USING LUGRE FRICTION MODEL TO SIMULATE THE DYNAMIC MOTION OF HYDRAULIC CYLINDERS

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ABSTRACT

Hydraulic cylinders are widely used in industry. Simply, hydraulic equipment is used as hydraulic jack or lifting machine. For more complex systems, hydraulic cylinders are equipped on machinery, mining machines or rudder control system, hydraulic governors, aerospace and autonomous vehicles. Friction force strongly affects to dynamic motion control. In same cases, friction fore may cause control errors or poor performance. Therefore, friction can not be ignored. Researchers have paid a lot of effort to deal with dynamic motion of hydraulic cylinder for 100 years. All researchers tried to illustrate the behavior of friction by mathematical model. Classical friction model is based on Coulomb friction. This model has been used for a long time. However, Coulomb friction couldn't fully capture the behavior of friction. Several efficient friction models such as steady – state friction model, Stribeck friction, LuGre friction model or Dahl model, Modified LuGre friction model. The researchers always try to complete the previous models. Using the more complex model for control system the better performance of cylinder is. In contrast, the full and complex mathematical friction models are so difficult to solve. Besides, experiments also show that each model is only adequate to several application. LuGre friction model has been widely used for modeling and predicting of dynamic motion of hydraulic cylinders. This paper represents the use of LuGre friction model to predict the performance of a hydraulic cylinder operating as a piston pump. The experiment is carried out by 70 mm bore, 30 mm diameter of rod, 300 mm length of stroke hydraulic cylinder. The cylinder is placed bottom up. Piston compresses and pushes hydraulic liquid by gravity. The motion behavior of piston then will be compared to experimental result to ensure the accuracy of LuGre model.

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1. Introduction

Hydraulic cylinders have been widely used in industry. For simple applications, hydraulic cylinder is used like a jack equipment or a lifting machine. Hydraulic cylinders also play an important role on mining machines, construction machines or machine tools, etc. Nowadays, hydraulic cylinders are widely used across vehicles and industrial applications such as rudder steering, flap control, autonomous vehicles, hydraulic governor system, robotic arms, CNC machines, etc. For high quality of control, friction force may cause of errors or poor performance of the system. It is evident that friction force can not be ignored. Friction always present in hydraulic cylinders and mechanism. It significantly affects the performance of the system. However, friction is a critical problem. Although friction has been studied from 16th century, researchers have focused at friction in hydraulic cylinders or pneumatic cylinders for 100 years (Yanada et al., 2014; Nguyen, 2021). Mathematical models is used to describe the characteristics of friction which represents the influence of normal force, deformation and speed of motion, etc. One of the well-known models are steady – state model, LuGre friction model, Modified LuGre friction model or Stribeck curve. LuGre friction model is the widest used for modeling the performance of hydraulic cylinders and predicting the dynamic motion of the system. Because LuGre model can simulate important nonlinear phenomena such as statiction, Stribeck effect, pre-sliding displacement and frictional lag (Ahmad et al., 2022; Tran, 2019). Nevertheless, LuGre model can not simulate well the dynamic behaviors of a hydraulic cylinder in the sliding regime (Yanada et al., 2014; Tran et al., 2014). This paper briefly presents the use of LuGre friction model for simulating the performance of hydraulic cylinder which counters a constant load. Matlab/Simulink is used to simulate the performance of cylinder to get predicted result of dynamic motion. Then the results will be compared to the experiment to ensure that the mathematical model is accurate enough. The paper is divided into 5 sections. The introduction recapitulates the hydraulic cylinder application and friction. The second section briefly illustrates the LuGre friction model and hydraulic dynamic model which includes friction force, compressible of hydraulic fluid. Those models are used to simulate by using Matlab/Simulink software. The simulink block diagram and the result of simulation are represented in section 3. The section 4 briefly depicts the experiment of a hydraulic cylinder which works as a displacement piston pump. The piston is pulled by gravity of steel plates. The counter MC-964 is used to measure the average velocity. The result of experiment is showed in section 4. Finally, the comparison between simulation results and experiment as a strong proof to propose LuGre friction model for predicting accurately the hydraulic cylinder's behaviors.

