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ANALYSIS OF ENERGY FLOW IN VIBRATIONS OF A BEAM SUBJECTED TO PERIODIC LOADINGS USING HARMONIC BALANCE METHOD

Abstract

In this study, characteristics of energy flow in vibrations of a beam structure subjected to periodic loadings are explored. To capture useful information of beam vibration based on an analytical approach, a single-mode governing differential equation obtained from the original equation of motion of beam is retained. Two expressions for energy input and dissipation due to the effect of geometrical nonlinearity of beam are derived using the harmonic balance method. The obtained results show that the nonlinear parameter of the system has a strong influence on energy flows in vibrating system when the effect of axial force of beam vibration is included. For comparison purpose, characteristics of energy flow in linear vibration of beam are also presented.

Key words:

beam vibration, energy flow, resosnance frequency, nonlinearity.

1. Introduction

The energy flow approach can be seen as a novel way that provides fundamental aspects to investigate behaviors of dynamical systems [1,2,3]. This approach has attracted more and more increasing interests of engineers and scientists in various fields of science and technology, especially in the field of structural mechanics [4,5,6]. In [4], Langley used the method of dynamic stiffness to explore a vibration control system with mean power flow and stored energy for a general framework in which the system subjected to harmonic and random excitations. Cuschieri [5] developed a mobility approach for analyzing structural power flow of an L-shape plate. Energy flow models studied in the framework of finite element method can be seen in [6]. Recently, in studying vibrations of mechanical systems, the approach of energy flow has been developed by some authors, for example in [7-9]. In [7], Yang et al. have investigated some dynamic characteristics and power flow behavior of a nonlinear vibration isolator system with a negative stiffness mechanism. They have used the method of averaging to obtain a frequency-response function of a system subjected to periodic loadings. The property of sub-harmonic resonance obtained from their study can provide effective information for designing nonlinear isolation systems. The research group of Yang also developed the energy flow approach to examine the dynamic performance of a nonlinear vibration absorber coupled to nonlinear oscillators in [8] and [9].

In continuous systems such as the beam and plate, the properties of energy flows are also explored [10-12]. The transmission of vibrational energy cross a structural joint [10] was derived using the Bishop's receptance method. Mechanical energy flow models of rods and beam were studied in [11] by Noiseux. An experimental method on energy flow was proposed in [12] to measure the vibrational intensity in uniform plates and beams vibrating in flexure.

The energy flow in nonlinear systems has been shown in several researches for discrete mechanical systems with one-, two-, and multi-degree-of-freedom [4-9]. In continuous systems with the presence of nonlinearity, however, our understandings about energy flow are very limited [1]. The purpose of this paper is to investigate featured characteristics of energy flow in vibrations of a beam subjected to periodic loading in case of nonlinear vibration due to the effect of axial force. We use method of harmonic balance method to find approximate solutions of system responses such as deflection, slope, energy flow quantities. The obtained results show the accuracy of the approximations in comparison with those obtained from the Runge-Kutta algorithm.

2. Governing equation of beam vibration

In this paper, we analyze vibrations of a simply supported beam (see Fig. 1) subjected to a periodic loading. The governing equation of the beam is as follows [13,14]

$$EI\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + \mu A\frac{\partial^2 w}{\partial t^2} + \beta\frac{\partial w}{\partial t} + K_f w = p(x,t) \quad (1)$$

where axial force N is given by

$$N = \frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \quad (2)$$

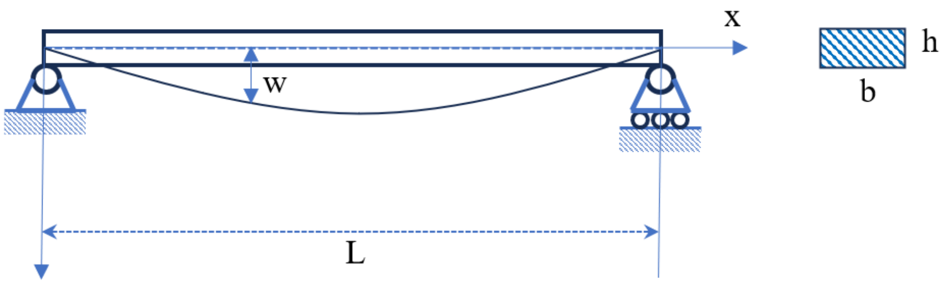

Fig. 1 – A model of simply supported beam

In Eq. (1), $w = w(x,t)$ is the deflection of the beam; E is the elastic modulus, I is the moment of cross-section area, μ is the mass density per unit length of the beam, $A = b\times h$ is the cross-section area of beam with width b and thickness h; β is viscous damping coefficient, K_f is the stiffness coefficient of elastic foundation. The external loading $p(x,t)$ is a function of two arguments x and t. Assume that the loading $p(x,t)$ can be expanded in series [13]

$$p(x,t) = \sum_{n=1}^{\infty} p_n(t)\phi_n(x) \quad (3)$$

where $\phi_n(x)$ ($n = 1,2,...$) are modal shape functions corresponding to free vibration of beam without the axial, viscous, elastic forces and external loading. The functions $\phi_n(x)$ ($n = 1, 2,...$) satisfy the following equations:

$$EI\frac{d^4\phi_n}{dx^4} = \mu A\omega_n\phi_n \tag{4}$$

$$\int_0^1 \phi_n\phi_m d\xi = \delta_{mn} = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases},\ \xi = \frac{x}{L} \tag{5}$$

where $\omega_n = \left(\frac{n^4 EI\pi^4}{\mu AL^4}\right)^{1/2}$ are natural frequencies of beam associated with modal shape function $\phi_n(x)$ ($n = 1,2,...$).

