

ANALYSIS OF STRESS TRANSFER AND DISTRIBUTION OF AN EMBANKMENT ON UNREINFORCED AND STONE COLUMN-REINFORCED SOFT SOILS

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ABSTRACT: The technology of stone column-reinforced soft soil (SCRS) has become widespread use in infrastructure projects including embankments, dikes, and dams, as well as industrial and civil constructions, with a particular presence in Western countries and North America. Compared to traditional reinforcement methods, SCRS offers numerous benefits which are the substantial enhancement of ground load-bearing capacity, the considerable decrease in building time, the increase in soil liquefaction resistance, and the utilization of locally sourced materials. The study establishes 3D numerical models of a four-meter high embankment on unreinforced and stone column-reinforced soft soils to investigate the stress transfer and distribution within the systems. The geological condition was derived from the project involving the approach road to the Ganh Hao bridge in Bac Lieu province, Vietnam. The soils, embankment, and stone columns were modeled using the linear elastic, perfectly plastic model. The stresses acting on the column head and soft soil, the stress concentration ratio, and the stress along column depth were described. The numerical results show the appearance of an arching effect for stress transfer within the embankment, characterized by a stress concentration ratio exceeding 3. Moreover, the assessment of negative skin friction around the column shaft points to the presence of an equilibrium plane for negative skin friction located at a depth of approximately 17 to 18 meters.

Keywords: Embankment, Stone column, Soft soil, Liquefaction, Stress transfer

1. INTRODUCTION

Ground improvement technique by stone columns is widely applied for its effectiveness in strengthening the soil beneath foundations and embankments. This technique involves the vertical drilling of holes into the ground, which are subsequently filled with compacted gravel stones. The concept of incorporating granular inclusions to enhance weak soils was initially introduced in 1839 in Bayonne, France, as an alternative to traditional wood piles. However, the widespread adoption of stone columns in Europe did not occur until the 1950s. With the significant advancements in construction technology, stone columns have become a prevalent choice for road and infrastructure projects.

Stone columns possessing higher stiffness, shear friction, and permeability have the potential to enhance the load-bearing capacity, stability of embankments, and acceleration of the consolidation process, while simultaneously diminishing the ultimate settlement. Reinforcing soft soil with stone columns is a cost-efficient approach due to the utilization of readily available local materials, which eliminates the necessity to wait for the consolidation process, thus reducing construction time.

Furthermore, stone columns are among the widely employed methods for mitigating liquefaction, as they serve to compact the adjacent soil, expedite drainage, and offer potential shear reinforcement. The compaction and drainage acceleration actions promote the expansion of the neighboring soil, impeding the buildup of excessive pore water pressure during seismic shaking. Consequently, this substantially decreases the likelihood of liquefaction and subsequent settlement after the shaking event [1].

In recent decades, many authors have conducted research to clarify the working principles as well as the stress transfer mechanism of the embankment over stone column-reinforced soft soil. Numerous experimental, analytical, and numerical methods have been conducted. Among the most important studies, it is necessary to mention the contributions of many authors [2-6].

The majority of research focuses on a unit cell model of a stone column, which relies on the assumption of uniform strain. An important parameter for assessing stress transfer in embankments built on soil reinforced with columns is the stress concentration ratio (n_c). This ratio is defined as the relationship between the stress acting on the top of column (σ_c) and the stress acting on

the adjacent soil (σ_s), as shown in Eq. (1). In previous field measurements and laboratory findings, the value of n_c for granular columns was found to vary from 2 to 6 [2].

