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LINEAR VIBRATION OF ZENER SYSTEM USING NEWMARK METHOD

Abstract

In this paper the dynamic response of Zener oscillator with third order derivative is researched by the Newmark method. First, the approximately analytical solution is obtained. Then, we calculate linear vibration of Zener viscoelastic system.

Keywords:

viscoelastic, third order, dynamic, Newmark, Zener.

1. Introduction

The Newmark scheme, originally introduced by Newmark [1959], is a classical time-stepping algorithm popular in structural mechanics codes. It has been modified and improved by many other researchers such as Wilson, Hilber, Hughes and Taylor… However, these methods are only used for the system of second order equations.

Many vibration problems in engineering lead the system of differential equations of third order. In this paper we calculate vibration of Zener system by using established Newmark integration method for calculating vibration of third order system.

2. The Newmark method for the third order systems

The Newmark method is a single-step integration formula. The state vector of the system at a time $t_{n+1} = t_n + h$ is deduced from the already-known state vector at time t_n through a Taylor expansion of the displacements, velocities and accelerations.

We get the approximation formulas of displacements, velocities and accelerations of system at time t_{n+1} to approach solving the system of third order differential equations.

$$\ddot{\boldsymbol{q}}_{n+1} = \ddot{\boldsymbol{q}}_n + \left(1-\alpha\right) h \dddot{\boldsymbol{q}}_n + \alpha h \dddot{\boldsymbol{q}}_{n+1}, \tag{1}$$

$$\dot{\boldsymbol{q}}_{n+1} = \dot{\boldsymbol{q}}_n + h\ddot{\boldsymbol{q}}_n + \left(\frac{1}{2} - \gamma\right)h^2\dddot{\boldsymbol{q}}_n + \gamma h^2\dddot{\boldsymbol{q}}_{n+1}, \tag{2}$$

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n + h\dot{\boldsymbol{q}}_n + \frac{h^2}{2}\ddot{\boldsymbol{q}}_n + \left(\frac{1}{6} - \beta\right)h^3\dddot{\boldsymbol{q}}_n + \beta h^3\dddot{\boldsymbol{q}}_{n+1}. \tag{3}$$

Let us then assume that the equations of dynamics

$$\boldsymbol{M}\dddot{\boldsymbol{q}} + \boldsymbol{B}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}(t), \tag{4}$$

are linear, i.e., that matrices $\boldsymbol{M}, \boldsymbol{B}, \boldsymbol{C}$ and $\boldsymbol{K}$ are independent of $\boldsymbol{q}$, and let us introduce the numerical scheme (1), (2) and (3) in the equations of motion at time t_{n+1} so as to compute $\dddot{\boldsymbol{q}}_{n+1}$

$$\begin{aligned}\left[\boldsymbol{M} + \alpha h\boldsymbol{B} + \gamma h^2\boldsymbol{C} + \beta h^3\boldsymbol{K}\right]\dddot{\boldsymbol{q}}_{n+1} &= \boldsymbol{f}_{n+1} - \boldsymbol{B}\left[\ddot{\boldsymbol{q}}_n + (1-\alpha)h\dddot{\boldsymbol{q}}_n\right] \\ -\boldsymbol{C}\left[\dot{\boldsymbol{q}}_n + h\ddot{\boldsymbol{q}}_n + \left(\frac{1}{2} - \gamma\right)h^2\dddot{\boldsymbol{q}}_n\right] &- \boldsymbol{K}\left[\boldsymbol{q}_n + h\dot{\boldsymbol{q}}_n + \frac{h^2}{2}\ddot{\boldsymbol{q}}_n + \left(\frac{1}{6} - \beta\right)h^3\dddot{\boldsymbol{q}}_n\right].\end{aligned} \tag{5}$$

By solving a system of linear equations (5) we obtain $\dddot{\boldsymbol{q}}_{n+1}$. Then, by using Newmark formulas (1), (2) and (3) we get accelerations, velocities and displacements $\ddot{\boldsymbol{q}}_{n+1}, \dot{\boldsymbol{q}}_{n+1}$ and $\boldsymbol{q}_{n+1}$. We determine the initial conditions of $\dddot{\boldsymbol{q}}(t_0)$ from the given values of $\boldsymbol{q}(t_0)$, $\dot{\boldsymbol{q}}(t_0)$ and $\ddot{\boldsymbol{q}}(t_0)$

$$\dddot{\boldsymbol{q}}(t_0) = \boldsymbol{M}^{-1}\left[\boldsymbol{f}(t_0) - \boldsymbol{B}\ddot{\boldsymbol{q}}(t_0) - \boldsymbol{C}\dot{\boldsymbol{q}}(t_0) - \boldsymbol{K}\boldsymbol{q}(t_0)\right]. \tag{6}$$

Let us assume that the non-linear dynamic equations of third order systems have the following form

$$\boldsymbol{M}(\boldsymbol{q})\dddot{\boldsymbol{q}} + \boldsymbol{k}(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = \boldsymbol{f}(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}), \tag{7}$$

