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ЭКОНОМИКА И УПРАВЛЕНИЕ

ФИЛОЛОГИЯ

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CHARACTERISTICS OF IN-PLANE FORCED VIBRATION OF A MASS - SPRING SYSTEM WITH STRIBECK FRICTION EFFECT

Abstract

In this article, the authors have developed a two-dimensional friction model with the Stribeck effect in calculating the low-velocity planar motion of a mass-spring system subjected to periodic forced excitation. The excitation is assumed to be kept a constant angle with respect to a fixed axis of motion plane. The Stribeck effect is known in unidirectional motion as a phenomenon in which the friction force decreases as the velocity increases in the region of relatively low velocity (i.e. velocity near zero). In the two-dimensional Stribeck model, the state variable vector in the microscopic motion description of the contact surface contains two components in two different directions, each of which obeys the known motion laws from the one-dimension model. The governing equation of the system is nonlinear and is solved numerically. To evaluate the influence of the external excitation force on the system response, different values of angle and amplitude of the excitation force are selected and explored. In case of relatively large excitation amplitudes, the LuGre friction model can revert to the known Coulomb friction model.

Key word

forced vibration, Coulomb friction, Stribeck effect, internal state variable, periodic solution.

1. Introduction

Friction is a phenomenon that appears in most fields of science related to the relative motion between two surfaces [1,2]. The occurrence of friction is intimately related to microstructures of two surfaces and relative velocity of motion between them. The earliest model of friction was probably discovered by Leonardo da Vinci in 1943 [3]. However, in a long time before that, the human used to take advantage of the phenomenon of adhesion between two surfaces to carry out the transportation of materials and goods, for construction, for war, etc. [4]. The friction can be classified into two types: static friction and dynamic friction. Static friction is the one that occurs between two surfaces in contact where there is no relative displacement between them. Dynamic friction is the state that occurs when two surfaces in contact move relatively to each other [5]. The most popular dynamic friction model is the Coulomb friction one. This model, however, is only suitable for motions with relatively large velocities. For motions at low velocity, the Coulomb friction model will no longer be suitable. Instead, other dynamic friction models have been proposed, including the appearance of new state variables, called *internal state variables*, to describe motion at the microscopic level between two surfaces. The dynamic friction model, called LuGre model, proposed in 1995 is a relatively "strong" model that is suitable for many different friction problems [6]. Its strength is that it can capture the Stribeck effect in low-velocity motion. Later, many other models were proposed based on the LuGre model and applied to many different problems of science and engineering, such as Al-Bender's friction model [7], the model of Gonthier [8], Saha's model [9]. In those models, the authors added new internal state variables (one or more) and a corresponding way to describe them in the friction laws. The previously investigated models usually applied for motions in one-dimensional space. For two-dimensional space, the friction model should be further developed to better capture many phenomena in which the one-dimensional model may not yet sufficiently reflect. In this study, the authors have developed the LuGre friction model from a known one-dimensional case to a two-dimensional problem for the motion of a mass - spring system moving in the plane and subjected to periodic external excitation. The effect of the external excitation on the response of the system is studied in detail.

2. Differential equations of motion

Consider a system model consisting of a mass m attached to two springs with the same stiffness K in the initial horizontal position configuration and with no contraction or expansion as shown in Figure 1. The mass m is subjected to a periodic force, $F_{ex} = F_0 \cos(\nu t)$, inclined an angle α to the horizontal.