$$n_c = \frac{\sigma_c}{\sigma_s} \quad (1)$$

As in [2], a simple theoretical study was conducted in which the free strain analysis was involved. The simple equation was introduced for assessing the behavior of soil reinforced with stone columns under a uniform load. The study presented findings related to stress distribution within the columns and the surrounding soil, as well as the stress concentration ratio. The research demonstrated that as depth and E_c/E_s increased, n_c increased, while n_c decreased with the increase of the radial distance ratio (r/a). Notably, the theoretical findings revealed that the n_c ranged from 3.5 to 8. Moreover, the results obtained through the proposed approach were effectively confirmed through comparison with results derived from numerical methods. Han and Ye [3] developed a closed-form solution for assessing the consolidation rate of foundations built on soil reinforced with stone columns, taking into account the modulus ratio between the stone column and the surrounding soil. The analytical findings demonstrated good agreement with previous results. Wang [4] presented an analytical method for evaluating the foundation on soft soil reinforced with stone columns, considering the consolidation process under time-dependent loads. The unit cell model was employed in this analysis. The resulting differential equations for the system encompassed the smeared zone and the resistance experienced under various applied loads. Closed-form equations were derived for pore water pressure and the average consolidation degree for different load types. Furthermore, this analytical solution was effectively validated against existing solutions.

In [5], numerous numerical simulations were performed to investigate the enhancement of soft soil through the use of stone columns. The response of soft soil reinforced with stone columns subjected to the load of an embankment was analyzed. The study took into account the effects of arching, clogging, and smear. The numerical results indicated that the transfer of vertical stress to both the column head and the surrounding soil within the unit cell was significantly influenced by the ratio of stiffness between the column and the soil. As the normalized vertical stress on the column increased from 2.3 to 6.3, the stress concentration ratio exhibited a sharp increase, rising from 2 to 14.

The scaled laboratory models were conducted, as presented in [6]. The two types of column materials tested were crushed sand and quasi-spherical glass beads, a coarser particle size from 0.7 to 1.0 mm. The researchers observed that as the

number of stone columns increased, the settlement of the soft soil that had been reinforced decreased. Moreover, the bearing capacity of the reinforced soil increased with a decrease in the spacing between the columns. This suggests that in soil improvement design, it is common to optimize the spacing of the stone columns. Additionally, the study recommended the use of the vibration method for better load-bearing capacity.

Based on the review of existing literature, it is evident that prior investigations have primarily focused on analyzing factors such as the stress concentration ratio, surface soil settlement, and the consolidation process. Most of these studies centered on the unit cell model, neglecting the influence of multiple columns acting together as a group. In this paper, a numerical method is employed to examine the mechanical response of embankments constructed on soft soils strengthened with stone columns. The numerical model developed for this study takes into consideration the combined effects of column groups. To illustrate this, a case study from a road project near the Ganh Hao bridge in Bac Lieu province, Vietnam, is employed. This case study is used to explore the stress transfer mechanisms within the embankment, focusing on factors such as the stress on soft soil, the stress at column head, the stress concentration ratio, and the stress distribution along column depth.

2. A CASE STUDY

2.1. The Project Information

To investigate the stress transfer mechanisms within the embankment, a case study on a forthcoming road project that links the Ganh Hao River bridge, connecting Dam Doi district in Ca Mau province with Dong Hai district in Bac Lieu province is conducted in this study. This road project spans a length of 6.08 kilometers as can be seen in Fig.1. It commences at the intersection of the East-West axis highway and the coastal road in Ca Mau province, extending to its termination at 1 Thang 3 Street in Dong Hai district, Bac Lieu province. The primary objective of this project is to incrementally enhance the transportation infrastructure system in alignment with the approved development plans of Bac Lieu and Ca Mau provinces. The design specifications for the Ganh Hao bridge's approaching road adhere to technical class III standards, featuring a designated speed of 80 km/h under the Vietnamese standard TCVN 4054-2005 [7]. The road cross-section comprises a surface width of 12 meters, with a 7-meter-wide pavement and two reinforced pavement sides, totaling 4 meters ($2 \times 2m = 4m$). The embankment's height, as per the designed

longitudinal profile, ranges from 2 to 5 meters [8].

