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Improved performance for antenna based on a combination of fractal geometry with CSRR

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ABSTRACT

In this paper, an antenna design method operating at 3.5 GHz for 5G system is presented to improve its performance. The antenna is designed using fractal geometry combined with an imperfectly structured ground plane. In which, the radiation surface has the form of a Minkowski island fractal geometry, and the removed part of the ground is a complementary split ring resonator unit cell. In this design, the substrate material is FR4-epoxy microwave laminates with dielectric constant $\epsilon = 4.4$, loss tangent ($\tan \delta$) of 0.02, and $h = 1.6$ mm thickness used to design the antennas. HFSS software is used in the simulation with the feeding method with a microstrip line. The proposed antenna has a significant performance increase compared to the original microstrip antenna such as reduced about 56% reduction in total size, enhanced 207% bandwidth, increased peak gain to 4.66 dB, and improved radiated efficiency to 89.3%. The physical model of the antenna has been fabricated and measured to verify the correctness of the design.

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KEYWORDS

Fractal antenna; metamaterials; CSRR; 5G application; microstrip antenna

1. Introduction

Today, with the great development of science and technology, the need to increase the performance of telecommunications equipment is very urgent. They not only need to integrate many new features, but also tend to reduce the size to be more flexible in use. Especially, for 5G systems, the data transmission rate is much faster than in other systems. Therefore, the issue of improving the performance of devices for 5G applications is of more significant concern to designers.

There are many factors that directly affect the performance of a radio transceiver system, one of them is the performance of the antennas in these devices. Therefore, improving the antenna's performance also determines the system's overall performance. In antenna design, performance is often evaluated through critical parameters such as

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size, bandwidth, gain, and radiation efficiency, etc. Currently, there have been many types of research and technical solutions to increase antenna performance such as antenna microstrip design, shorting pin, and shorting walls, Using fractal geometry and meta-material structures in the designed antenna is one of the solutions being considered and selected (Moghariya et al., 2013).

Fractal geometry is understood as a rough fragment that can be separated into similar parts but at a smaller scale. Fractal geometry can be found in nature such as branches of rivers, branches of a tree, mountains, thunders, etc., or they could be generated by mathematical equations, such as the Sierpinski carpet, Minkowski island, Hilbert curve ... Fractal geometry is applied in many fields of life such as information technology, medicine, art, etc. In addition, the study of electromagnetic theory based on fractal geometry has created new radiation devices. Self-similarity and space-filling are the two most important properties of fractal geometry, especially in antenna design. The fractal geometries are used in planar antenna design as the radiation surface of the antenna. Where the self-similarity property is used in the design to create multi-band antennas and space-filling is often used to reduce the dimension of antennas (Cao & Krzysztofik, 2016; Cohen, 1997; Krzysztofik & Cao, 2017; Parmar & Singh, 2017).

Metamaterials are understood as a type of material, which has properties that do not exist in nature. They are created by the arrangement of microstructures to create the desired macro-physical properties. Therefore, metamaterials are also called artificial materials, their properties depend on the structure more than their composition. Microstructures can be made of electromagnetic materials, no electromagnetism or dielectric and are called 'atoms'. The structure of 'atoms' can be symmetrical, isotropic or anisotropic. The arrangement of 'atoms' can be chaotic or in a certain order, so that they are having simultaneously negative permittivity (ϵ) and permeability (μ), leading to a negative refractive index. Therefore, metamaterials structures can support backflows, i.e. the phase velocity of metamaterials and group velocities are parallel. Metamaterials have properties that no natural material can have, such as disobey to Snell's law, Doppler effect, Vavilov – Cerenkov radiation, ... so, it can change the nature of the electromagnetic wave as it passes through it (Caloz & Itoh, 2006; Dong & Itoh, 2012).

In antenna engineering, metamaterials are used for the purpose of improving the parameters or reducing the size. To satisfy that, they can be applied as an antenna environment or as part of an antenna. Depending on the design objective of the antenna, the metamaterial application method is chosen differently. In the case of wanting to improve the performance of an antenna, metamaterials are applied as its environment by arranging the unit cells of the metamaterials around the radiation surface or placing them on one or more other substrates (that also called superstrate) (Krzysztofik & Cao, 2018; Sarkar, 2015). In addition, these metamaterials can also be used as part of the antenna, the unit cell can be loaded into the radiation surface, and the ground plane can also be partially removed by the ground plane (it is called defected ground structure – DGS). This case can be applied to reduce the size or increase the bandwidth of the planar antenna (Chkraborty et al., 2017).