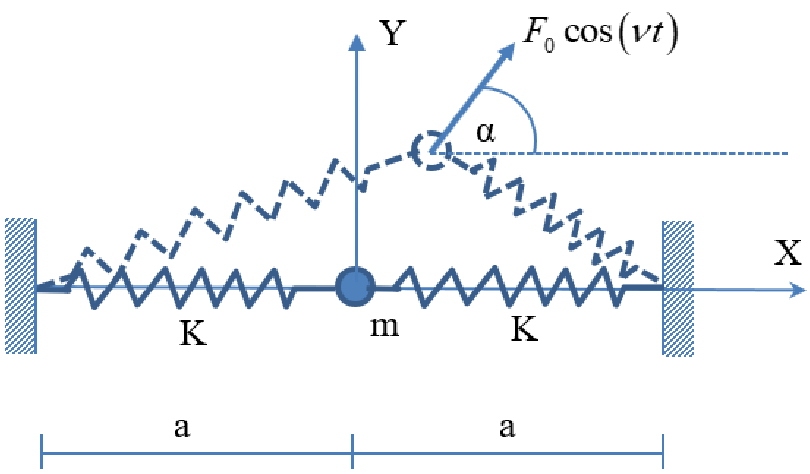


Figure 1 – Model of a mass-spring system under the action of external force

The mass m moving in a horizontal plane is affected by the friction force from the contact between two surfaces. To capture the Stribeck effect in low-velocity motion, the LuGre friction model is used. This model, first proposed in 1995 by Canudas de Wit et al., can explain many experimentally observed phenomena in the field of engineering related to tribology. The friction force is assumed depending on a state variable, called the internal state variable, denoted by *Z*, which characterizes the roughness of the surface and is modeled as the average deflection of bristles. In this study, because the mass moves in two-dimensional space, the authors have proposed a two-dimensional LuGre friction model to describe the motion of the system. The friction force is separated into two components in two different directions, $\mathbf{F}_{fr} = \left(F_{fr,X}, F_{fr,Y}\right)$, where

$$F_{fr,X} = \Sigma_0 Z_X + \Sigma_1 \dot{Z}_X + \Sigma_2 \dot{X},$$
$$F_{fr,Y} = \Sigma_0 Z_Y + \Sigma_1 \dot{Z}_Y + \Sigma_2 \dot{Y}. \quad (1)$$

where Σ_0, Σ_1, Σ_2 are the stiffness coefficient of bristles, internal damping and viscous damping coefficient in relative motion between the two surfaces; (X, Y) is the coordinates of the position of the mass m in the OXY coordinate system with O being the origin placed at the middle point betweent two springs, coincident with the initial position at which the springs does not contract or expand; $\dot{X}, \dot{Y}$ are the two velocity components of the object. The state variable Z_X, Z_Y is determined from the following equations:

$$\dot{Z}_X = \dot{X} - \frac{\|\mathbf{V}\|}{G(\mathbf{V})} Z_X,$$
$$\dot{Z}_Y = \dot{Y} - \frac{\|\mathbf{V}\|}{G(\mathbf{V})} Z_Y \tag{2}$$

where $\|\mathbf{V}\| = \sqrt{\dot{X}^2 + \dot{Y}^2}$ is the magnitude of velocity vector $\mathbf{V}$ of the body m; $G(\mathbf{V})$ is the Stribeck function describing the Stribeck effect in low velocity motion, is defined as:

$$G(\mathbf{V}) = \frac{1}{\Sigma_0}\left(F_c + (F_s - F_c)\exp\left\{ -\frac{\|\mathbf{V}\|^2}{V_s^2} \right\} \right) \tag{3}$$

where $F_c = \mu_c N^*$, $F_s = \mu_s N^*$ are the Coulomb friction force and static force, respectively. They are calculated through the Coulomb coefficient of friction μ_c, the coefficient of static friction μ_s and the normal force N^*. V_s is the Stribeck velocity. It can be seen that if $\|\mathbf{V}\| \to +\infty$ then the Stribeck function $G(\mathbf{V})$ has the form $G(\mathbf{V}) = F_c / \Sigma_0$, that is, the Stribeck friction model returns to the well-known Coulomb friction model. When $\|\mathbf{V}\| \to 0$ we get $G(\mathbf{V}) = F_s / \Sigma_0$, i.e. the static friction model. In the near-zero velocity domain, the Stribeck effect is revealed, we can see that, as velocity increases, friction decreases, in contrast to Coulomb friction where friction increases with increasing velocity.