2. LuGre friction model, mathematical friction model of hydraulic cylinders

2.1. LuGre friction model

For hydraulic cylinders, friction force is critical problems. Friction force is result of normal force between moving parts and stationary parts which is known as Coulomb friction. Firstly, the assembly dimensions causes of primary deformation of sealant parts. Therefore, seal rings apply normal forces to cylinder bore and piston rod (*called F_1*). So, F_1 friction force is influenced by mechanical wear. F_1 nearly is constant. Therefore, it's considered as static friction. Then, high pressure leads to secondary deformation of sealant parts. Consequently, it is necessary to add friction force F_2. Different from static friction force, F_2 is strongly influenced by pressure and moving speed, elastic. The higher pressure is, the greater deformation is, the greater value of F_2. It is evident that the pressure in the cylinder chambers is not a constant value. In additions, there is hydraulic oil film between the seal ring - cylinder bore, seal ring – piston rod and guiding rings – cylinder bore, piston rod. Shear stress and viscosity of the hydraulic oil issues dynamic viscosity friction (Yanada, et al., 2014). Dynamic friction behavior is confusing and complex. There are several well-known models to describe friction behavior. However, it is very difficult to depict fully friction behavior. LuGre model was proposed by Canudas de Wit and his partners in 1995 (Nguyen, 2021; Ahmad et al., 2022). Canudas de Wit et al. combined Dahl model and Stribeck effect, Coulomb friction, also elastic bristles to simulate friction behavior.

It is assumed that two matting surfaces contact at several asperities through elastic bristles as shown in Figure 1. When a tangential force is applied to a surface, the bristles will deflect like springs; and when the force is sufficiently large, some of the bristles will break and then slip. The mean deflection of the elastic bristle is denoted as z and is defined as (Tran et al., 2019):

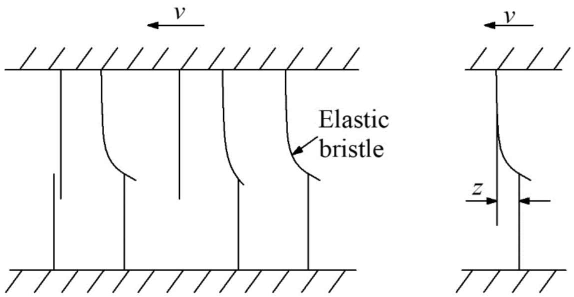


Figure 1 – Elastic bristles model (Tran et al., 2019)

$$\frac{dz}{dt} = v - \frac{\sigma_0 z}{g(v)} v \qquad (1)$$

Where: σ_0 is the stiffness of the elastic bristle, N/m; z is the deflection of the elastic bristles, m; v is the velocity, m/s; t is time, s; g(v) is the Stribeck function, N. g(v) is defined as:

$$g(v) = F_c + (F_s - F_c)e^{-(v/v_s)^n} \qquad (2)$$

Where: F_c is Coulomb friction force, N; F_s is maximum static friction force, N; v_s is the Stribeck velocity, m/s; n is an appropriate exponent; v is the velocity, m/s.

The dynamic behavior of friction is depicted as Figure 2:

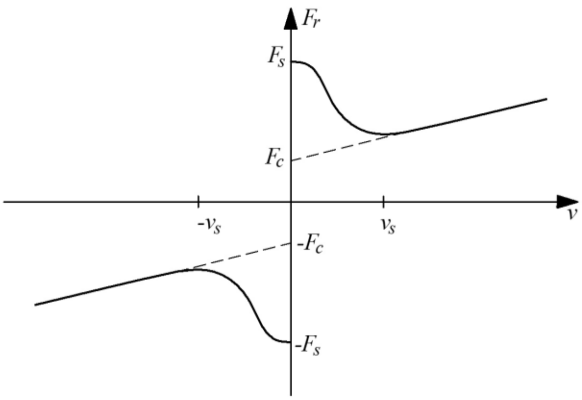


Figure 2 – Steady-state friction model (Tran et al., 2019)