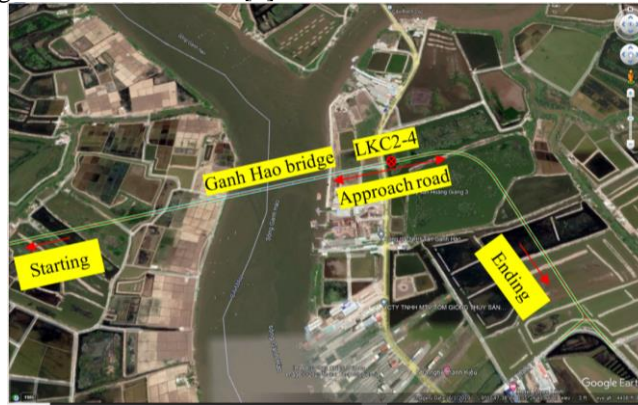


Fig.1 The location of Ganh Hao Bridge [8]

2.2. Soil Profile

The geological profile is obtained from a borehole located on the approach road to the Ganh Hao Bridge in Bac Lieu province, Vietnam. According to the geological survey report in the feasibility study phase provided by the Transport Engineering Design Joint Stock Incorporated South - Tedi South, the LKC2-4 borehole revealed a soil foundation consisting of four distinct layers: namely, layer K, layer 1, layer 2, and layer 3, with respective thicknesses of 1.5 meters, 21 meters, 2.5 meters, and more than 5 meters. Based on the physical and mechanical properties listed in Table 1, it is determined that layer K, layer 1, and layer 2 constituted soft soil layers, amounting to a combined thickness of 25 meters. These layers are characterized by soft soil properties, featuring high compressibility and low shear strength. Layer 3 was identified as a relatively stable soil layer suitable for placing the column toe. The groundwater level in the surveyed area was relatively shallow, 0.2 meters below the ground surface [8].

Table 1 The physical and mechanical properties of soils in the LKC2-4 borehole [8].

Parameters	Unit	Layers K and 1	Layer 2	Layer 3
Soil moisture, W	%	60.5	35.0	34.4
Unit weight, γ_w	kN/m ³	16.1	18.1	18.5
Specific weight ρ	-	2.67	2.70	2.73
Void ratio e_0	-	1.670	1.011	0.984
Liquid limit of soil	%	67.8	44.0	58.2
Plastic limit of soil	%	27.3	22.2	23.3
Plastic index	%	40.5	21.8	34.9
Soil consistency	-	0.82	0.59	0.32
Young's modulus, E	MPa	1.5	6.5	20.0
Friction angle, ϕ	degree	4°36'	9°57'	12°04'
Cohesion, c	kN/m ²	5.9	19.0	25.6
N of SPT test	pcs	0 ÷ 2	5 ÷ 7	9 ÷ 12

3. NUMERICAL MODELING

3.1. Geometry of Model

To assess the efficiency of reinforcing soft soil with stone columns, 3D numerical full-scale models to represent embankments constructed on both unreinforced and stone column-reinforced soft soil are conducted. These models featured dimensions including a 12-meter width, a 4-meter height for the embankment, and a ground profile depth of 29 meters. The stone columns, each having a diameter of 0.8 meters and a length of 25 meters to ensure comprehensive soil reinforcement, were modeled and positioned in a square grid pattern with a 2-meter spacing, as shown in Fig.2.

3.2. 3D Modeling Description

Based on [9], various geometric models have been employed for simulating stone columns reinforced soft soil. These models included the unit cell model, planar model, slide model, and homogenous ground model, etc. In the study, the slide model was selected because this was a fully correlated model, allowing us to consider the simultaneous working of column groups, the influence of the soils around the columns, and the influence of the embankment zone.

The 3D numerical slide model was created using the FLAC3D software, which is based on the finite difference method [10]. To account for symmetry, the model was developed for half of the embankment and included six stone columns (see Fig.2). The mesh division utilized polyhedron elements that were interconnected at nodes, forming a comprehensive mesh. Volumetric elements were employed for the soils, stone columns, and embankment, enabling the observation of stress and displacement within these components. The 3D numerical mesh of the model is depicted in Fig.3.

Monitor points, denoted as A, B, C and D, and columns numbered 1 to 6, were strategically placed to observe settlement, displacement, and stress.