Using Lagrange equation combined with the friction force expression from (1), we get the equation of motion of the system as follows:

$$\ddot{x} = -\omega^2(1+x)\left(1 - \frac{1}{\sqrt{(1+x)^2 + y^2}}\right) + \omega^2(1-x)\left(1 - \frac{1}{\sqrt{(1-x)^2 + y^2}}\right) + \frac{1}{am}\left(\Sigma_0 Z_X + \Sigma_1 \dot{Z}_X + \Sigma_2 \dot{X}\right) + \frac{F_0\cos\alpha}{am}\cos(\nu t) \tag{4.1}$$

$$\ddot{y} = -\omega^2 y\left(1 - \frac{1}{\sqrt{(1+x)^2 + y^2}}\right) - \omega^2 y\left(1 - \frac{1}{\sqrt{(1-x)^2 + y^2}}\right) + \frac{1}{am}\left(\Sigma_0 Z_Y + \Sigma_1 \dot{Z}_Y + \Sigma_2 \dot{Y}\right) + \frac{F_0\sin\alpha}{am}\cos(\nu t) \tag{4.2}$$

where

$$x = \frac{X}{a},\ y = \frac{Y}{a},\ \omega = \sqrt{\frac{K}{m}}, \tag{5}$$

The time derivatives $\dot{Z}_X$, $\dot{Z}_Y$ are determined from equations (2), and (3); $F_0\cos(\nu t)$ is the periodic excitation force with amplitude F_0 and frequency ν. Equations (2) and (4) establish a system of differential equations describing the dynamics of the mass m with spring constraints and the friction forces between the two surfaces. Note that the system of equations (4) and (2) is a complex system because it is nonlinear due to both of contribution of geometrical nonlinearity and that of the friction force with the Stribeck effect. In addition, system behavior is also affected by external forces. To solve this system, the authors used the

Runge-Kutta numerical algorithm, the results obtained will be presented in Section 4.

3. Equilibrium point

The equilibrium point of the system is obtained when the velocity and acceleration of the system in time vanish, including the rate of change in time at the tip of the of bristles. That leads to the following system of algebraic equations:

$$-\omega^2(1+x)\left(1-\frac{1}{\sqrt{(1+x)^2+y^2}}\right)+\omega^2(1-x)\left(1-\frac{1}{\sqrt{(1-x)^2+y^2}}\right)=0$$

$$-\omega^2 y\left(1-\frac{1}{\sqrt{(1+x)^2+y^2}}\right)-\omega^2 y\left(1-\frac{1}{\sqrt{(1-x)^2+y^2}}\right)=0 \quad (6)$$

where it is noted that, since the derivatives $\dot{Z}_x, \dot{Z}_y$ are vanishied, relying on Eq. (2) leads to the bristle deflection is zero, i.e. the vertical bristle at the equilibrium state of the system. Solving the above algebraic system, we get the result that the equilibrium point has coordinates (0,0), coincides with the initial position when the spring does not contract or expand and mass lies on the straight line connecting the two springs. When subjected to a periodic external force, the system may have a periodic response to a given set of system parameters.

4. System response analysis

Using the Runge-Kutta method with the system parameters given in Table 1, we obtain the response results illustrated in Figures 2-12. We consider the following three cases corresponding to three different angles of α : (i) $\alpha = 90^0$, (ii) $\alpha = 60^0$; (iii) $\alpha = 0^0$.

Table 1

Parameters used in simulation

Description/Unit	Notation	Value
Internal stiffness (N/m)	Σ_0	100000
Internal damping (Ns/m)	Σ_1	316.2278
Viscous damping (Ns/m)	Σ_2	0.4
Mass (kg)	m	0.2
Stribeck velocity (m/s)	V_s	0.001
Static friction coefficient	μ_s	0.61
Coulomb coefficient	μ_c	0.47
Spring initial length (m)	a	0.2