In recent years, many researchers have been interested in designing planar antennas operating in the 3.5 GHz frequency band for 5G applications. The authors mainly

proposed planar antennas with rectangular radiating surfaces and used the probed feeding method (Murugan, 2021). The antenna parameters of interest are the overall size ($25.3 \times 26.8 \text{ mm}^2$), return loss S_{11} , gain (4.2 dB), and bandwidth (100 MHz) without mentioning the radiated efficiency of the antenna. In general, with this design method, the overall size of the antenna is small, and the gain is relatively high, but the bandwidth is narrow. In addition, another research also presented rectangular microstrip antenna designs operating at the 3.5 GHz frequency band on different substrate materials such as FR-4, RT-5880, and TLC-30 (Ramli et al., 2020) with the feeding by microstrip line. The results of the antennas are compared to see the consequences when using different substrate materials. In this design, the antenna parameters of interest are mainly size, radiated coefficient, and gain. Corresponding to each type of background material, the parameters of each antenna will be different. The results of this research show that the size and gain of the antenna are inversely proportional to the dielectric constant of the substrate used to design it. Therefore, depending on the technical requirements of each design, it is possible to choose the appropriate substrate material. In general, the results of recent studies focus on the design of antennas with basic shapes as rectangles with probed and microstrip line methods without mentioning the solution to improve antenna performance. The main parameters of interest are operating frequency, gain, and bandwidth. Moreover, the results achieved by these studies are only shown in simulation models that have not been fabricated and measured for verification.

In this paper, we present a compact antenna flat design, operating at 3.5 GHz for 5G systems. The antenna is designed based on a combination of fractal geometry and metamaterial structure to greatly improve its performance such as reducing its size, enhancing its bandwidth, and improving its gain. In which, the inverted Fractal Minkowski geometry is used for the radiation plane, while the metamaterial structure is a complementary split-ring resonator (CSRR), which is loaded into the antenna's ground plane to generate DGS (Jangid et al., 2015).

2. Antenna design

This section describes the design process from a microstrip antenna to an antenna that uses fractal geometry combined with metamaterial cells. This work aims to improve the performance of the antenna such as reducing its size and increasing its bandwidth by designing the antenna with the radiating surface of the Minkowski island fractal geometry and the ground plane using the Complementary split ring resonators (CSRR) unit.

The original design was a rectangular microstrip antenna operating at 3.5 GHz. The antenna is designed on an FR4-epoxy microwave laminates with a thickness (h) of 1.6 mm dielectric constant (ϵ) of 4.4, loss tangent ($\tan \delta$) of 0.02 and fed with a microstrip line. From the obtained results of the microstrip antenna, we proceed to form the Minkowski island fractal antenna by removing parts of the copper layer on the radiating surface of the antenna. This will increase the electrical length of the antenna, so it will operate at lower frequencies. Therefore, we must decrease the size of the fractal antenna so that it operates at the desired frequency. The resulting Minkowski island fractal antenna is used as the basis for the next step of the design by removing the

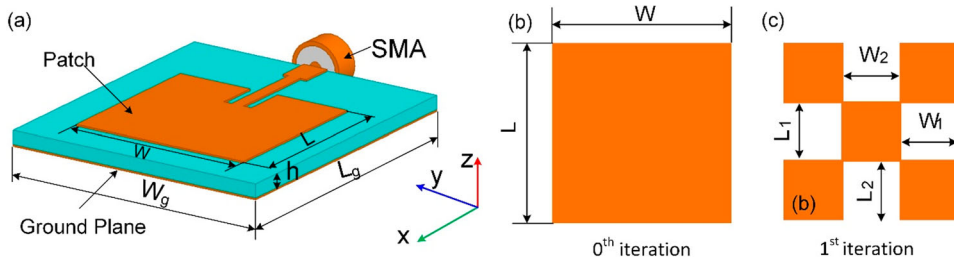


Figure 1. (a) Microstrip antenna configuration, (b) Iterations of Minkowski island fractal geometry: 0th iteration, (c) 1st iteration.

part of the copper layer of the ground plane corresponding to a unit cell CSRR so that the antenna's ground plane is a defected ground structure (DGS).

