

# Combining GNSS with terrestrial measurement systems for modernizing the coordinate systems in Vietnam

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## SUMMARY

Currently in Vietnam, GNSS technology and CORS stations are widely applied. One of the effective applications is to provide information to modernize the terrestrial and height network. The report presents the theoretical development process and practical issues in recent times. Introducing the author's research results in building a mixed correction theory for terrestrial geodetic networks and combining with GNSS to determine parameters for coordinate specialization and modernize coordinate systems serving cities. create topographic maps and cadastral maps.

**Keywords:** GNSS application, Adjustment computation, National geodetic network, terrestrial measurement systems.

## 1 INTRODUCTION

Traditional terrestrial surveying technology includes building terrestrial surveying networks and height systems. However, these networks were built in different stages so their accuracy is very different. GPS technology allows for hardening geodetic networks with a length of thousands of kilometers [1]. Today GNSS technology is applied as common sense. However, in Vietnam, the WGS-84 coordinate system is not used, which is VN2000, so the problem of applying GNSS technology is associated with coordinate transformation. In this article, we introduce the history of construction and development of coordinate systems in Vietnam, and especially the phase of building geodetic networks associated with GPS/GNSS technology. From there, the problem of handling mixed types of measurement values in different coordinate systems arises. This may be the experience for countries with similar characteristics to GNSS applications. These jobs actively contribute to solving problems serving infrastructure development, climate change monitoring and other natural resource protection.

## 2 The process of modernizing coordinate systems in Vietnam applying GNSS technology

### 2.1. The process of building the National geodetic network

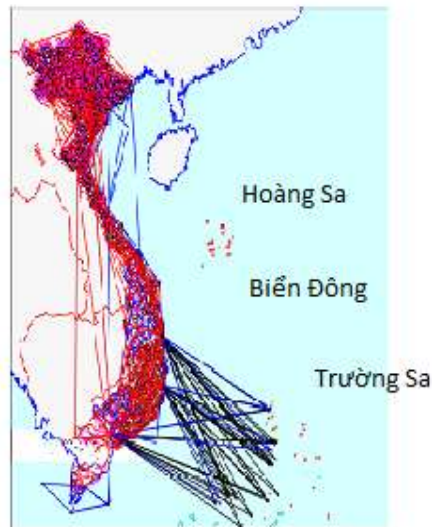
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The geodetic network in Northern Vietnam was built starting in 1959 as a triangular network consisting of 330 class I points and 106 class II points. From 1977 to 1983, the combined geodetic network in the Binh area was carried out. -Tri-Thien is connected to the astronomical and geodetic network of the North. The network consists of 25

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points. In 1983, a class II triangular network was built in the Central region. Class II triangular network includes 351 points.

After 1975, in the South, the geodetic control network was built as a class II pass. The Class II pass net in the Southwest region has 124 points, the Eastern region has 50 points.



## 2.2. Modernizing the Height geodetic network

Building a network in the North (1959 - 1964), determining the average sea level with a 10-year tidal data series (1955 - 1964)

- Announcing the Hai Phong altitude system - 1972. Building a network in the South according to the Ha Tien altitude system (before 1975)
- Building a unified network across the country (1981 - 1991), determining the average sea level with a 43-year tidal data series (1950 - 1992), adjusting the overall network level I, II (1996)
- Complete the national height network class I, II including restoring lost or destroyed landmarks, measuring the original height network, measuring the entire network (2001 - 2004), adjusting the overall network class I, II and the original network with the starting height being the height of the Do Son Origin Point determined in 1992.

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Height nets of classes III and IV are built on the basis of height nets of classes I and II and are adjusted separately. Since 1995, when the basic cadastral networks were built, a number of

points in the basic cadastral networks (class III coordinate networks) have had their height measured and determined according to the GPS method - Level standard used. global geoid model EGM-96.

Currently, the Department of Surveying, Mapping and Geographic Information is implementing the Project to Modernize the National Height System.

### 2.3. 3D spatial GNSS network

GNSS network The short-sided GPS network has been built in areas with particularly difficult terrain conditions (in Minh Hai, Central Highlands, Song Be (old)) and has applied GPS technology to cover the coordinate network in the areas. this area.

Long-sided GPS network on land and sea. In 1992, a marine geodetic network was built using GPS technology to measure and connect the coordinates of points on the island with the coordinate system on the mainland. The network consists of 36 points.