4.1. Response of the system to angle 90° of the applied force

We first consider the case where the angle of the external force is perpendicular to the line connecting the two springs at initial position. The initial position of the mass has coordinates (X, Y) = (0, 0.04) (m), i.e. $(x, y) = (0, 0.2)$. For this case, it is possible to intuitively feel that the mass will vibrate in the direction perpendicular to the line connecting the two springs, i.e. vibrating in the Y direction of the coordinate system. The spring stiffness is taken as $K = 1000\ (\text{N/m})$, with the mass of the body $m = 0.2\ (\text{kg})$, the frequency ω of the reference system is $\omega = \sqrt{K/m} =$ 70.7107 (rad/s). Assume the excitation frequency is taken as $\nu = 70\,(\text{rad/s})$ and excitation amplitude is $F_0 = 5\,(\text{N})$. The time-varying responses of the ratio $x = X/a$ and $y = Y/a$, with $a = 0.2\,(\text{m})$ are illustrated in Figure 2. It can be seen that the x and y response ratios of the system are almost periodic. However, the value of x is quite small, about below 2×10^{-5}, while the motion of y is much larger. This shows that the displacement in the x direction is negligible. The non-zero values of x reflect the fact that the motion of the system is still slightly vibrating in the x direction due to the effect of two-dimensional LuGre friction. In the Y direction, the system's oscillations are much larger, the ratio $y = Y/a$ falls between -0.04 and 0.01, and the oscillation of y has a non-zero mean value. The trajectory graph will therefore be nearly in the Y direction as illustrated in Figure 3.

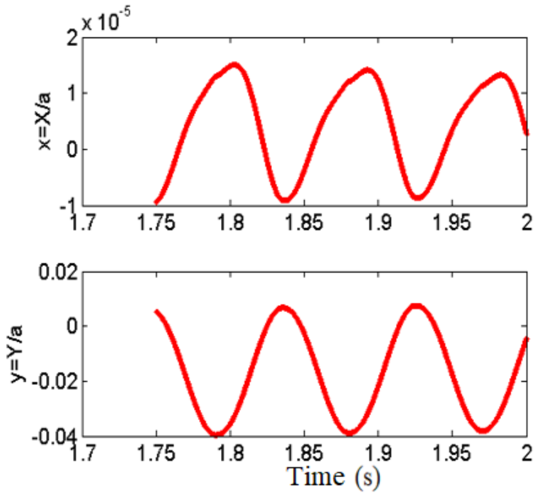


Figure 2 – Time evolution of ratios $x = X/a$ and $y = Y/a$ ($\alpha = 90^0$)

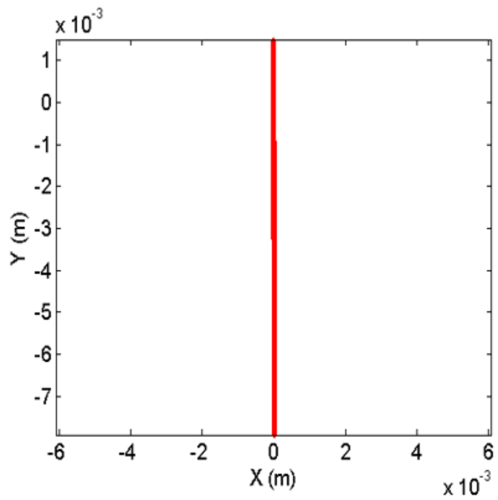


Figure 3 – Graph of the motion of the object in the case of incline angle of 90^0

Figure 4 shows the time evolution of the velocity and friction force components. It can be seen that, at locations where the velocity is close to zero, the value of the friction force changes markedly. That is because the Stribeck effect has been taken into account. In Figure 5, the phase trajectory of the displacement Y and the velocity $\dot{Y} = dY/dt$ is illustrated, showing that the phase trajectory is almost a closed curve, reflecting that the object's motion trajectory is almost a periodic one, due to the effects of periodic exciation, $F_{ex}(t) = 5\cos(70t)\,(\text{N})$.

It is worth mentioned here that the authors emphasize and focus on the calculation to show the Stribeck effect in the low-velocity domain. Figure 6(a) shows a theoretical Stribeck curve based on equation (3) and the calculated values of the velocity response obtained from Figure 4 and values of the corresponding Stribeck function at this velocity. It is clear that, for large velocity domain, the Stribeck function is almost constant, while for small velocity domain, the Stribeck function has a distinct height. To clearly see the Stribeck effect, Figure 6(b) illustrates an enlargement for the velocity domain with the range $(-5,5)\times10^{-5}$ (m/s).