2.1. Microstrip patch antenna

The configuration of the rectangular microstrip antenna is shown in Figure 1 a. In which, the antenna parameters are calculated by the formulas (1) – (6) and optimized to operate in the frequency range.

$$W = \frac{C}{2f_r \sqrt{\epsilon_r + 1}} \quad (1)$$

where: W is the length of the patch; f_r is the resonant frequency; ϵ_r is the relative dielectric constant of the substrate; c is the speed of light in a vacuum, respectively.

In the microstrip antenna, a transmission line model applies to the case of the infinite dimension of the ground plane. However, in fact, the size of the ground plane has finite size. In that case, the permittivity of the substrate must be modelled by an equivalent value of permittivity, ϵ_{reff} . The effective dielectric constant ϵ_{reff} is determined by (Balanis, 2005; Cao & Krzysztofik, 2018; Meena & Kannan, 2018):

$$\epsilon_{\text{reff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 12 \frac{h}{W} \right]^{-\frac{1}{2}} \quad (2)$$

Modification of ϵ_r makes the size expansion of the microstrip antenna.

$$\Delta L = 0.412h \frac{(\epsilon_{\text{reff}} + 0.3) \left(\frac{W}{h} + 0.262 \right)}{(\epsilon_{\text{reff}} - 0.258) \left(\frac{W}{h} + 0.813 \right)} \quad (3)$$

So, the effective length of the antenna L_{eff} is determined from

$$L_{\text{eff}} = \frac{c}{2f_r \sqrt{\epsilon_{\text{reff}}}} \quad (4)$$

and, the resulting length of the radiation patch L is equal to

$$L = L_{\text{eff}} - 2\Delta L \quad (5)$$

It is necessary to adjust the finite size of the antenna ground so that it matches the device, which will be applied. The commonly used approximation is the six times of the substrate thickness all around the periphery of the patch. In this design, the dimension of the ground plane is limited to.

$$W_g = W + 6h \tag{6}$$

For the antenna microstrip to operate close to the desired frequency, we have to optimize its dimensions during simulation. The overall size of the microstrip antenna operating at 3.5 GHz is $34.3 \times 30.9 \text{ mm}^2$.

2.2. Minkowski island fractal antenna

This section presents the fractal antenna formation process by transforming the shape of the patch from a rectangular to a Minkowski island fractal geometry. A mathematical process is used to form the Minkowski fractal geometry, which can be described based on a series of affine transformations through the Iterative Function Systems (IFS) algorithm. Equation (7) is used to describe affine transformations in planes (Caloz & Itoh, 2006; Krzysztofik, 2013). The IFS algorithm creates the shape of the Minkowski fractal with different iterations, after each iteration, the Minkowski fractal creates its subsets, with the shape of each subset being the same as the fractal's shape in the previous iteration.

$$w(x, y) = Ax + t = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \tag{7}$$

where x, y are coordinates of a point x ; t is a translation factor; a, b, c, d are coefficients controlling the rotation and scaling of fractal parts, while e and f are the linear translation. All six parameters are real numbers (Kubacki et al., 2018; Luo et al., 2009). The generic square patch and three iterations of Minkowski fractal island geometry are shown in Figure 1 b and c.

An important parameter of fractal geometry is the fractal dimension (DS), which is the ratio that provides a statistical indicator of the complexity of the details in a pattern at different stages of iterations. It is given by the formula (Krzysztofik, 2013)

$$DS = \frac{\text{Log}N}{\log \frac{1}{\delta}} \tag{8}$$

where, N is the number of hyper cubes (similar sections), of side length δ (scaling factor) required to cover the object. For the Minkowski fractal, these quantities are $N = 5, \delta = 1/3$, and $DS = 1.465$ (Dhar et al., 2013; Krzysztofik, 2008; Vargas, 2014).

Formulas (1) and (2) are used to calculate the size of the rectangular microstrip antenna, then use the PSO algorithm (Particle Swarm Optimization) to select the dimensions appropriate to the operating frequency. To reduce the overall size, the radiating surface of the antenna is changed to the Minkowski Island fractal geometry with the first iteration. The overall size of the microstrip antenna operating at 3.5 GHz is $30.4 \times 26.6 \text{ mm}^2$. The result obtained after applying the Minkowski island fractal geometry for the surface radiation of the antenna is that the overall size is reduced by about 25% compared to the original overall size.

