Application of Robust Estimation for Adjustment Analysis in Geodetic Networks with Outliers in the Control Points

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Abstract

In the field of data analysis, the method of least squares has been a go-to approach when dealing with measurements that contain random errors. However, this method shows its limitations when faced with real-world data, which, in addition to accidental error, often contains outlier. These outlier can significantly skew the results, leading to inaccurate conclusions if not properly addressed. In response to this challenge, Robust Estimation has emerged as an effective method for handling outlier. Unlike traditional methods, Robust Estimation is designed to be less sensitive to outliers in the data, providing a more reliable and accurate estimate by reducing the impact of outlier on the final result. One of the key features of Robust Estimation is its flexibility. The outcome of each robust estimation method is influenced by the choice of its weight function, allowing the method to be tailored to the specific characteristics of the data. This paper applies the principles of Robust Estimation to the analysis of geodetic networks, which often contain original data errors. By doing so, it aims to provide a more accurate and reliable analysis of these networks, contributing to their improved utilization and management.

Keywords: *Robust Estimation, outliers, Geodetic Network*

1. Introduction

Currently, Vietnam's Ministry of Natural Resources and Environment is building and implementing the project "Modernizing the national geodetic network system for planning, construction, socio-economic development and response to climate change in some large cities and coastal areas". The goal of important national work is to complete and modernize the sustainable and stable national coordinate and elevation system to ensure accurate and consistent determination of elevations of points and locations throughout the country and meet the increasing requirements of socio-economic development, national security and defense, and response to climate change. Tasks that need to be implemented include calculating and adjusting the national coordinate and elevation network and announcing the new National Elevation System. The network has various types of measurement values such as complete measurements of the modern national elevation network connecting "century benchmarks", GNSS CORS points, repeated measurements of the vertical displacement monitoring network in cities of Hai Phong, Hanoi, Da Nang, Ho Chi Minh, Can Tho and the Mekong Delta region using leveling method to analyze, evaluate and determine the causes of subsidence.

Robust estimation method is used in many applications such as GPS positioning [1, 2], GPS coordinate transformation [3], polynomial modelling [4], study of relationship between lake's water area and water level [5], data reconciliation [6], robotics [7], evaluation of the air quality area [8], etc. Various robust estimating techniques are employed to identify minor outliers that could exist in geodetic measurements. Comparative assessment of several resilient estimating techniques for deformation analysis was presented in [9]. Among these techniques are the Danish approach, the Least Absolute Sum, and robust M-estimators.

Thus, when modernizing the construction of the geodetic network, it is natural to make measurements connected to points with higher accuracy and then there is a case where the original points contain errors. Therefore, it is necessary to research data processing methods in this case. On the other hand, the measurement data of the control network cannot avoid outliers. For such a control network system, a data processing tool with coarse error detection and network reliability analysis is needed. In this article, the application of the Robust estimation method is chosen as a solution to solve the problem of data processing and network analysis.

The construction and development of Robust estimation theory is mentioned in many publications [10-12]. The essence of Robust estimation theory is that in cases wh`ere crude errors cannot be avoided, choose an appropriate estimation method so that the estimated values of the parameters are not affected. One of the robust estimation methods is robust estimation M proposed by Huber in 1964 and applied to geodetic data processing. Besides, some scientists have researched and successfully applied robust estimation methods such as Professor Zhow Jiangwen (1980) proposed the principle of estimating M in case the measured values are not of the same accuracy. The method of changing the weights during the adjustment process to achieve parameter stability is called the weight replacement method. Today, a number of scientists have been researching and applying robust estimates in different fields, including surveying. However, Vietnam's geodetic network has its own characteristics, so we research and modify a robust estimation method to process geodetic network data effectively in Vietnamese conditions.