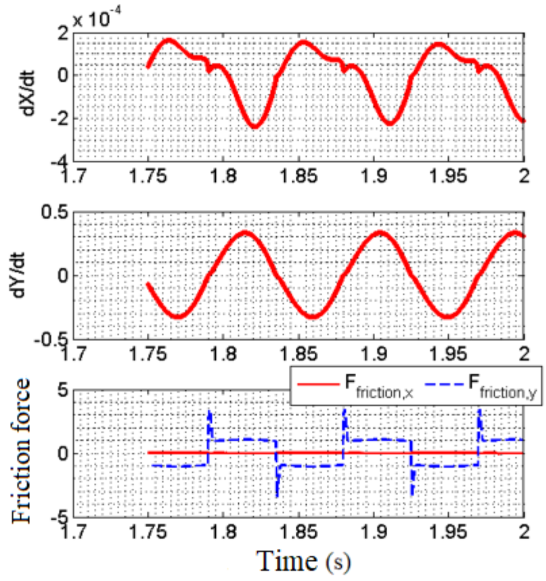


Figure 4 – Time evolution of the velocity and friction force components ($\alpha = 90^0$)

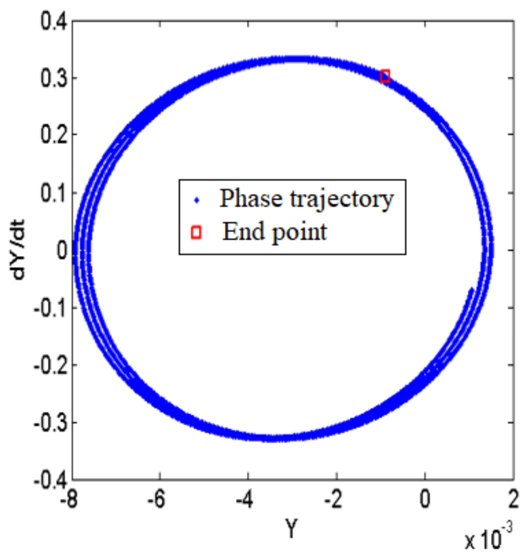


Figure 5 – Phase trajectory of the displacement Y and the velocity $\dot{Y} = dY / dt$ ($\alpha = 90^0$)

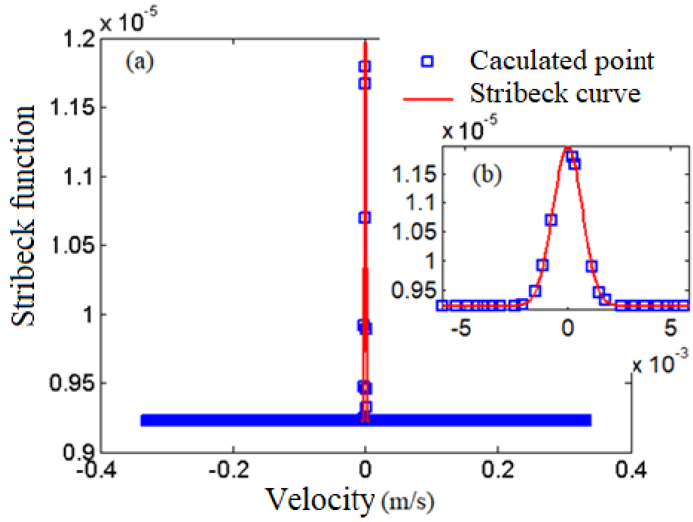


Figure 6 – Theoretical and calculated Stribeck curves ($\alpha = 90^0$)

4.2. Response of the system to angle of the applied force

The second situation is considered here that the angle between the applied force and OX axis is 0°, that is, the force acts in the X direction. The initial position is taken as (X,Y)=(0.04, 0), i.e (x, y)=(0.2, 0). It can be observed that system oscillations will follow the X direction like that of the conventional mass - spring system, the resulting motion trajectory is illustrated in Figure 7, where the Y-direction motion is almost zero. The time evolutions of velocity and friction are illustrated in Figure 8. It can be seen that oscillations in the X-direction are quasi-periodic, while in the Y-direction, the oscillations are close to zero. We still observe the Stribeck effect in this case, the friction force changes significantly in the vicinity of zero velocity. Figure 9 exhibits the Stribeck effect through the calculated points from the system model; this effect is evident in the very narrow region of the velocity close to zero.