The friction force is given by:

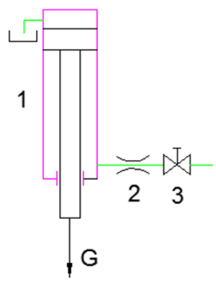
$$F_r = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \qquad (3)$$

Where: σ_0 is the stiffness of the elastic bristle, N/m; σ_1 is micro-viscous friction coefficient, N×s/m; σ_2 is the viscous friction coefficient, N×s/m; z is the deflection of the elastic bristles, m; t is the time, s; v is the velocity, m/s.

2.2. Mathematical friction model of hydraulic cylinders

In this study, LuGre friction model is chosen for simulating the dynamic motion of hydraulic cylinders. The mathematical model is based on LuGre friction model which is defined in section 2.1.

The cylinder bore D = 70 mm, the diameter of piston rod d = 30 mm, two piston seal rings are labeled USH 60 – 70 – 6 (*NOK corporation, Japan*), two rod seal rings are labeled USH 30 – 40 – 6 and one dust wiper ring is labeled LBH 30 – 40 – 8. The cylinder is mounted at the top of a frame, the piston chamber is upside. The piston chamber is vented. Therefore, the pressure p_1 in the piston chamber is always atmospheric. A constant load is apply to piston rod end by holding square steel plates. Figure 3 briefly illustrates it's schematic diagram. The motion of piston is caused by gravity (positive *action*) and back pressure of oil in rod end chamber (*negative action*). The piston rod chamber is filled with hydraulic oil (*viscosity 46 cSt*). By weight of load and piston – rod weight (*tare weight*) the pressure in piston rod chamber p_2 raises up. So, the hydraulic oil can flow through a throttling valve to the tank. The bottom up cylinder operates as a displacement pump (Figure 4).



1 – Hydraulic cylinder, 2 – Throttling valve, 3 – Ball valve

Figure 3 – Schematic diagram of source cylinder

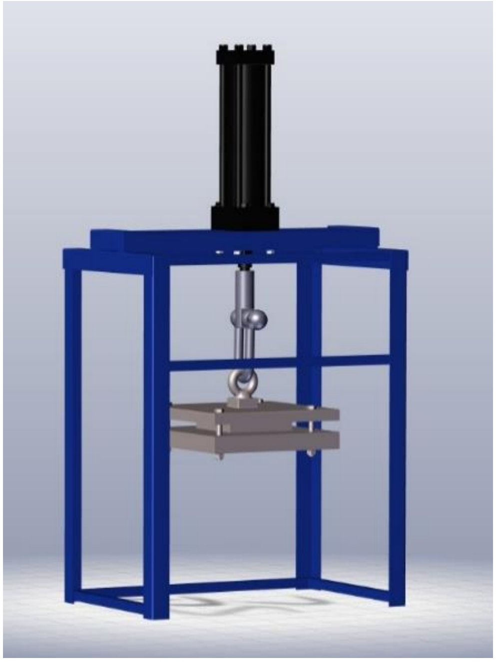


Figure 4 – A bottom up hydraulic cylinder plays as a displacement pump (source cylinder)

$$G_0 + G - F_{ms} - p_2 \cdot A_2 = (m_0 + m) \cdot a \quad (4)$$

Where: G_0 is the tare weight of piston and piston rod, N; G is the total weight of steel plates, N; F_{ms} is the friction force, N; p_2 is the liquid pressure in rod chamber, Pa; A_2 is the cross section of rod chamber, m^2; m_0 is the tare mass of piston and rod, kg; m is the total mass of steel plates, kg; a is the acceleration of piston, m/s^2.

