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ESTABLISHING NONLINEAR VIBRATIONAL DIFFERENTIAL EQUATION OF CAR WITH FRACTIONAL DAMPING

Abstract

Based on the theory of fractional derivative, Newton's second law of motion and some transformation, the present study aims to establish nonlinear vibrational differential equation of car with integer damping. Then, nonlinear vibrational differential equation of car with fractional damping is obtained.

By establishing nonlinear vibrational differential equation of car with fractional derivative, complex structures can be designed logically, technical standard assurance.

Keywords

vibration, nonlinear, car, fractional order, damping.

1. Introduction

Cars are a means of transport that play a very important role in the national economy and are currently widely used in all fields of economy and life. Car vibrations not only affect people (drivers and passengers), transported goods, durability, and safety of movement of cars, but also affect the life of the road. Especially during movement, when the car vibrates, it generates very large dynamic loads that impact the car's chassis system, details, overall structure... affecting the durability and longevity of the car. they. Therefore, studying car vibrations is necessary and useful. One of the important tasks of car's vibration research is to establish and solve differential equations to determine vibration parameters.

The generalization of the concept of derivative $D^{\alpha}[f(x)]$ to noninteger values of α goes back to the beginning of the theory of differential calculus. In fact, Leibniz, in his correspondence with Bernoulli, L'Hopital and Wallis (1695), had several notes about the calculation of $D^{1/2}[f(x)]$. Nevertheless, the development of the theory of Fractional Calculus is due to the contributions of many mathematicians such as Euler, Liouville, Riemann, and Letnikov [1-3].

In recent years Fractional Calculus has been a fruitful field of research in science and engineering [1–4]. In fact, many scientific areas are currently paying attention to the Fractional Calculus concepts and we can refer its adoption in viscoelasticity and damping, diffusion and wave propagation, electromagnetism, chaos and fractals, heat transfer, biology, electronics, signal processing, robotics, system identification, traffic systems, genetic algorithms, percolation, modeling and identification, telecommunications, chemistry, irreversibility, physics, control systems as well as economy, and finance…

2. Establishing nonlinear vibrational differential equation of car with fractional damping

Linear viscoelasticity is a combination of models: linear elasticity (Figure 1), integer linear viscosity (Figure 2) and fractional linear viscosity (Figure 3). Where σ is the stress, ε is the strain, E is the elastic modulus when tensile or compressive (characterizing the stiffness of the material), η and c are the viscous resistance coefficients, D_t^{α} which are fractional derivatives with time t.

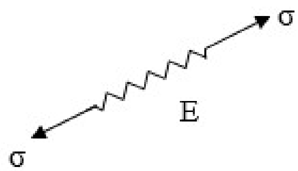

Fig 1 – Linear elasticity model $\left(\sigma = E.D^0\varepsilon\right)$

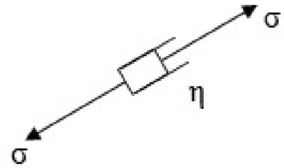

Fig 2 – Integer linear viscosity model $\left(\sigma = \eta.D^1\varepsilon\right)$

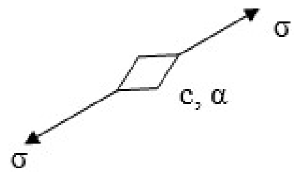

Fig 3 – Fractional linear viscosity model $\left(\sigma = c.D_t^\alpha\varepsilon\right)$

2.1. Integer derivative model

Let *y* and *u* be the displacements of the object and the wheel. It is assumed that the stiffness of the shaft lining can be represented by a spring equivalent to the stiffness k_3 with the displacement denoted by *z*.

We have Newton's second law of motion

$$m\ddot{y} = -R\left(t\right), \tag{1}$$

Where *m* is the mass of the object, $y\left(t\right)$ is the displacement, and $R\left(t\right)$ is the internal force generated inside the viscoelastic object.

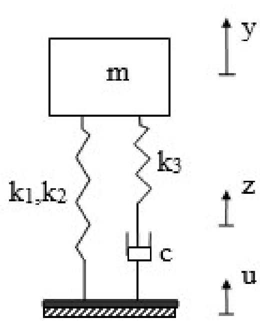

Fig 4 – Integer derivative model

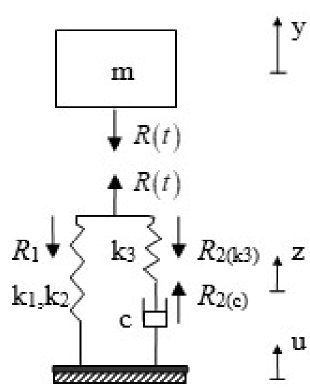

Fig 5 – Force analysis

The force value $R\left(t\right)$ is equal to the total value of the force R_1 acting on the spring k_1 and the force R_2 acting on spring k_2 or damping *c*

$$R\left(t\right) = R_1 + R_2, \tag{2}$$

$$R_2 = R_{2\left(k_3\right)} = R_{2\left(c\right)}, \tag{3}$$

Where

$$R_1 = k_1(y-u) + k_2(y-u)^3, \qquad R_{2(k_3)} = k_3(y-z), \qquad R_{2(c)} = c(\dot{z} - \dot{u}), \tag{4}$$

$$\begin{aligned} \Rightarrow R(t) &= R_1 + R_2 = R_1 + R_{2(k_3)} \\ &= k_1(y-u) + k_2(y-u)^3 + k_3(y-z), \end{aligned} \tag{5}$$