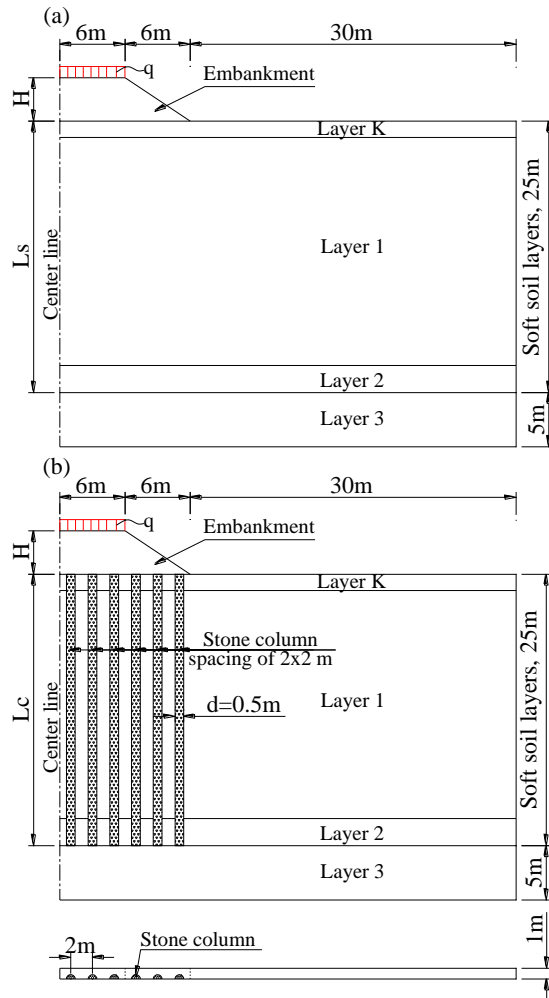


Fig.2 Geometric elevation: (a) without reinforcement; (b) with reinforcement

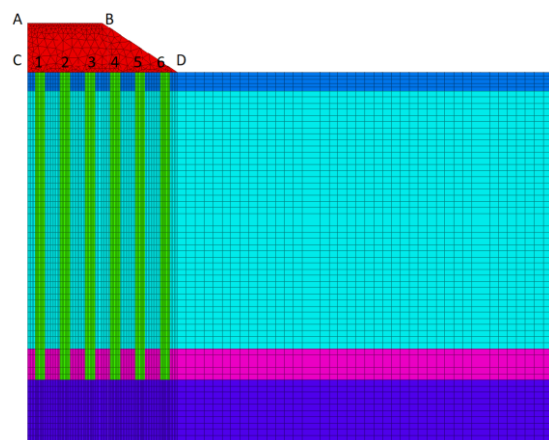


Fig.3 3D mesh for numerical analyses

For constitutive models, the linear elastic, perfectly plastic model, Mohr-Coulomb (MC model) allowed considering plastic behavior and limit states. The model has widely been applied thanks to its simplicity, ease of use and reliable results [10]. It was suggested to characterize the behavior of the embankment, soil layers, and stone columns. The model parameters could be defined based on data obtained from laboratory experiments. Additionally, the model accounted for the interaction between the columns and the surrounding soil through interfaces.

In terms of boundary conditions, the model took into account the complete thickness of the soil layers. The lower boundary was positioned at a consistent depth of -30 meters. To maintain stability, all displacements at the bottom were set to zero. Given the model's symmetry, the horizontal displacement along the central plane was set to zero. To minimize the impact of the model size, the length of the model was set to three times the width of the embankment, resulting in a total of 30 meters. At this plane, displacement was also constrained in the x-direction. The boundary planes perpendicular to the y-direction were similarly set to zero displacement in the y-direction. The 3D numerical models were analyzed under drained conditions.

3.3. Constitutive Models and Parameters

As previously stated, it was advised to employ the MC model for characterizing the soil properties. The required input parameters of the model consist of Young's modulus (E), Poisson's ratio (ν), friction angle (ϕ), cohesion (c) and unit weight (γ). The values for E , ϕ , c and γ were derived from the data of experimental tests, as shown in Table 1. The Poisson's ratio normally ranged from 0.3 to 0.4 [11,12]. In the numerical analyses, it is assumed to be 0.3 for all soils. There is no testing data for layer K, but due to its small thickness, the model parameters of this layer are assumed to be the same as layer 1. The embankment parameters are designed based on the guidelines in Vietnamese standard TCVN 4054-2005 [7]. Stone columns are constructed from crushed stone material, and those adhering to the Mohr-Coulomb model are deemed appropriate. After analyzing the results of numerical modeling for soft soil strengthened with stone columns, Castro [9] introduced several sets of parameters for these columns. However, due to a lack of sufficient experimental data to determine the shear resistance and bearing capacity parameters for the stone columns, the parameter set by Ali et al [13] was adopted for this study. The model parameters of the soils, embankment and stone columns used for the numerical analyses are listed in Table 2.