The flow rate Q supplied by hydraulic fluid source. Q is defined by two equations below. The first equation represents the relationship between the flow rate Q and the movement of piston when hydraulic fluid is considered as compressible liquid. The other shows the flow rate goes through an orifice.

$$A_2 \cdot v - Q = \frac{V}{E} \cdot \frac{dp_2}{dt} \quad (5)$$

Where: A_2 is the cross section of rod chamber m^2; v is the velocity of piston motion m/s; Q is the flow rate of hydraulic fluid, m^3/s; V is volume of the hydraulic fluid in the rod chamber, m^3; E is the bulk module of hydraulic fluid, Pa; p_2 is liquid pressure in the rod chamber, Pa; t is the running time, s.

The flow rate of fluid:

$$Q = C_v \cdot A \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}} \quad (6)$$

Where: Q is the flow rate of hydraulic fluid, m^3/s; C_v is the discharge coefficient; A is the cross section m^2; Δp is the differential pressure between up stream and down stream, Pa; ρ is the density of hydraulic fluid, kg/m^3.

However, for the flow returns to the tank ($p_T \approx 0$), the friction loss of the downstream line and the throttling valve is considered as C. The flow seems to be the function of upstream pressure. The simpliied formulation as below:

$$Q = C \cdot \sqrt{p_2} \quad (7)$$

Where: Q is the flow rate of hydraulic fluid, m^3/s; C is the equivalent discharge coefficient; p_2 is the upstream pressure (it is also rod chamber pressure), Pa.

3. Predictive dynamic motion of piston

Matlab is an useful software for researchers and engineers to model and simulate the machinery, dynamic motion and behavior. Matlab helps researchers and engineers to analyse the system behavior by using virtual models. Therefore, the time consumption is reduced as well as the cost is lower. As mentioned, Matlab is chosen as a dedicated tool to model the dynamic motion of piston. The result of modeling as predictive behavior. Figure 5 shows the block diagram model of hydraulic cylinder plays as a displacement pump which supplies a stability flow of hydraulic oil to hydraulic system.

All parameters of mathematical friction model is showed by Table 1.

Table 1: Model parameters

Param eter	Val ue	Param eter	Value	Param eter	Value
F_S (N)	180	0 (N/m)	5×10^6	A_2 (m^2)	31.416×10^{-4}
F_C (N)	56	1 (N×s/m)	0.1	m (kg)	82
v_s (m/s)	0.0125	2 (N×s/m)	30	m_0 (kg)	3
n	0.05	E (Pa)	10^8	V_0 (m^3)	628.32×10^{-6}
		C	0.337×10^{-6}		

Note: Regarding the drawback of Matlab language, those number 0, 1 and 2 denote the σ_0, σ_1 and σ_2 respectively. C is defined by experiment.