From equations (3) and (4) we have

$$k_3(y-z) = c(\dot{z} - \dot{u}) \tag{6}$$

Substituting equation (5) into (1) we obtain the motion equation

$$m\ddot{y} = -k_1(y-u) - k_2(y-u)^3 - k_3(y-z), \tag{7}$$

Derive *z* from above equation

$$z = \frac{m\ddot{y} + k_1(y-u) + k_2(y-u)^3}{k_3} + y, \tag{8}$$

From equations (8) and (6) we get

$$k_3(y-z) = c(\dot{z} - \dot{u})$$

$$\Leftrightarrow -m\ddot{y} - k_1(y-u) - k_2(y-u)^3 = c(\dot{z} - \dot{u})$$

$$\Leftrightarrow -m\ddot{y} - k_1(y-u) - k_2(y-u)^3 = c\left[\frac{m\dddot{y} + (k_1+k_3)(\dot{y}-\dot{u}) + 3k_2(y-u)^2(\dot{y}-\dot{u})}{k_3}\right]$$

Let $y - u = x$ we have the differential equation of motion

$$\Leftrightarrow \dddot{x} + \frac{k_3}{c}\ddot{x} + \frac{3k_2}{m}x^2\dot{x} + \frac{k_1+k_3}{m}\dot{x} + \frac{k_1k_3}{mc}x + \frac{k_2k_3}{mc}x^3 = -\left(\dddot{u} + \frac{k_3}{c}\ddot{u}\right) \tag{9}$$

2.2. Fractional derivative model

Let *y* and *u* be the displacements of the object and the wheel. It is assumed that the stiffness of the shaft lining can be represented by a spring equivalent to stiffness k_3 with the displacement denoted by *z*.

We have Newton's second law of motion

$$m\ddot{y} = -R(t), \tag{10}$$

Where *m* is the mass of the object, $y(t)$ is the displacement, and $R(t)$ is the internal force generated inside the viscoelastic object.

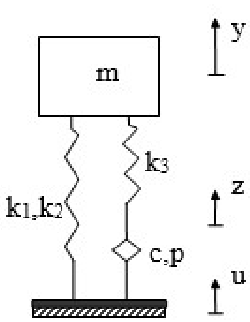

Fig 6 – Fractional derivative model

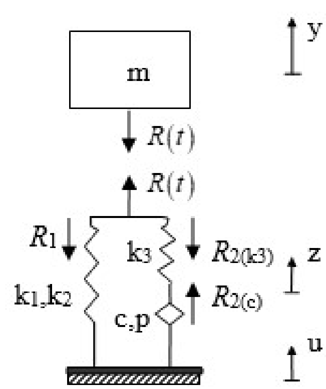

Fig 7 – Force analysis

The force value $R(t)$ is equal to the total value of the force R_1 acting on the spring k_1 and the force R_2 acting on spring k_2 or damping c

$$R(t) = R_1 + R_2, \tag{11}$$

$$R_2 = R_{2(k_3)} = R_{2(c)}, \tag{12}$$

Where

$$R_1 = k_1(y-u) + k_2(y-u)^3, \quad R_{2(k_3)} = k_3(y-z), \quad R_{2(c)} = cD_t^p(z-u), \tag{13}$$

$$\begin{aligned} \Rightarrow R(t) &= R_1 + R_2 = R_1 + R_{2(k_3)} \\ &= k_1(y-u) + k_2(y-u)^3 + k_3(y-z), \end{aligned} \tag{14}$$

From equations (12) and (13) we have

$$k_3(y-z) = cD_t^p(z-u) \tag{15}$$

Substituting (14) into (10) we have the motion equation

$$m\ddot{y} = -k_1(y-u) - k_2(y-u)^3 - k_3(y-z), \tag{16}$$

Derive *z* from above equation

$$z = \frac{m\ddot{y} + k_1(y-u) + k_2(y-u)^3}{k_3} + y, \tag{17}$$

From equations (17) and (15) we get

$$\begin{aligned} & k_3(y-z) = cD_t^p(z-u) \\ & \Leftrightarrow -m\ddot{y} - k_1(y-u) - k_2(y-u)^3 = cD_t^p\left(\frac{m\ddot{y} + k_1(y-u) + k_2(y-u)^3}{k_3} + y\right) - cD_t^p u, \\ & \Leftrightarrow D_t^p\ddot{y} + \frac{k_3}{c}\ddot{y} + \frac{k_2}{m}D_t^p(y-u)^3 + \frac{k_1}{m}D_t^p(y-u) + \frac{k_3}{m}D_t^p y + \frac{k_1k_3}{mc}(y-u) \\ & \quad + \frac{k_2k_3}{mc}(y-u)^3 - \frac{k_3}{m}D_t^p u = 0. \end{aligned}$$

Let $y - u = x$ we have the differential equation of motion

$$D_t^p\ddot{x} + \frac{k_3}{c}\ddot{x} + \frac{k_2}{m}D_t^p x^3 + \frac{k_1 + k_3}{m}D_t^p x + \frac{k_1k_3}{mc}x + \frac{k_2k_3}{mc}x^3 = -\left(D_t^p\ddot{u} + \frac{k_3}{c}\ddot{u}\right) \tag{18}$$

3. Conclusions

In this paper, we used the force analysis method and Newton's second law to establish the nonlinear vibrational differential equation of a car in the case of integer and fractional damping. In the case of integer damping, the nonlinear vibrational differential equation takes the form of a third-order differential equation. In the case of fractional damping, the nonlinear vibrational differential equation contains fractional derivatives.

The authors will use the definitions and properties of fractional derivatives, through analytical and numerical methods to find solutions of established differential equations in subsequent studies. Thanks to that, it is possible to survey the vibrations of car, contributing to setting directions and measures to improve the quality of produced cars.

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