According to [13], in the presence of measurement errors, robust estimation produces "less-biased" estimates than the least squares approach. The least median squares (LMS) approach is a reliable estimating technique. In other study, [14] conducted a study on application of robust estimation in geodetic networks. The obtained results indicated that least-squares adjustment of a geodetic network is the best alternative when neither gross nor systematic errors affect the observations nor the mathematical model. The most likely network solution is then provided by the least-squares adjustment. In order to address large mistakes in close-range photogrammetric data sets that call for a photo bundle correction solution, [15] evaluated the efficiency of robust estimate models. The findings of investigation show that, in comparison to the least squares approach, all robust approaches have the advantage of being able to identify and eliminate gross flaws, particularly when the observation contains large-scale errors. In addition, [16] used this method to detemine displacements in geodetic control network of dam. Then, the advantage of M-estimation over ordinary least squares adjustment in minimizing the impact of outliers on estimated network parameters is demonstrated in this study. Also related to identifing deformation, [17].

In Vietnam, there are not many studies mentioning this method. [18] applied a robust estimation method with a reasonable weight function to adjust and analyze the spatial ground network - GPS. The obtained results show that this method allows searching for raw measurement values, Therefore, "clean" measurement data can be generated and adjusted using the usual method. Also using this method, [19] detected coarse errors in the Son La hydropower construction grid based on robust estimates according to the posterior variance. The calculation results show that this method is effective. In addition to being able to determine the measurement value containing raw error, it can also determine the approximate value of the raw error in the measurement value, and moreover determine the exact value of the measurement value. get not just one but many gross errors in the measurement set. However, Vietnam's geodetic network has its own characteristics, so we research and modify a robust estimation method to process geodetic network data effectively in Vietnamese conditions.

2. Method of Robust estimation applied network adjustment containing the errors of control points

2.1 The Robust Estimation method

Assuming that the matrix of independent measurement called L_{nx1} and uknown vector is $\hat{}_{tx1}$; the observation equations can be rewritten that include residuals. The resulting set of equations is residual equation as [20]:

$$V_{nx1} = A_{nxt}\hat{X}_{tx1} + L_{nx1} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix}_{nxt} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \dots \\ \hat{X}_t \end{bmatrix}_{tx1} + \begin{bmatrix} l_1 \\ l_2 \\ \dots \\ l_n \end{bmatrix} \tag{1}$$

where: A_{nxn} is the design matrix (coefficients matrix), a_i is the individual coefficients of ith, L_{nxl} is the vector of observation (freedom matrix).

From equation (1), the function of robust estimation has form as equation (2) [20]:

$$\rho\,(l_i, \hat{X}) = \rho\,(v_i) \tag{2}$$

Due to the measurements do not equal accuracy; so the weight P can be expressed in matrix form as equation (3):

$$P_{nxn} = \begin{pmatrix} p_1 & & & & \\ & p_2 & & & \\ & & \dots & & \\ & & & \dots & \\ & & & & p_n \end{pmatrix}_{nxn} \tag{3}$$

On the other hand, the ρ function must be minimized: = min

$$\sum_{i=1}^{n} p_i \rho(v_i) = \sum_{i=1}^{n} p_i \rho(a_i \hat{X} + l_i)_{= \min} \tag{4}$$

Taking the derivative of expression (4) with respect to X, denoting $\varphi(v_i) = \frac{\partial \rho}{\partial v_i}$, setting the resulting equation equal to zero yields:

$$\sum_{i=1}^{n} p_i \varphi(v_i) a_i = 0_{= \min} \tag{5}$$

Let $\overline{p}_i = p_i w_i$, $w_i = \frac{\varphi(v_i)}{v_i}$ we have:

$$\sum_{i=1}^{n} a_i^T \overline{p} v_i = 0 \tag{6}$$

or we can write equation (6) as equation (7):

$$A^t PV = 0 \tag{7}$$

Besides, replaceing the V matrix with given in Equation (7) yields the normal matrix of robust estimation M as equation (8):

$$A^T \overline{P} A\hat{X} + A^T \overline{P} L = 0 \tag{8}$$

where: $\overline{P}$ is the weight matrix, $\overline{p}_i$is the weight element, w_i is weight coefficients.