In 1993, long-sided GPS network measurements were conducted to connect a number of points in triangular networks and passes from North to South to strengthen coordinate points for the State network. The network consists of 10 points coinciding with the points on the terrestrial that have been built.

Since 2018, CORS stations have been built. To date, there have been 65 stations, including 24 Geodetic CORS stations evenly distributed throughout the territory of Vietnam with distances between stations from 150 km to 200 km and 41 NRTK CORS stations in key areas with densities ranging from 50 km to 80 km/station.

## 2. DATA PROCESSING METHODS

### 2.1. Combining traditional measurement and GNSS

We give symbols  $\Delta r^S = (\dots \Delta x_{ij}^S \ \Delta y_{ij}^S \dots)^T$  - vector of planar coordinate increments of GPS points.

$$\Delta r^S + \Delta B U + V = \Delta r^{(0)} + A \delta r \quad (1)$$

$\delta r$  - Vector of numbers correcting the x and y coordinates of overlapping points;

$\Delta r$  - vector of coordinate difference calculated in approximate coordinates.

$$U = (\alpha \ m)^T$$

Matrix C is determined from the following block matrices [ 4 ]:

$$C_i = \begin{pmatrix} \Delta Y & \Delta X \\ -\Delta X & \Delta Y \end{pmatrix} \quad (2)$$

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$$A_i = (\dots -E \dots E \dots) \quad (3)$$

$E$   
 $2 \times 2$  - unit matrix

$$V = A \delta r - C U + L^S \quad (4)$$

$$L^S = (\Delta r^{(0)} - \Delta r)$$

with the weight matrix being  $Q_{\Delta r}$

Establish the following system of equations

$$V = \delta r + l; \text{ với } P_t = Q_t \quad (5)$$

From the system of equations (4) and (5), establish a system of normal equations with the following conditions

$$V^T \cdot Q^{-1} \Delta r V + V^T P_t V = \min \quad (6)$$

We have a system of equations:

$$\begin{pmatrix} A^T P \Delta r A & -A^T P \Delta r C \\ & C^T P \Delta r C \end{pmatrix} \begin{pmatrix} \delta r \\ \delta U \end{pmatrix} + \begin{pmatrix} A^T P \Delta r L \\ -C^T P \Delta r L \end{pmatrix} = 0 \quad (7)$$

$$P \Delta r = Q^{-1} \Delta r.$$

Finally we have

$$\begin{pmatrix} A^T P \Delta r A + R_t & -A^T P \Delta r C \\ & C^T P \Delta r C \end{pmatrix} \begin{pmatrix} \delta r \\ \delta U \end{pmatrix} + \begin{pmatrix} A^T P \Delta r L \\ -C^T P \Delta r L \end{pmatrix} = 0 \quad (8)$$

### 3.2. Mixed processing of heght network and GNSS

Assuming there are n GNSS points with height H determined in the terrestrial coordinate system and with height and level, we have a system of conditional equations with additional unknowns [ 6]:

$$BV + A\Delta x + W = 0 \quad (9)$$

$$B_{nx3n} = ( E_{nxn} - E_{nxn} - E_{nxn} )$$

$$Q_V = \begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix}$$

$\Delta x$ - unknown vector

$Q_H, Q_\zeta, Q_h$  is the inverse weight matrix of the vectors

$H, \zeta, h$ .

$$0 = H - \zeta - h = (E - E - E) \begin{pmatrix} H \\ \zeta \\ h \end{pmatrix} \quad (10)$$

$$Q_y = BQB^T$$

$$= (E - E - E) \begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix} (E - E - E)^T$$

$$= (Q_H + Q_\zeta + Q_h) \quad (11)$$

System (9) is solved with the following conditions:

$$\Phi = V^T Q_V^{-1} V = V_H^T Q_H^{-1} V_H + V_\zeta^T Q_\zeta^{-1} V_\zeta + V_h^T Q_h^{-1} V_h = \min$$

$$\Phi = V^T P V + 2K^T (B V + A \Delta x + W) = \min$$

(12)

$$\frac{\partial \Phi}{\partial V} = 2V^T P + 2K^T A = 0 \quad (13)$$

$$\frac{\partial \Phi}{\partial \Delta x} = -2K^T A = 0$$

$$V = P^{-1} B^T K \quad (14)$$

$$A^T K = 0 \quad (.15)$$

Substituting expressions (14) and (15) into (9) we have:

$$\begin{cases} NK + A \Delta x + W \\ A^T K = 0 \end{cases} \quad (16)$$

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Here:  $N = BQB^T$

$$Q = P^{-1}$$

Expression (16) can be rewritten as:

$$\begin{pmatrix} N & A \\ A^T & O \end{pmatrix} \begin{pmatrix} K \\ \Delta x \end{pmatrix} + \begin{pmatrix} W \\ O \end{pmatrix} = 0 \quad (3.10.17)$$