We have $\dddot{\boldsymbol{q}}_{n+1}$ from equation (3) and substitute into equations (1) and (2), we realize that $\dddot{\boldsymbol{q}}_{n+1}, \ddot{\boldsymbol{q}}_{n+1}, \dot{\boldsymbol{q}}_{n+1}$ are represented by $\boldsymbol{q}_{n+1}$ and the known values of $\boldsymbol{q}_n, \dot{\boldsymbol{q}}_n, \ddot{\boldsymbol{q}}_n, \dddot{\boldsymbol{q}}_n$. By substituting $\dddot{\boldsymbol{q}}_{n+1}, \ddot{\boldsymbol{q}}_{n+1}, \dot{\boldsymbol{q}}_{n+1}$ into (7), we obtain the system of non-linear algebraic equations with unknown $\boldsymbol{q}_{n+1}$. We have values of $\boldsymbol{q}_{n+1}$ through the iteration method Newton. Then, we determine values of $\dot{\boldsymbol{q}}_{n+1}, \ddot{\boldsymbol{q}}_{n+1}$ and $\dddot{\boldsymbol{q}}_{n+1}$ with the initial conditions of $\dddot{\boldsymbol{q}}(t_0)$ derived from the equations of dynamics (7)

$$\dddot{\boldsymbol{q}}_0 = \boldsymbol{M}^{-1}(\boldsymbol{q}_0)\left[\boldsymbol{f}(t_0, \boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \ddot{\boldsymbol{q}}_0) - \boldsymbol{k}(t_0, \boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \ddot{\boldsymbol{q}}_0)\right]. \tag{8}$$

3. Calculating linear vibrations of Zener system

According Newton's second law of motion, we have motion equation of system

$$m\,\ddot{x}(t) + p(t) = F(t) \tag{9}$$

Where $x(t)$ is displacement of mass m, $F(t)$ is external force, $p(t)$ là internal force in viscoelastic materials.

Property of Zener model: total displacement is sum of ingredient displacements (spring k_1 and spring k_2). Thus, we have

$$x = x_1 + x_2 \tag{10}$$

$$p = p_1 = p_{2(lx)} + p_{2(n)} \tag{11}$$

And $$p_1 = k_1 x_1, \quad p_{2(lx)} = k_2 x_2, \quad p_{2(n)} = c\,\dot{x}_2 \tag{12}$$

Derivative of expression (10) $$\dot{x} = \dot{x}_1 + \dot{x}_2 \tag{13}$$

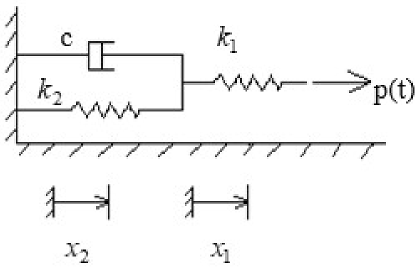

Fig. – 1a

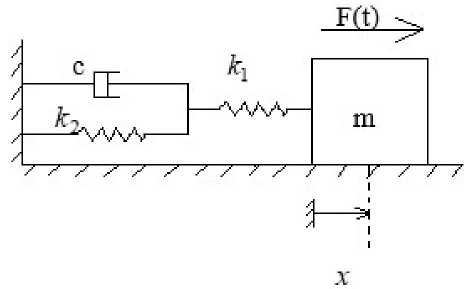

Fig. – 1b

Equation (12) yeilds $\dot{x}_1 = \frac{\dot{p}_1}{k},\ \dot{x}_2 = \frac{p_{2(n)}}{c}$. Substituting into (13), we obtain

$$\dot{x} = \frac{\dot{p}_1}{k_1} + \frac{p_{2(n)}}{c} = \frac{\dot{p}}{k_1} + \frac{p - p_{2(lx)}}{c} = \frac{\dot{p}}{k_1} + \frac{p}{c} - \frac{k_2}{c} x_2 \tag{14}$$

From (10) and (14) we deduce relation between internal force and displacement

$$c\frac{\dot{p}}{k_1} + p + \frac{k_2}{k_1} p = k_2 x + c\dot{x} \tag{15}$$

We have force *p* from equation (9)

$$p = F(t) - m\ddot{x} \tag{16}$$

Substituting equation (16) into equation (15) we obtain the motion equation of Zener model

$$cm\dddot{x} + (k_1 + k_2)m\ddot{x} + k_1 c\dot{x} + k_1 k_2 x = c\dot{F} + (k_1 + k_2)F \tag{17}$$

Assume at time t_n we have the motion differential equation of system

$$cm\dddot{x}_n + (k_1 + k_2)m\ddot{x}_n + k_1 c\dot{x}_n + k_1 k_2 x_n = c\dot{F}_n + (k_1 + k_2)F_n \tag{18}$$