Table 2 The parameters for the soils, embankment and stone columns used for the numerical analyses

Soil	Model	Parameters
Layer K	MC	$E = 1.5 \text{ MPa}$, $\nu = 0.3$, $\phi = 4^\circ 36'$, $c = 5.9 \text{ kPa}$, $\gamma = 16.1 \text{ kN/m}^3$
Layer 1	MC	$E = 2.5 \text{ MPa}$, $\nu = 0.3$, $\phi = 4^\circ 36'$, $c = 5.9 \text{ kPa}$, $\gamma = 16.1 \text{ kN/m}^3$
Layer 2	MC	$E = 6.5 \text{ MPa}$, $\nu = 0.3$, $\phi = 10^\circ 04'$, $c = 19 \text{ kPa}$, $\gamma = 18.1 \text{ kN/m}^3$
Layer 3	MC	$E = 20 \text{ MPa}$, $\nu = 0.3$, $\phi = 12^\circ 6'$, $c = 25.6 \text{ kPa}$, $\gamma = 18.5 \text{ kN/m}^3$
Embankment	MC	$E = 30 \text{ MPa}$, $\nu = 0.3$, $\phi = 35^\circ$, $c = 5.0 \text{ kPa}$, $\gamma = 19.0 \text{ kN/m}^3$
Stone column	MC	$E = 100 \text{ MPa}$, $\nu = 0.3$, $\phi = 45^\circ$, $c = 0.0 \text{ kPa}$, $\gamma = 22.0 \text{ kN/m}^3$

To model the interaction between the stone columns and the soils, the interaction elements were applied at the interface between the structure and the soil. Following the guidelines provided in the FLAC3D software documentation, the interaction elements were attributed with shear and normal stiffness values of 10^8 kN/m/m . Cohesion values were determined from experimental data. The friction angle of these interaction elements was established as two-thirds of the friction angle of the surrounding soil [10].

3.4. Applied Loading

Before applying any surcharge loads, it is necessary to establish the initial stress conditions within the model. This involves defining the initial stress state of the soil in all three directions, namely, the x, y, and z. The initial stress state, which varies with depth, was initially determined using the formulas outlined in Eq. (2) [10].

$$\sigma_{zz} = \gamma z \quad (2)$$

$$\sigma_{xx} = \sigma_{yy} = K_0 \gamma z$$

In which, γ - unit weight (kN/m^3); z - depth of soil (m); K_0 - lateral earth pressure coefficient.

A uniform surcharge load, denoted as q , was applied across the entire top surface of the embankment. The surcharge load incrementally increased by 20 kPa, which is 10, 30, and 50 kPa. This approach was adopted to account for the impact of the surcharge load on the stress transfer mechanism within the embankment and the soft soil.

4. RESULTS AND DISCUSSIONS

A 3D numerical simulation is carried out to model an embankment constructed on stone column-reinforced soft soil. The model accounts for both the embankment gravity load and the uniformly distributed surcharge. This study aims to assess the efficacy of stone columns by examining

the stress transfer mechanisms within the embankment, focusing on the stresses at the column heads and in the soft soil. The influences of applied load on the stress transfer are also investigated.

4.1 Stress Transfer and Distribution within Embankment

4.1.1. Under gravity load of embankment

The research examines the stress transfer and distribution within the embankment on unreinforced and stone-column-reinforced soft soils subjected to the gravity load of the embankment. Fig.4 (a) shows the stress distribution within the embankment on the unreinforced soft soil. As a result of the system subjected to embankment gravity load, the vertical applied stress displays a linear increase with depth, following the equation $\sigma_z = \gamma z$, which results in a value of 76 kN/m^2 . The stress on soft soil is uniformly distributed across the entire base of the embankment.