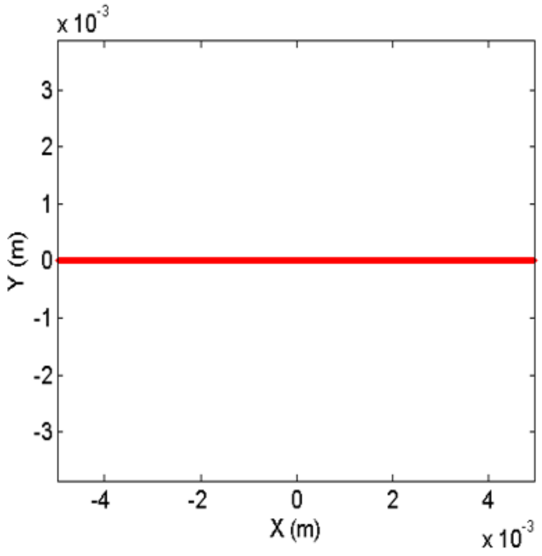


Figure 7 – Motion trajectory in case of angle 0° of applied force

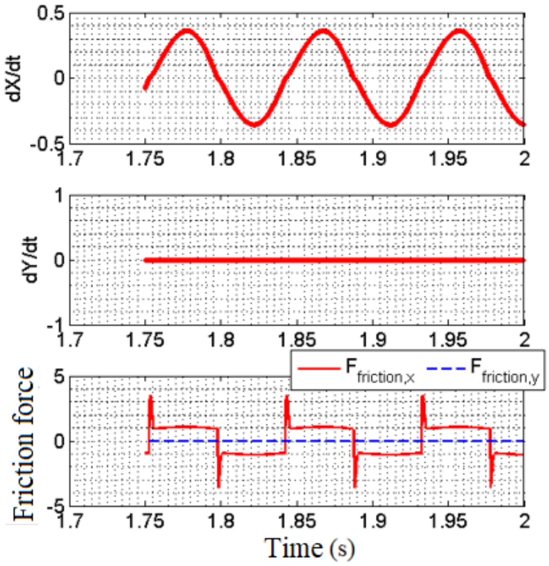


Figure 8 – Time evolution of the velocity and friction force components ($\alpha = 0^{\circ}$)

4.3. Response of the system to two-dimensional friction in the case of an arbitrary angle of the applied force

The considered case here is the angle $\alpha = 60^{0} \in \left(0, 90^{0}\right)$. With this angle, motion in both directions is clearly seen, for example, in Figure 10, which illustrates the motion of the object. To clearly see the Stribeck effect, Figure 11 shows the Stribeck function in the X, Y directions. We see that the calculated points coincide with the constructed Stribeck curve. Thus, in the case of the two-dimensional LuGre model, we also obtain a response where there exists a low-velocity region near zero that makes the friction force change its properties comparing to the Coulomb friction, i.e. friction decreases as speed increases.

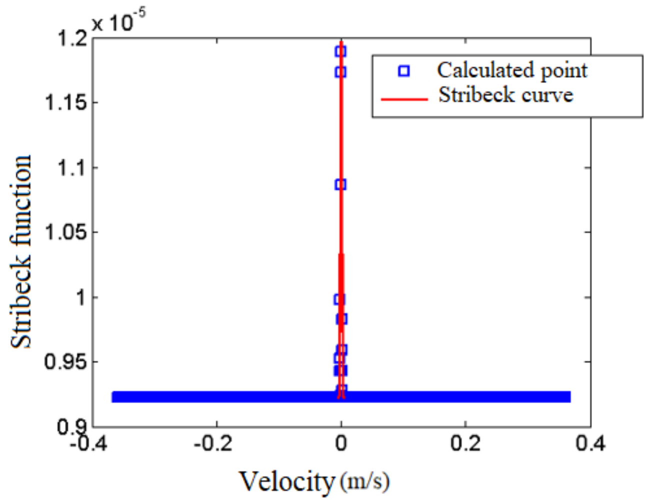


Figure 9 – Theoretical and calculated Stribeck curves ($\alpha = 0^{\circ}$)