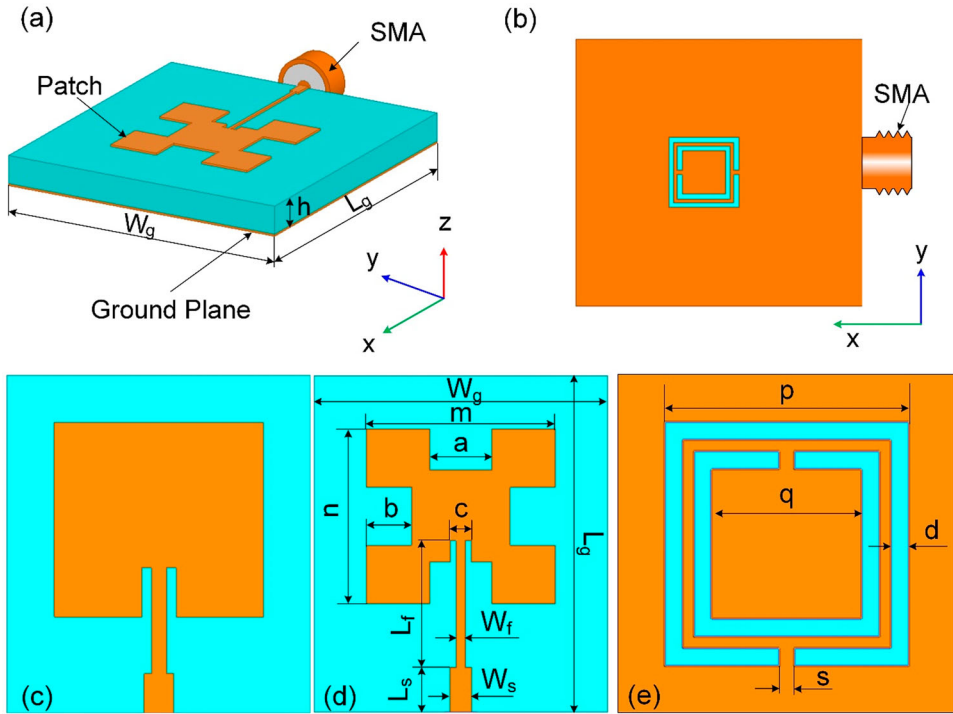


Figure 2. Schematic diagram of the proposed antenna: (a) 3-D view, (b) bottom-view, (c) top-view of microstrip (d) top-view of Minkowski island fractal surface, and (e) enlarged views of CSRR unit cell.

2.3. Proposed antenna configuration

From the achievement of using island fractal geometry in antenna design, the authors designed a fractal antenna combined with the use of CSRR structure, which is loaded into the ground to create a defected ground structure. The result of this combination is to make the antenna operate at a lower frequency than the antenna fractal. Therefore, the overall size of this antenna has to be adjusted smaller for it to work at the desired frequency. The size adjustment is performed simultaneously on both the radiation side and the unit cell size (Mitra et al., 2016; Phan et al., 2016; Raval et al., 2015). This work can be done by optimizing with an algorithm, which is built on the relationships between the antenna dimensions. The overall size of the antenna after combining the use of Minkowski island fractal geometry for the radiation surface with the CSRR metamaterial unit as the DGS for the ground plane is $22.7 \times 20.4 \text{ mm}^2$. It has been reduced by 56% compared to the size of the original antenna. The configuration of this antenna parameter are shown in Figure 2a, d,e and Table 1.

Table 1. The dimensions of of Minkowski island fractal antenna with CSRR for 5G at 3.5 GHz.

Parameters	L_g	W_g	m	n	a	b	L_s
Value [mm]	22.7	20.4	10.8	10.1	3.6	2.4	2
Parameters	W_s	L_f	W_f	p	q	d	s
Value [mm]	1	9.5	0.3	10.2	7.6	0.5	0.4

Table 2. The comparison of the antenna parameters.

