $mes(B) = \frac{1}{3}$. The measure here is Lebesgue measure which will be discussed later Textbook on measurement and integration. That's possible explains the increased number of visits to Ms. A 2 times the number of visits to Ms. B.

Example 8. (See [4]. Illustration sample space has possible elements appear unequally). In a residential area, the incidence of disease X is $1/1000$. If a resident has disease X, when going The test result will be definitely positive (ratio 100%). But if a citizen does not infected with disease X, when tested, it was due to error of test machine, the result will likely be positive (rate 5%). Select a resident at random Tested, the result was positive. Ask How likely is it that that person has disease X?

This problem was proposed by three mathematicians Cassels, Shoenberger, Crayboys quiz 60 doctors and nurses member of Harvard Medical School. Answer most are 95% (because they take $100\% - 5\% = 95\%$).

But actually the answer is approximately 2%. Uncle The explanation can be as follows: Let A be the event that the person is selected for consideration The test is positive, set B_1 as a human event is selected not to have disease X and B_2 is an event the selected person has disease X. According to the full probability formula we have

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) = 99,9\%.5\% + 0,1\%.100\% = 509,5\%$$

The probability to find according to the Bayes formula is

$$P(B_2|A) = P(B_2)P(A|B_2)/P(A) = 0,1\%.100\%/509,5\% = 10/509,5 < 10/500 = 2\%$$

Someone tried to compare mathematics to music and said that Algebra-Topology-Geometry is like symphony and opera, and Probability & Statistics is like ballet.

4 Conclude

The research has been done and tried many years of experience teaching Probability and Statistics for many classes of many schools, such as: i) People's Security Academy; ii) University of Technology, Hanoi National University; iii) University of Economics, National University Hanoi; iv) University of Social Sciences and Humanities, Hanoi National University; v) University of Natural Resources and Environment Ho Chi Minh City; vi) Department of Geography, University of Natural Sciences, University Hanoi National; vii) University of Law, Ha Noi national university. Students and students Students showed interest and deeper understanding of Probability and Statistics.

• In Vietnam, the theory of Probability & Statistics was first studied and taught at Hanoi University. The pioneer is Professor Nguyen Bac Van, a respected teacher of many generations of students majoring in Probability & Statistics.

Nguyen Bac Van (July 11, 1935 - August 30, 2022) was the Head of Probability & Statistics Department of the Department of Mathematics - Mechanics, Hanoi University from 1960 to 1981. After that, Mr. Van moved to work. at the University of Ho Chi Minh City and has lived with his family in Saigon ever since. Mr. Van is a great pedagogue, living modestly, exemplary in working and teaching, the first person to teach Probability & Statistics at university level in Vietnam. Mr. Van is the translator of many very precise terms in Probability & Statistics such as: sample space, random event, expectation, variance, correlation coefficient, reliability, unbiased estimate, Hypothesis testing, stochastic process, queuing theory,... Thanks to that, teachers as well as learners Probability & Statistics easily work in Vietnamese with this subject.

In addition to teaching mathematics at the university level, Mr. Van also teaches mathematics at the high school level for high school students specializing in Mathematics (A_0) of Hanoi University. The lectures on spatial geometry (classical) of Mr. Van (according to French documents) were very attractive and captivated many math students from 1970-1980.

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