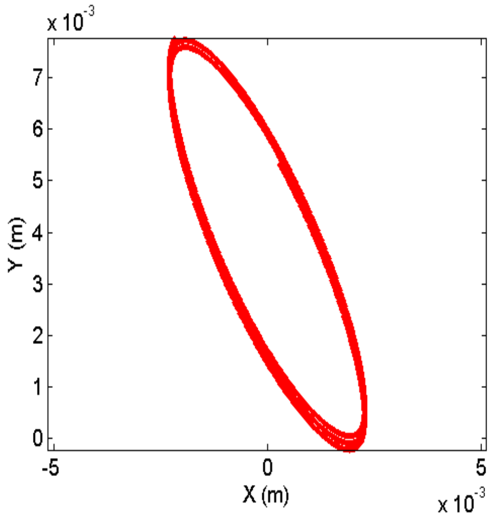


Figure 10 – Motion trajectory of mass m in the XOY plane

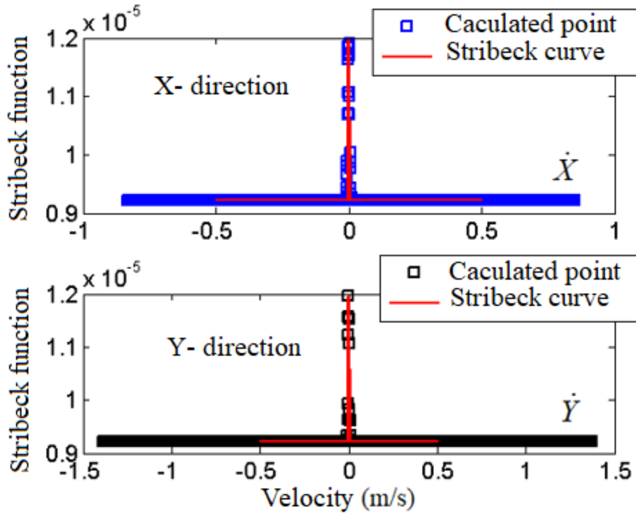
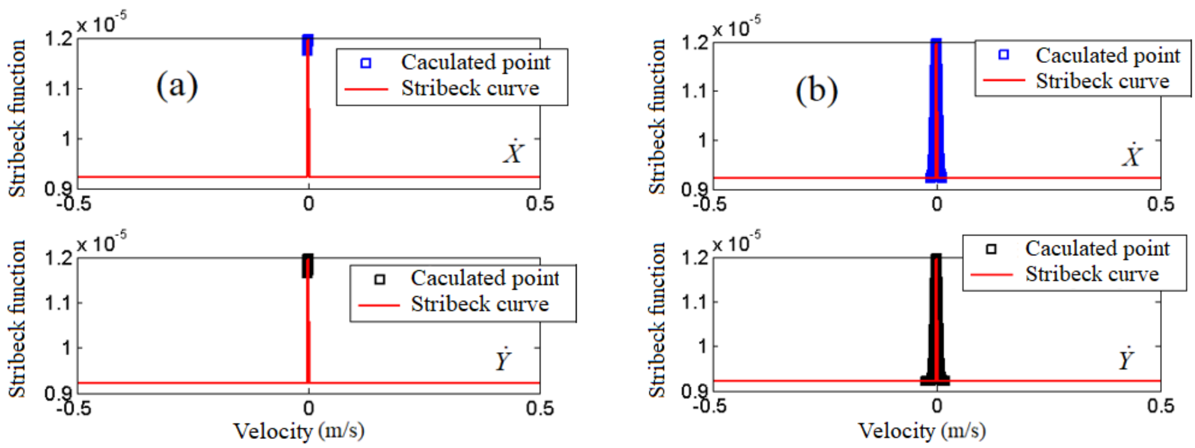