Types of antenna	Ant.1	Ant.2	Ant.3
Overall size (mm × mm)	34.3 × 30.9	30.4 × 26.6	22.7 × 20.4
Bandwidth (MHz)	65	120	200
Gain (dB)	2.269	3.21	4.46
Efficiency (%)	67.8	69.43	89.3

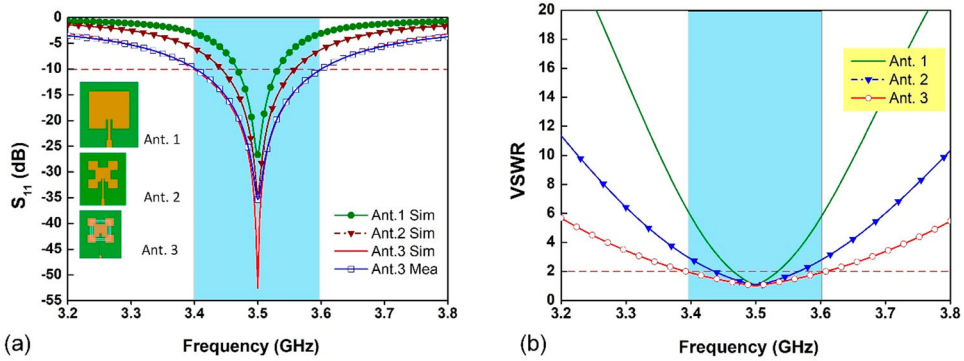


Figure 3. Comparison of antenna parameters (a) Return loss S_{11} , (b) VSWR.

3. Results and discussion

This section describes the results achieved in improving antenna performance through the application of Minkowski island fractal geometry in combination with the CSRR unit cell in the design. After each design improvement, the performance of the antenna is greatly improved as size reduction, bandwidth enhancement, gain improvement, and radiation efficiency >80%. The improvement of antenna performance is shown by comparing its parameters in Table 2, Figures 3 and 4. The antenna symbols in Figure 3 are conventionally named Ant.1 – Microstrip antenna, Ant.2 – Minkowski island fractal antenna, and Ant.3 – proposed antenna.

Figure 4 shows the gain (Figure 4a) and radiated efficiency (Figure 4b) of the proposed antenna, it is clear that the simulation results and measurement results are

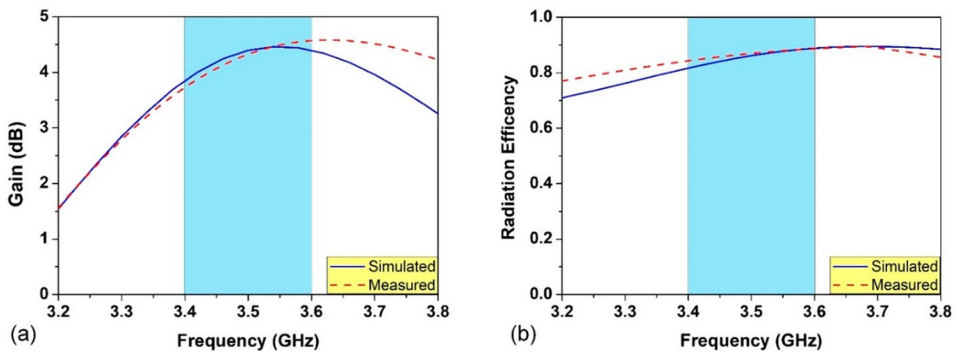


Figure 4. (a) Gain and (b) radiation efficiency of the proposed antenna.

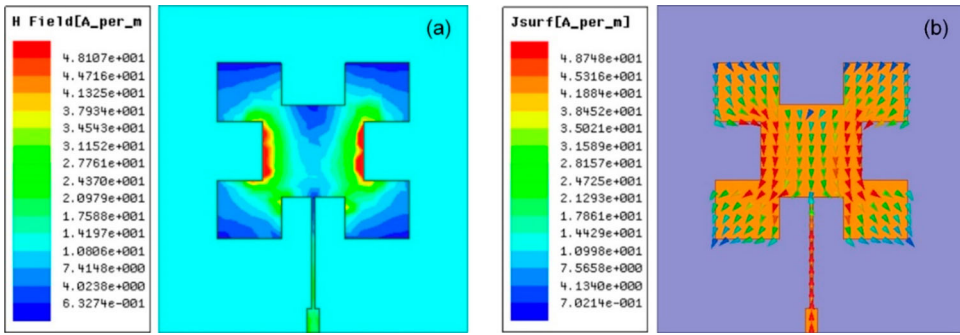


Figure 5. (a) Simulated electric field distributions and (b) surface current at 3.5 GHz of the proposed antenna.