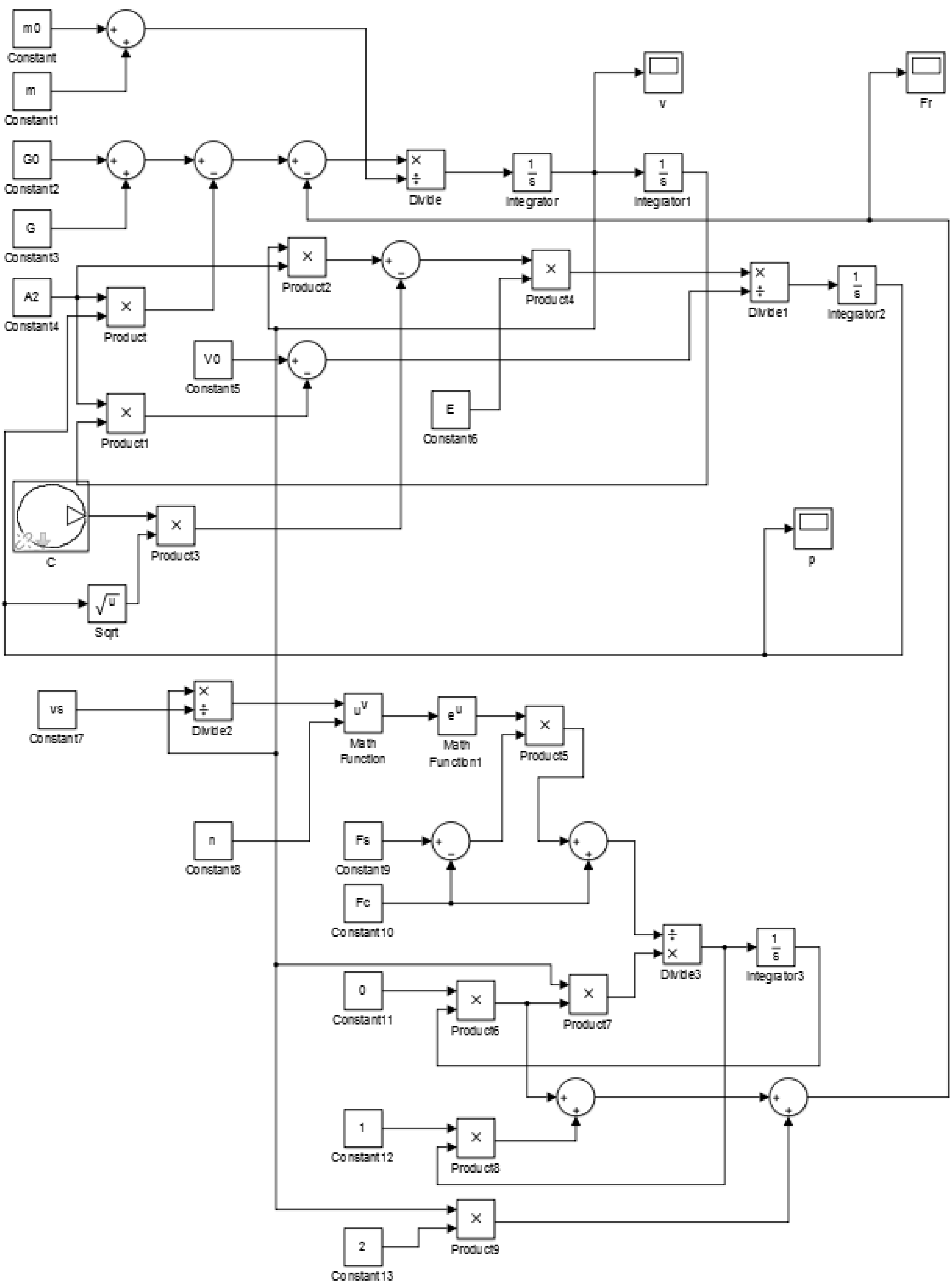


Figure 5 – Matlab block diagram of friction model

The acceleration curve and velocity curve above represent the predictive motion of piston. Firstly, piston begins to move by gravity of load. The motion is caused by the difference between gravity and friction. Currently, the pressure in the rod chamber hasn't been built up yet. The character of friction in pre-sliding phase strongly influences to the motion.

Therefore, the acceleration has a positive value and the velocity rises quickly. Then the acceleration starts to go down to negative value because of the higher back pressure. The greater velocity is, the greater flow rate is. The equation 6 shows that the drop pressure is proportion to the flow rate flows through the throttling valve. That means the back pressure is built up (equation 7). Back pressure acts like a brake. Consequently, the velocity can not keep going up. Finally, the velocity almost doesn't change when the gravity balance to friction and counter force (*back pressure*). The Figure 6 depicts the fluctuation of acceleration curve, the amplitude of oscillation is approximately 0.05 m/s^2. The velocity curve seems to be plateau. The constant velocity is about 0.061 m/s (*Figure 7*).