From the first equation of the system (16) we have the expression

$$K = -N^{-1}A\Delta x - N^{-1}W = -N^{-1}(A\Delta x + W) = -N^{-1}W_1 \quad (17)$$

Here vector

$$W_1 = A\Delta x + W \quad (18)$$

substituting the expression (17) into the second equation of the system (16) we get

$$A^T N^{-1} A \Delta x + A^T N^{-1} W = 0$$

$$\Delta x = -(A^T N^{-1} A)^{-1} A^T N^{-1} W$$

$$= -[A^T (Q_H + Q_\zeta + Q_h)^{-1} A A^T (Q_H + Q_\zeta + Q_h)^{-1} W] \quad (19)$$

$$\begin{aligned} V = \begin{pmatrix} V_H \\ V_\zeta \\ V_h \end{pmatrix} &= QB^T K = \begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix} \begin{pmatrix} E \\ -E \\ -E \end{pmatrix} N^{-1} W_1 \\ &= \begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix} \begin{pmatrix} E \\ -E \\ -E \end{pmatrix} N^{-1} W_1 \end{aligned}$$

$$= - \begin{pmatrix} Q_H \\ Q_\zeta \\ Q_h \end{pmatrix} (Q_H + Q_\zeta + Q_h)^{-1} W_1 \quad (20)$$

To evaluate the accuracy after correction we have

Here the matrix

$$T = \begin{pmatrix} Q_H \\ Q_\zeta \\ Q_h \end{pmatrix} (Q_H + Q_\zeta + Q_h)^{-1}$$

$$T = \begin{pmatrix} Q_H \\ Q_\zeta \\ Q_h \end{pmatrix} N^{-1} \quad (22)$$

$$Q_{\Delta x} = (A^T N^{-1} A)^{-1}$$

Symbol  $R = A^T N^{-1} A$  t substituting formula (3.10.21) into formula (3.10.20) we have

$$W_1 = A\Delta x + W = (-AR^{-1}A^T N^{-1} + E)W$$

Here E is the unit matrix.

$$Q_{w1} = (-AR^{-1} + E)N(-N^{-1}A R^{-1}A^T + E)$$

$$= (-AR^{-1}A^T + N)(-N^{-1}AR^{-1}A^T + E)$$

$$= AR^{-1}A^T AR^{-1}A^T - N^{-1}AR^{-1}A^T + AR^{-1}A^T + N = N - AR^{-1}A^T$$

Như vậy chúng ta có biểu thức

$$Q_{w1} = N - AR^{-1}A^T \quad (23)$$

$V_{Hi}, V_\zeta, V_{hi}$  Used to calibrate values  $H_i, \zeta_i, h_i$  at GNSS points and heght networks.

#### 4. SUMMARY AND CONCLUSION

Nowadays, the application of GNSS technology combined with traditional terrestrial measurement results has become an inevitable trend. In Vietnam, in addition to accessing new technology in terrestrial measurement and updating technological advances, GNSS also has to develop the data processing theory of the mixed error correction problem. Practical results of the problem of level correction have demonstrated the clear superiority of accuracy when using a combination of these two technologies.

Applying GNSS technology and terrestrial measurement systems is not only applied in the

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National geodetic control network but also in construction surveying. Geodetic projects in  
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Vietnam serving construction, planning and land management also widely apply the  
combination of GNSS technology with terrestrial measurement to take advantage of the  
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advantages of the two technologies and have achieved results. Proven over decades with many projects.

A big problem is applying GNSS technology to perfect the altitude system in Vietnam. To improve the accuracy of height determination, it is necessary to accurately model the Geoid and Quasigeoid in the territory of Vietnam. This work is currently underway and combined with gravity data to modernize the height system.

To solve practical problems in the field of geodetic mapping in particular and earth sciences in general, which is the determination of spatial data in both international and national coordinate and reference systems. National coordinates and Local coordinate systems, need to solve the problem of processing terrestrial and GNSS data, while also determining the displacement parameters.

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