Figure 11 – Stribeck function in terms of two velocity components

To evaluate the effect of external exciation, different excitation amplitudes are used in our calculations. Figure 12 illustrates the Stribeck function for four different values of the amplitude F_0: (a): $F_0 = 0.5$ (N); (b): $F_0 = 1.0$ (N); (c): $F_0 = 10$ (N); (d): $F_0 = 50$ (N). Figure 12(a) reveals that, for small amplitude excitation, the system's motion velocity is relatively small, so the LuGre model captures the Stribeck effect quite clearly. The calculated points ly on the curve part, not on the horizontal line part of the Stribeck curve. Figure 12(b) corresponds to a larger excitation amplitude, i.e. $F_0 = 1.0$ (N), shows that the captured Stribeck points are denser than Figure 12(a) and the velocity value is still small, lying almost entirely on the part of the curve . As the excitation amplitude is increased $F_0 = 10$ (N), as shown in Figure 12(c), the number of calculated points in the low-velocity domain begins to decrease, replaced by relatively large velocity points located on the near "straight" of the Stribeck curve. Finally, in Figure 12(d), for a relatively large excitation amplitude, $F_0 = 50$ (N), the low-velocity domain with the Stribeck effect is almost invisible. This value of excication force is much larger than the static friction force of the object in contact with the floor, which seems to cause the object to move at a relatively fast velocity. In this situation, the Stribeck curve will return to a straight line reflecting the Coulomb friction force where the friction force increases as the velocity of the system increases in the field of view.



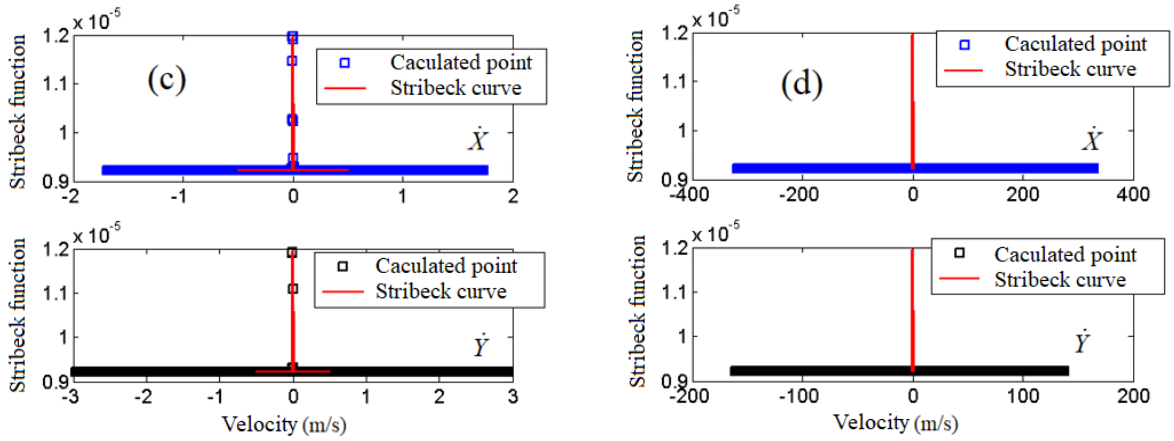


Figure 12 – Effect of external excitation amplitude on the occurrence of the Stribeck friction effect

5. Conclusion

The oscillation of a system that takes into account friction is a complex problem. The novelty of this study is the inclusion of a 2D LuGre friction model to describe the motion of a system with two springs and a mass in two-dimensional space. A relatively natural reason to use a 2D LuGre model is that the system is subjected to an excitation of an external force with inclination angle to the reference direction is an arbitrary angle. Here the effect of periodic exciation on the response of the system is investigated. The obtained results show that the inclination angle and magnitude of the excitation force amplitude affect the low-velocity motion of the system. In the case of a relatively large external excitation amplitude, the system moves with a relatively large velocity and thus the LuGre friction model reverts to the Coulomb friction model. In the case of small amplitude external exciation, low velocity motion is observed and thus the proposed 2D LuGre model captures the Stribeck effect.

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