Fig.4 (b) illustrates a significant difference in the stress transferred to the column head compared to that transferred to the soft soil. The stress distribution within the embankment takes on a pattern resembling stress arches. This phenomenon, characterized by an increase in the applied stress on the column head, results in a reduction of the applied stress on the soft soil, leading to a decrease in the total and differential embankment settlements.

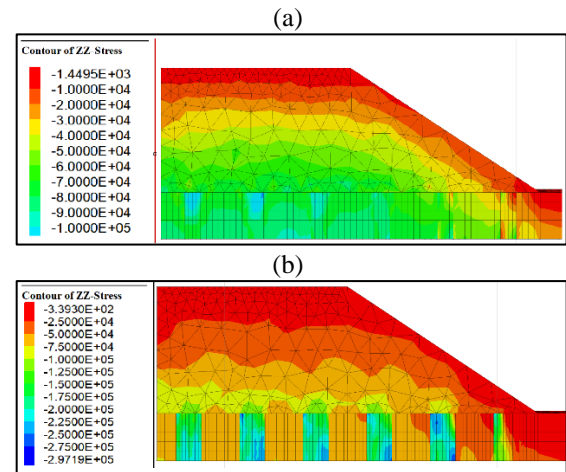


Fig.4 Stress distribution within the embankment under gravity load: (a) unreinforced soft soil; (b) stone column-reinforced soft soil.

Table 3 presents the numerical values for the stresses applied to both the soft soil and the column head. The data shows that when stone columns are present in the soft soil, the stress acting on the soft soil is only about 88% compared to the case of unreinforced soft soil. The analysis result indicates a stress concentration ratio of 2.4.

Table 3 Stress acting on the soft soil and the column head in the cases of the embankment on unreinforced and stone column-reinforced soft soils

Study cases	Stress on soft soil (kPa)	Stress on column head (kPa)
Embankment on unreinforced soil	72.9*	-
Embankment on stone column-reinforced soil	64.1*	152.0**

Note: * stress on soft soil at point C; ** stress on column head No. 1

4.1.2. Under surcharge load

In order to account for the impact of the surcharge load on stress transfer within the embankment, the applied load was varied from 10 kPa to 50 kPa. All displacements were set to zero before applying the load. Fig.5 illustrates the stress distribution within the embankment and the stresses transferred to the column head and soft soil when the embankment is subjected to a uniform load of 30 kPa. It is evident that the arching effect within the embankment becomes more pronounced as the load is applied. In addition, columns No. 1 to No. 5 under full-height embankment bear a greater load than column No. 6 under the embankment slope.

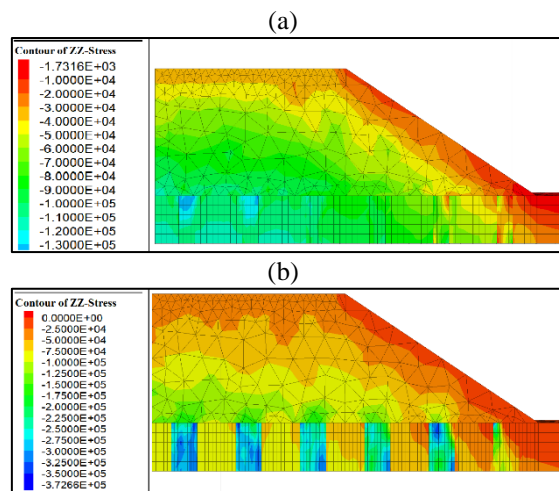


Fig.5 Stress distribution within the embankment under a surcharge of 30 kPa: (a) unreinforced soft soil; (b) stone column-reinforced soft soil

Fig.6 illustrates the relationship between the stress at column head No. 1 and the stress on the soft soil at point C under varying surcharge loads. Generally, as the surcharge load rises, both the stresses on the column and the soft soil increase. Particularly, the rate of increase in the stress on the column head is more rapid than the rate of increase in the stress on the soft soil.