The $\hat{}$ parameters of robust estimation M can be determined as:

$$\hat{X} = (A^T \overline{P} A)^{-1} x A^T \overline{P} L \tag{9}$$

The Function of weight Robust is described as the equation (10), where, c is constant value and select with c of 1.5.

$$\begin{array}{ll} w_i = 1 & |\overline{v}_i| \le c \\ w_i = \frac{c}{|\overline{v}_i|}; |\overline{v}_i| = \frac{v_i}{s_{v_i}}; \overline{p}_i = p_i w_i; & |\overline{v}_i| > c \\ w_{ii} = w_{jj} = 1; & |\overline{v}_i| \le c; |\overline{v}_j| \le c; \\ w_{ii} = \frac{c}{|\overline{v}_i|}; |\overline{v}_i| = \frac{v_i}{s_{v_i}}; w_{jj} = \frac{c}{|\overline{v}_j|}; |\overline{v}_j| = \frac{v_j}{s_{v_j}}; \overline{p}_{ij} = p_{ij} w_{ij}; & |\overline{v}_i| > c; |\overline{v}_j| > c; \end{array} \tag{10}$$

2.1 Network adjustment containing the errors of control points

In this case, the control points are held fixed which is not perfect, meaning that they contain errors. Therefore, the unknown value is included coordinate of the control point.

For adjustment containing the errors of control points, we can choose B=0; β=E, the residual equation is [20]:

$$\begin{array}{l} V = A.X + a\hat{X} + L \\ V = E.X + L \end{array} \tag{11}$$

Where: $\hat{}$ is the unknown vector of the control point; X is the residual vector of approximate value $X^{(0)}$. If $X^{(0)}$ equal to L and to zero, the equation shows:

Tab. 1. Angle measurement

Tab. 1. Pomiar kąta

No	Angle			Value (0 ' ")		
		Occupy	Backsight	Backsight	Occupy	second
1	II	I	III	33	27	26
2	V	II	I	28	34	41
3	IV	II	V	50	51	39
4	III	II	IV	46	35	36
5	I	III	II	20	30	44
6	IV	III	I	40	44	55
7	II	IV	III	72	8	46
8	V	IV	II	71	13	18
9	II	V	IV	57	55	01
10	I	V	II	32	50	07
11	III	I	V	85	8	00

Tab. 2. Distance measurement

Tab. 2. Pomiar odległości

No	Distance		S(m)
	Occupy	Sight	
1	I	II	404 919
2	I	III	934 485
3	I	V	357 225
4	II	III	637 048
5	II	IV	586 833
6	II	V	655 743
7	III	IV	486 229
8	IV	V	537 194

Tab. 3. Baseline components ΔX, ΔY and their weight in rectangular coordinate system

Tab. 3. Składniki bazowe ΔX, ΔY i ich waga w prostokątnym układzie współrzędnych

No	Baselines		Observe		Weight		
	Occupy	Sight	ΔX	ΔY	Occupy	Sight	ΔX
1	I	II	-216.141	342.407	890914.68	-262093 59	323744 99
2	I	III	-851.811	384.289	682879.24	-195924 51	166977 63
3	I	V	-174.003	-311.984	667251.90	-229191 31	471605 13
4	II	III	-635.682	41.860	801837.10	-244621 72	197906 39
5	II	IV	-430.401	-398.913	1075012.79	-326412 03	341196 31
6	II	V	42.152	-654.398	2205098.64	-720459 46	1034072 38
7	III	IV	205.274	-440.776	216375.89	-113497 00	196044 45
8	IV	V	472.540	-255.501	1199357.84	-349210 59	403780 78

Tab. 4. The difference of residual angle between including outliers and without outliers

Tab. 4. Różnica kąta resztkowego pomiędzy uwzględnieniem wartości odstających i bez wartości odstających

No	Angle			v_i (") without outliers	Residual Difference $\Delta v_i = \lvert v_i - v_i' \rvert$ (second)	Including outliers
	Backsight	Occupy	Foresight	**1 07**	**0 84**	
1	II	I	III	-5 64	0 12	
2	V	II	I	-6 08	0 50	
3	IV	II	V	1 41	0 26	
4	III	II	IV	3 23	0 03	
5	I	III	II	-7 45	2 47	
6	IV	III	I	1 81	2 70	
7	II	IV	III	7 45	2 86	
8	V	IV	II	0 63	**15 63**	**+ 18"**
9	II	V	IV	3 33	1 50	
10	I	V	II	-0 77	0 84	
11	III	I	V	1 07	0 12	