Applying formulas (1), (2) and (3), we obtain $\dddot{x}_n$

$$\begin{aligned}&\left[a_1 + \alpha h a_2 + \gamma h^2 a_3 + \beta h^3 a_4\right]\dddot{x}_n = f_n - a_2\left[\ddot{x}_{n-1} + (1-\alpha)h\dddot{x}_{n-1}\right]\\ &-a_3\left[\dot{x}_{n-1} + h\ddot{x}_{n-1} + \left(\frac{1}{2}-\gamma\right)h^2\dddot{x}_{n-1}\right] - a_4\left[x_{n-1} + h\dot{x}_{n-1} + \frac{h^2}{2}\ddot{x}_{n-1} + \left(\frac{1}{6}-\beta\right)h^3\dddot{x}_{n-1}\right]\end{aligned} \tag{19}$$

Where $a_1 = cm, a_2 = (k_1 + k_2)m,\ a_3 = k_1 c, a_4 = k_1 k_2, f_n = (k_1 + k_2)F_n + c\dot{F}_n.$

By substituting the above value of $\dddot{x}_n$ into equations (1-3), we get the approximation formulas of displacements, velocities and accelerations of system at time t_n

$$\begin{aligned}\ddot{x}_n &= \ddot{x}_{n-1} + (1-\alpha)h\dddot{x}_{n-1} + \alpha h\dddot{x}_n,\\ \dot{x}_n &= \dot{x}_{n-1} + h\ddot{x}_{n-1} + \left(\frac{1}{2}-\gamma\right)h^2\dddot{x}_{n-1} + \gamma h^2\dddot{x}_n,\\ x_n &= x_{n-1} + h\dot{x}_{n-1} + \frac{h^2}{2}\ddot{x}_{n-1} + \left(\frac{1}{6}-\beta\right)h^3\dddot{x}_{n-1} + \beta h^3\dddot{x}_n.\end{aligned} \tag{20}$$

With the initial conditions of $x(0), \dot{x}(0), \ddot{x}(0)$, we determine values of $\dddot{x}(0)$:

$$\dddot{x}(0)=\frac{1}{cm}\left[c\dot{F}(0)+(k_1+k_2)F(0)-(k_1+k_2)m\ddot{x}(0)-k_1c\dot{x}-k_1k_2x(0)\right].$$

4. Numerical example

We have tried out the algorithm on an example. We have chosen the initial conditions $m=5(\mathrm{kg}), c=2(\mathrm{Ns/m}), k_1=5(\mathrm{N/m}), k_2=7(\mathrm{N/m}), h=0.01(\mathrm{s}), \alpha=0.5,$

$$\gamma=0.25,\ \beta=1/12,\ F=0.5\sin\left(\pi t+\frac{\pi}{6}\right)(\mathrm{N}),$$

$$x(0)=0(\mathrm{m}),\dot{x}(0)=1(\mathrm{m/s}),\ddot{x}(0)=0(\mathrm{m}/s^2)\Rightarrow\quad \dddot{x}(0)=\frac{\pi\sqrt{3}-14}{20}(\mathrm{m/s^2}).$$

The differential equation of motion has following form

$$10\dddot{x}+60\ddot{x}+10\dot{x}+35x=6\sin\left(\pi t+\frac{\pi}{6}\right)+\pi\cos\left(\pi t+\frac{\pi}{6}\right) \tag{21}$$

The solution of Eq. (21) obtained by the Newmark method is represented in Fig.2.

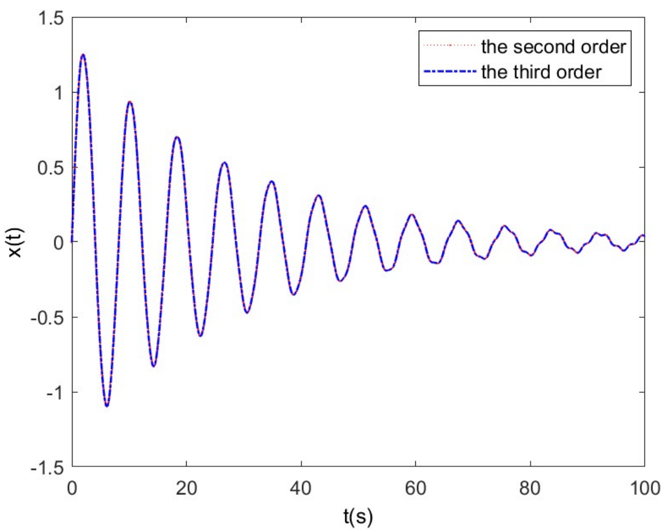

Fig. 2 – Time histories of the displacement in Eq. (21)

5. Conclusions

Using the Newmark integration scheme, a numerical algorithm is developed to calculate dynamic response of third order systems. The motion differential equation of Zener model is established and numerical solution is obtained. In the example, a good agreement is obtained by the Newmark method between second order system and third order system.

The single-step Newmark numerical integration algorithm presented here for Zener third order systems is effective and successful. According to this algorithm, a computer program is developed using MATLAB software.

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