Fig.7 shows the relationship between the stress concentration ratio and the surcharge load. In

general, the stress concentration ratio increases as the surcharge load rises. This tendency is consistent with findings from the previous studies [11-15]. The numerical analyses indicate that the stress concentration ratio stands at approximately 2.3 under the embankment load. It subsequently rises to 3.1 as the surcharge load ranges from 10 kPa to 30 kPa, and increases to 3.4 when the surcharge load reaches 50 kPa. The stress concentration ratio in numerical analyses is in close agreement with that in the former studies [2,5], where the stress concentration ratio typically ranged from 2 to 6, with the majority of values falling within the range of 3 to 4.

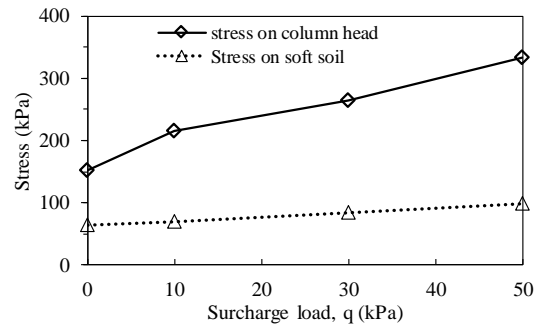


Fig.6 Stress on column head No. 1 and soft soil at point C with the different surcharge loads

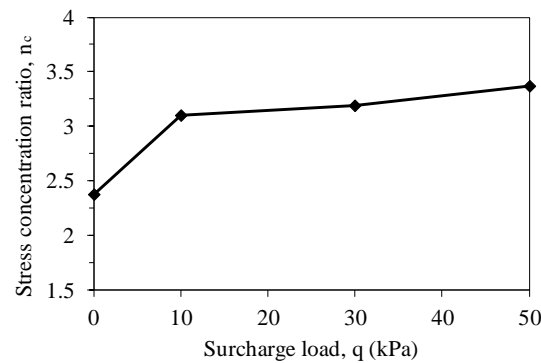


Fig.7 Stress concentration ratio on column head No. 1 with different surcharge loads

When compared to the case of soft soil reinforced with rigid piles, where the stress concentration ratio falls within the range of 6 to 30, the stress concentration ratio for stone columns is considerably lower, typically around 20% to 30%. This difference is evident because rigid piles like concrete, reinforced concrete, and concrete-filled steel pipes exhibit significantly greater stiffness, resulting in a more substantial transmission of stress to the pile head.

As indicated in previous research, the stress beneath the column head is partially distributed due to the arching effect and the shear resistance of the

granular embankment material. As a result, the stress transferred to the soft soil is reduced. Fig.8 illustrates the stress transmitted to the soft soil in cases of both unreinforced and reinforced embankments. Particularly, in the case of stone column reinforcement, the stress applied to the soft soil is substantially lower than in the absence of reinforcement. When the surcharge load is set to 10 kPa, the stress on the soft soil in the reinforced case is 69.7 kPa, while it is approximately 81.1 kPa in the unreinforced case, representing a reduction of about 14%. Under a surcharge load of 50 kPa, the stress transferred to the soft soil is 126.6 kPa in the reinforced case and 98.9 kPa in the unreinforced case, indicating a decrease of 22%. This reduction in the stress transmitted to the soft soil significantly mitigates settlement in both the soft soil and the embankment.

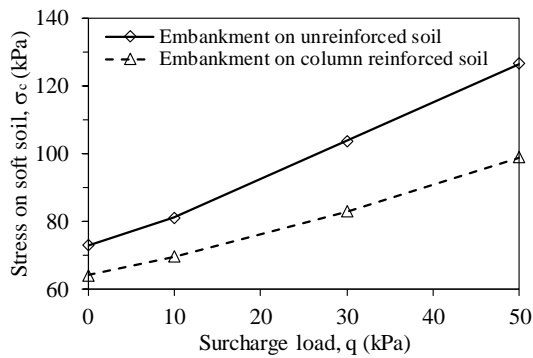


Fig.8 Stress on soft soil at point C under different surcharge loads

Fig.9 shows the stress transferred to the column heads as the embankment is subjected to the different surcharge loads. In general, the stress on the column heads escalates with increasing applied loads. There is a negligible difference in stress among columns No. 1, 2, 3, 4, and 5, with variations of around 10%, while column No. 6 experiences significantly lower stress. Columns No. 1 to 5 are situated beneath a four-meter high embankment, where the arching effect is fully formed. Consequently, the stress exerted on these columns is substantially greater compared to column No. 6, which is located under an embankment of approximately 1-meter height.