Tab. 5. The difference of residual distance between including outliers and without outliers

Tab. 5. Różnica odległości resztkowej pomiędzy uwzględnieniem wartości odstających i bez wartości odstających

No	Distance		v_i (mm) without outliers	Residual Difference $\Delta v_i = \lvert v_i - v_i' \rvert$ (mm)	Including outliers (mm)
	Occupy	Sight			
1	I	II	-2 94	1 58	
2	I	III	-1 12	1 12	
3	I	V	-4 39	2 25	
4	II	III	-3 30	1 23	
5	II	IV	-5 17	5 17	
6	II	V	-1 94	2 01	
7	III	IV	-1 71	1 71	
8	IV	V	-0 58	**98.08**	**+100mm**

Tab. 6. The difference of residual baseline between including outliers and without outliers
Tab. 6. Różnica resztowej linii bazowej pomiędzy uwzględnieniem wartości odstających i bez wartości odstających

No	Baselines		V_i (mm) without outliers		Residual Difference $\Delta v_i = \lvert v_i - v_i' \rvert$ (mm)		Including outliers
	Occupy	Sight					
1	II	I	-0.38	-2.62	**97.62**	0.38	**Δx +100mm**
2	III	I	-3.29	-5.61	1.17	0.14	
3	V	I	0.08	2.54	0.08	2.54	
4	II	III	3.33	3.77	1.21	0.24	
5	II	IV	2.10	4.12	3.21	4.15	
6	II	V	-2.46	-2.16	2.56	2.6	
7	IV	III	-3.77	1.65	4.42	3.91	
8	V	IV	-4.44	-5.72	5.66	6.31	

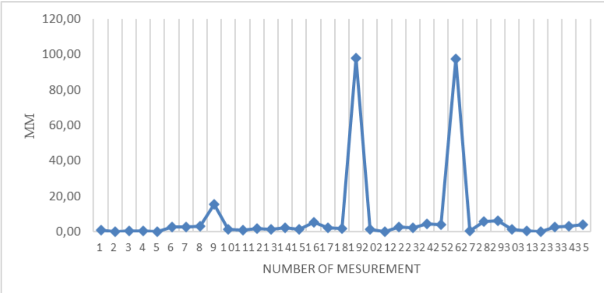

Fig. 1. Difference of residual between two cases: without and including outliers
Rys. 1. Różnica reszt pomiędzy dwoma przypadkami: bez wartości odstających i z uwzględnieniem wartości odstających

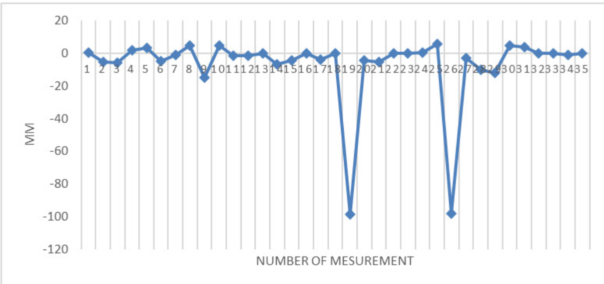

Fig. 2. Detecting outliers using robust estimates
Rys. 2. Wykrywanie wartości odstających przy użyciu solidnych szacunków

$$\begin{aligned} &RX + Ra\hat{X} + b = 0 \\ &R_a^T X + R_X X + b_a = 0 \end{aligned} \tag{12}$$

Where: $R_\alpha = A^t P\alpha$; $R_X = R_{\alpha\alpha} + P$; $R_{\alpha\alpha} = \alpha^t P\alpha$; $b_\alpha = \alpha^t PL$