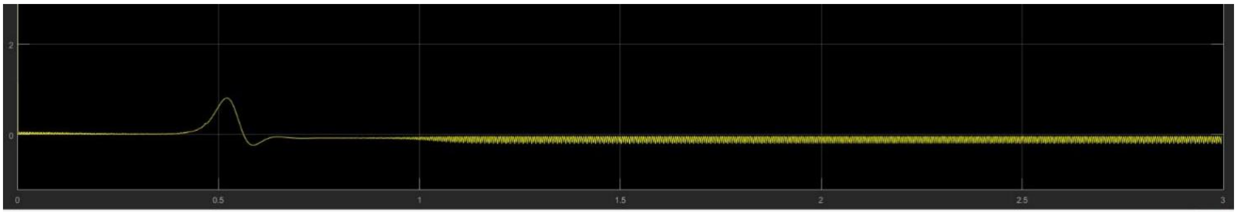


Figure 6 – The predictive acceleration curve

Figure 7 – The predictive velocity curve

4. Experiment

An experiment has been carried out to approve the simulation results. The cylinder is mentioned in section 2.2. To measure the velocity of piston, we used a Digital Multipurpose Counter MC-964 (*Figure 8*). For measuring the linear velocity, operator needs to select A to B mode. The average velocity is defined the distance traveled per time. The distance traveled is distance between the two fiber transducer gates. When the indicator object reaches the first gate (*point A*) the time-counter is activated then it is stopped when the indicator object reaches the second gate (*point B*). These two gates are placed at 60 mm distance. The experiments are carried out at three positions. For each position, the gates are consequently placed at beginning, the middle and the end of stroke. For each position, the experiment was carried out at least 5 times. The first gate was put 50 mm far from the upper dead point. The Figure 9 shows the real bottom up hydraulic cylinder operates as a piston pump and the principle working of fiber transducer gates is represented by figure 10. The obtained time consumption and average linear velocities are shown below (*Table 2, Table 3 and Table 4*).

Figure 8 – Digital counter MC-964

Figure 9 – The real hydraulic cylinder

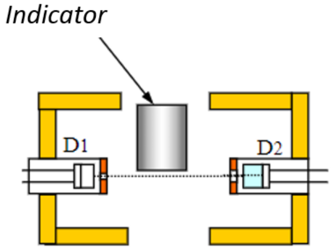


Figure 10 – Principle working of fiber transducer gate

Table 2: The beginning of stroke

Experiment	Time consumption, s	Velocity, mm/s
1	1.040	57.692
2	1.125	53.333
3	1.070	56.074
4	1.062	56.497
5	1.088	55.147

Table 3: The middle of the stroke

Experiment	Time consumption, s	Velocity, mm/s
1	0.982	61.100
2	0.963	62.305
3	0.975	61.538
4	0.946	63.425
5	0.955	62.827

Table 4: The end of stroke

Experiment	Time consumption, s	Velocity, mm/s
1	0.960	62.500
2	0.952	62.025
3	0.966	62.112
4	0.986	60.852
5	0.967	62.047

5. Conclusions

The three tables above show the obtained average velocities of each experiment. They are similar at all three positions. For the beginning of stroke the velocity is smallest. It can be illustrated by the appearance of static friction at early phase. At rest where the velocity is zero, the higher friction force is also called static friction (*Figure 2*). Both simulation results and experiment results show the average velocity approximately 0,057 mm/s at the first stage of cylinder's stroke.

The experimental velocity is homogeneous to predictive velocity curve form. It is a proof to strongly approve the LuGre friction model to predict the hydraulic cylinder's behavior. LuGre friction model can represent almost features of dynamic friction of hydraulic cylinder. The result is consistent with previous studies of other researchers.

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Contributions of authors

The manuscript is written by Tung Son Nguyen. The experiment was take placed by both of three authors. Thuy Thi Pham has commented and edited the original manuscript and provided a precious reference. Minh Hoang Bui has commented and edited the original manuscript.

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