4.2 Stress Along Stone Column Depth

The stress distribution along column No. 1 at various depths under different loads is depicted in Fig.10. The column's stress gradually increases with depth, reaches a peak at the specific depth, and then gradually decreases towards the column tip. This phenomenon is known as negative skin friction, which was initially observed in cases involving

embankments over rigid pile-reinforced soft soil. When the soil's displacement surpasses that of the column, a negative frictional force component is generated around the column, directed toward the column tip, leading to an increase in column stress. At a level where the displacement differential is zero, the negative skin frictional force becomes zero, and the column's stress reaches its maximum value [16].

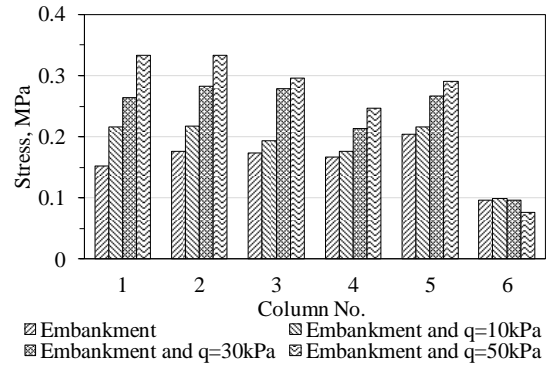


Fig.9 Stress on column heads with different surcharge loads

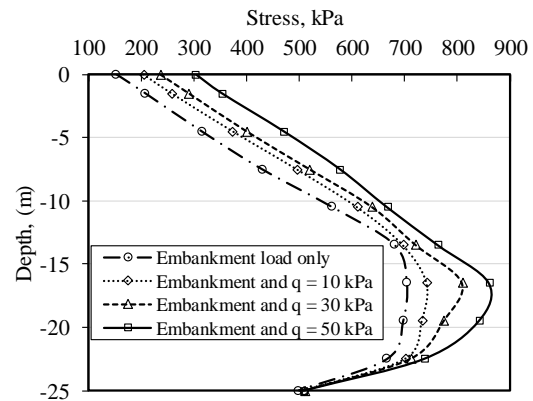


Fig.10 Stress distribution along the depth of column No. 1 under different surcharge loads

Moreover, it can be seen from Fig.10, that as the applied load on the embankment increases, both the stress at the column head and the stress along the column increase. The equilibrium plane of displacement appears at a depth of 17 to 18 meters, where the stress within the column reaches its peak value. In the case of an embankment load, the maximum stress in the column is 69.7 kPa. When the applied load is 50 kPa, the maximum stress in the column reaches 86.0 kPa. Typically, this maximum column stress value is considered for design purposes in terms of geometry and column material.

5. CONCLUSIONS

Ground improvement technique by stone columns is commonly applied in strengthening the soil beneath foundations and embankments with the advantages of significantly increasing the load-bearing capacity of the ground, decreasing the construction time, prevailing liquefaction mitigation, and using local materials. Based on the 3D numerical analysis results, the following conclusions are drawn:

1. The stress transferred to the column head is significantly greater than that transferred to the soft soil. The stress transferred to the soft soil in the case of reinforcement is approximately 88% of that in the case of non-reinforcement.

2. The stress concentration ratio at the column head increases with higher applied loads. The stress concentration ratio rises from 3.1 to 3.4 as the surcharge load varies from 10 to 50 kPa. This indicates a more pronounced arching effect within the embankment. Furthermore, the columns under full height embankment bear greater load than those under embankment slope.

3. There is an occurrence of negative friction around the column, causing the stress within the column to gradually increase towards the equilibrium plane. This equilibrium plane is situated at a depth of approximately 17 to 18 meters.

6. ACKNOWLEDGMENTS

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