From equation (12), it can be written as:

$$\begin{bmatrix} R & R_\alpha \\ R_\alpha^T & R_X \end{bmatrix} \begin{bmatrix} X \\ \hat{X} \end{bmatrix} + \begin{bmatrix} b \\ b_\alpha \end{bmatrix} = 0 \tag{13}$$

On the other hand, equation of residual equation (11) shows the block matrix:

$$\begin{bmatrix} V \\ \hat{V} \end{bmatrix} = \begin{bmatrix} A & \alpha & X \\ 0 & E & \hat{X} \end{bmatrix} + \begin{bmatrix} L \\ \hat{L} \end{bmatrix} \tag{14}$$

The problem of equation (14) can be solved with condition as:

$$V^T PV + V^T Q_G^{-1} V = \min \tag{15}$$

Besides, the weight matrix has the form of a diagonal matrix as:

$$\overline{P} = \begin{bmatrix} P & \\ & Q_g^{-1} \end{bmatrix} \tag{16}$$

Therefore, the coefficient matrix of a system of linear equations can be written:

$$\begin{bmatrix} A^T & 0 \\ \alpha^T & E \end{bmatrix} \begin{bmatrix} P & \\ & Q_g^{-1} \end{bmatrix} \begin{bmatrix} A & \alpha \\ 0 & E \end{bmatrix} = \begin{bmatrix} A^T PA & A^T P\alpha \\ \alpha^T PA & \alpha^T P\alpha + Q_g^{-1} \end{bmatrix} \tag{17}$$

Finally, the standard deviation was calculated (mo) as:

$$m_0 = \sqrt{\frac{[PVV]}{n-t}} \tag{18}$$

Where:

$$[PVV] = V^t PV + V^t Q_G^{-1} V \tag{19}$$

2.4 Steps to calculate robust estimates in adjustment containing the errors of control points

Step 1: Setting up the residual equation

Assume that $w_1 = w_2 = w_3 = \ldots = w_n = 1$

$W = I$, $P = E$, so $\overline{P}^{(0)} = \begin{bmatrix} P & 0 \\ 0 & Q_g^{-1} \end{bmatrix}$

Where: P is the weight matrix.

Step 2: Solving problem of equation (13), we can obtain the estimated value the first time X, $\hat{}$ and equation (14) calculates the residual V flowing:

$$\hat{V}\begin{bmatrix} X \\ \hat{X} \end{bmatrix}^{(1)} = -\begin{bmatrix} A^T PA & A^T P\alpha \\ \alpha^T PA & \alpha^T P\alpha + Q_g^{-1} \end{bmatrix}^{-1} \begin{bmatrix} b \\ b_\alpha \end{bmatrix}$$

$$\begin{bmatrix} V \\ \hat{V} \end{bmatrix}^{(1)} = \begin{bmatrix} A & \alpha & X \\ 0 & E & \hat{X} \end{bmatrix} + \begin{bmatrix} L \\ \hat{L} \end{bmatrix}$$

Step 3: From this $V^{(1)}$, the weight matrix can be estimat- ed based on equation (10), and thén solving general equation (13) to obtain a $\hat{}$ atrix of $X^{(2)}$ and $X^{(2)}$. Calculating the residual matrix $V^{(2)}$ and $V^{(2)}$ by equation (20).

Step 4: Similar to step 3, from matrix of $X^{(2)}$, $\bar{}^{(2)}$ the equation of (13) can be calculated. This process was repeated and stopped until the different values twice in a row were less than 10^{-6} mm.

Step 5: Final result is determined from equation (21):