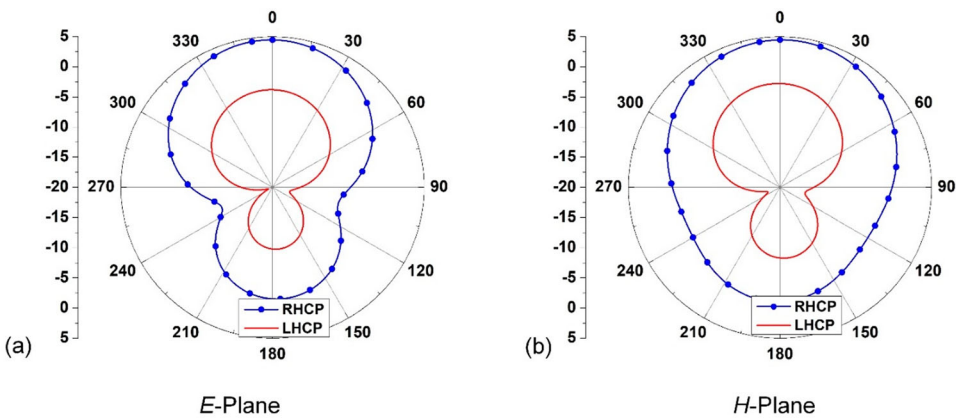


Figure 6. Radiation patterns of the proposed antenna in (a) E-plane and (b) H-plane at various frequencies of 3.5 GHz, respectively.

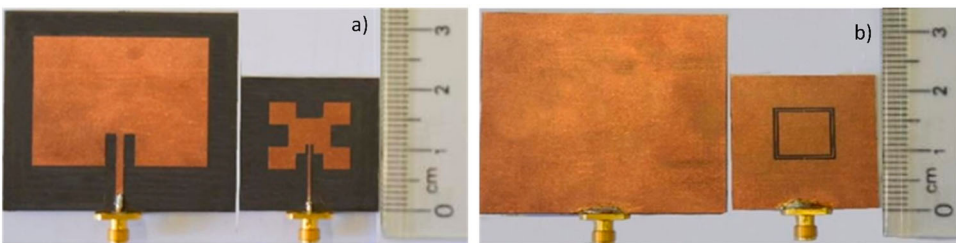


Figure 7. The photos of a fabricated rectangular microstrip antenna and proposed antenna: (a) top – view (a), and (b) bottom – view.

compatible with each other as well as in accordance with the requirements of antennas operating at 3.5 GHz for 5G applications. The simulated electric field distributions and radiation patterns for the proposed antenna at 3.5 GHz are shown in Figures 5 and 6. The simulated electric field distributions and radiation patterns for the proposed antenna at 3.5 GHz are shown in Figure 5 and Figure 6. From the results, it is shown

Table 3. The performance comparison of the proposed antenna with other antennas operating at 3.5 GHz

Refs.	Substrate	Total size (mm^3)	BW (MHz)	Gain (dB)	Efficiency (%)
(Murugan, 2021)	FR4-epoxy	$25.3 \times 26.8 \times 1.6$	100	4.2	–
(Ramli et al., 2020)	FR4-epoxy	$36 \times 28 \times 1.6$	130	3.38	60.13
(Ferdous et al., 2019)	FR4-epoxy	$25.2 \times 48 \times 1.6$	–	5.1	–
This work	FR4-epoxy	$22.7 \times 20.4 \times 1.6$	200	4.46	89.3

that the parameters achieved by the antenna are satisfactory. The fabricated microstrip antenna and fabricated Minkowski island fractal antenna combined CSRR are shown in Figure 7.

From the above-obtained results, it shows that the efficiency of combining both fractal geometry with metamaterial structure in planar antenna design has significantly improved the performance of antennas operating at 3.5 GHz for 5G application. The comparison of this design with other previously published designs is shown in Table 3.

4. Conclusions

An antenna design process using Minkowski island fractal geometry combined with a CSRR unit cell at 3.5 GHz for 5G application was investigated and designed. The antennas are designed on an FR4-epoxy microwave laminate with a thickness $h = 1.6$ mm dielectric constant $\epsilon = 4.4$, loss tangent ($\tan \delta$) of 0.02, and fed with a microstrip line. The antenna design process in turn goes through the stages of changing the antenna's radiating surface to have a Minkowski island fractal shape, then establishing the antenna's ground plane as a defect with the removed part corresponding to a CSRR cell unit.

The result of the study is to create a compact antenna and increased bandwidth. Its size was reduced by about 56% and its bandwidth enhanced by 207%. This is an attractive result, which should be taken care of and applied in the design of other systems. In addition, this design method can be used with different dielectric materials, having higher dielectric coefficients that will produce more compact antennas.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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