$$\begin{bmatrix} X \\ \hat{X} \end{bmatrix}^{(2)} = -\begin{bmatrix} A^T P^{(1)} A & A^T P^{(1)}\alpha \\ \alpha^T P^{(1)} A & \alpha^T P^{(1)}\alpha + Q_g^{-1} \end{bmatrix}^{-1} \begin{bmatrix} b \\ b_\alpha \end{bmatrix} \begin{bmatrix} V \\ \hat{V} \end{bmatrix}^{(2)} = \begin{bmatrix} A & \alpha & X^{(2)} \\ 0 & E & \hat{X}^{(2)} \end{bmatrix} + \begin{bmatrix} L \\ \hat{L} \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} X \\ \hat{X} \end{bmatrix}^{(k)} = -\begin{bmatrix} A^T P^{(k-1)} A & A^T P^{(k-1)}\alpha \\ \alpha^T P^{(k-1)} A & \alpha^T P^{(k-1)}\alpha + Q_g^{-1} \end{bmatrix}^{-1} \begin{bmatrix} b \\ b_\alpha \end{bmatrix} \begin{bmatrix} V \\ \hat{V} \end{bmatrix}^{(k)} = \begin{bmatrix} A & \alpha & X^{(k)} \\ 0 & E & \hat{X}^{(k)} \end{bmatrix} + \begin{bmatrix} L \\ \hat{L} \end{bmatrix} \quad (21)$$

3. Data and experiment

The Lang Son geodetic network consists of two control points and three unknown points. In this network, there are 11 angles, 8 distances, and 8 baseline vectors. Besides, the weight of the control point is equal to PX = PY = 50.000. Note that, the network will adjust in the rectangular coordinate system. Network adjustment containing the errors of control points based on least squares. Assuming that the blunder in angle is (II-V-IV) + 18" and distance is S(IV-V)+ 100 mm, while baseline is ΔX(I-II) +100 mm. To detect the blunders, we use adjustments combined containing the errors of control points and robust estimates. Creating the data without blunder to adjust using the principle of least squares or robust estimates to possess the outliers (errors) in order to calculate the most probable value.

Note that, the Total point has error of measurements are mS = ±(2mm + 1ppm.D) and $m_{\beta i}$=3" for distance and angles, respectively.

From the result of the experiment shown in Tables 4, 5, 6, and Figure 1 and Figure 2; it can be shown that The outliers were detected accurately, and the magnitude of the estimated outlier is approximately equal to the raw error value included in the experimental model.

4. Conclusion

The Robust estimation method is investigated in this paper, with the selection of a reasonable weight function to detect measurements containing blunders, processing and analyzing the quality of the geodetic network containing the errors of control points for reliable results. Combining the Robust estimation algorithm with the model of network adjustment containing the errors of control points, brings dual efficiency as it detects outliers while also addressing the impact of the errors in control points on the estimation results. This method allows for the identification of outliers, thereby assisting data processors in generating 'clean' measurement data and adjusting network according to conventional methods.

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Zastosowanie Robust Estimation do analizy korekty w sieciach geodezyjnych z wartościami odstającymi w oryginalnych punktach kontrolnych

W dziedzinie analizy danych, metoda najmniejszych kwadratów była podstawowym podejściem przy radzeniu sobie z pomiarami zawierającymi błędy losowe. Jednak ta metoda pokazuje swoje ograniczenia w obliczu rzeczywistych danych, które oprócz błędów losowych często zawierają błędy grube. Te błędy grube mogą znacznie zniekształcić wyniki, prowadząc do nieprecyzyjnych wniosków, jeśli nie zostaną odpowiednio uwzględnione. W odpowiedzi na to wyzwanie, Robust Estimation pojawiła się jako skuteczna metoda radzenia sobie z błędami grubymi. W przeciwieństwie do tradycyjnych metod, Robust Estimation jest zaprojektowana tak, aby była mniej wrażliwa na wartości odstające w danych, dostarczając bardziej niezawodne i precyzyjne oszacowanie poprzez zmniejszenie wpływu błędów grubych na końcowy wynik. Jedną z kluczowych cech Robust Estimation jest jej elastyczność. Wynik każdej metody Robust Estimation ma wpływ przez wybór jej funkcji wagowej, co pozwala dostosować metodę do specyficznych cech danych. Ten artykuł stosuje zasady Robust Estimation do analizy sieci geodezyjnych, które często zawierają błędy pierwotnych danych. Robiąc to, ma na celu dostarczenie bardziej precyzyjnej i niezawodnej analizy tych sieci, przyczyniając się do ich lepszego wykorzystania i zarządzania.

Słowa kluczowe: *Robust Estimation, wartości odstające, sieć geodezyjna*