Let the beam deflection $w = w(x,t)$ be expanded in terms of orthogonal modal shapes $\phi_n(x)$ as follows

$$w(x,t) = \sum_{n=1}^{\infty} w_n(t)\phi_n(x) \tag{6}$$

where $w_n(t)$ is the modal amplitude corresponding to *n*-th modal shape function $\phi_n(x)$.

Using the Galerkin method [2] associated with expansions (3) and (6), the governing equation (1) is transformed to a set of equations of modal amplitudes w_m [13]:

$$\begin{aligned} &\ddot{w}_m + \frac{\beta}{\mu A}\dot{w}_m + \omega_m^2\left(1 + \frac{1}{m^4}\frac{K_f}{\mu A\omega_0^2}\right)w_m \\ &+ \frac{\omega_m^2}{2R^2m^2}\sum_{n=1}^{\infty} n^2 w_n^2 w_m = \frac{1}{\mu A}p_m(t) \end{aligned} \tag{7}$$

where

$$\omega_0 = \left(\frac{EI\pi^4}{\mu AL^4}\right)^{1/2},\ \omega_m^2 = m^4\omega_0^2,\ R = \sqrt{\frac{I}{A}} \tag{8}$$

Eq. (7) is a system of nonlinear differential equations of modal amplitudes. Depending on different forms of external loading functions p_m ($m = 1,2,...$), one may find appropriate strategies for solutions of Eq. (7). Here, we assume that p_m ($m = 1,2,...$) are periodic functions in expanding the external loading $p(x,t)$ from Eq. (3). The system (7) is equivalent to the governing equations of a multi-degree-of-freedom system if the first modes of beam vibration are retained. For purpose of analytical calculations, one may consider several first modes of vibration because the contribution of these modes on vibrational amplitudes is significant in comparison with remaining higher nodes [13]. The previous works in the literature (see [13,14]) shown that the first vibrational mode plays an important role in investigating vibration characteristics of beam. In our present study, the first mode of beam vibration is considered. From Eq. (7), the equation for the first mode is written as follows

$$\ddot{w}_1 + \frac{\beta}{\mu A}\dot{w}_1 + \left(\omega_0^2 + \frac{K_f}{\mu A}\right)w_1 + \frac{\omega_0^2}{2R^2}w_1^3 = \frac{1}{\mu A}A_p \sin\left(\omega_p t\right) \tag{9}$$

where external component $p_1(t)$ is assumed to be a periodic function with amplitude A_p and excitation frequency ω_p, i.e. $p_1(t) = A_p \sin\left(\omega_p t\right)$. For the single-mode equation (9), the complicated nonlinear component in Eq. (7) is reduced to a simple form with a cubic term known as that of Duffing oscillator. We introduce to the following dimensionless quantities

$$\eta = \frac{w_1}{h},\ \Omega_0 = \frac{\omega_0}{\omega_p},\ \zeta = \frac{\beta}{2\mu A\omega_0},\ \alpha = \left(\frac{K_f}{\mu A\omega_p^2}\right)^{1/2},\ p_0 = \frac{A_p}{\mu A h\omega_p^2},\ n\tau = \omega_p t \tag{10}$$

where n is a non-zero arbitrary natural number. Using (6), Eq. (9) is transformed to the corresponding single-mode equation in new dimensionless variable η as follows

$$\begin{aligned} &\eta'' + 2n\zeta\Omega_0\eta' + n^2\left(\Omega_0^2 + \alpha^2\right)\eta + 6n^2\Omega_0^2\eta^3 \\ &= n^2 p_0 \sin\left(n\tau\right) \end{aligned} \tag{11}$$

where $\eta' = d\eta/d\tau$ denotes the derivative of η with respect to the dimensionless time τ, called dimensionless slope. It represents "velocity" of mode, i.e. the rate of change of deflection in time. In Eq. (11), η is the ratio of the first modal amplitude w_1 to the h thickness of the beam. If the thickness h remains constant, vibrational characteristics of the dimensionless quantity η and that of the physical quantity w_1 are the same but have different values. The dimensionless frequency Ω_0 is the ratio of the primary frequency of the beam vibration to the frequency of the external loading. ζ is called damping factor. The value ζ is assumed to be smaller than unity, i.e. $\zeta < 1$. The quantity α represents for stiffness coefficient of elastic foundation. Its value also depends on parameters μ, A and ω_p. Eq. (11) may be solved numerically to obtain the solution η in dimensionless time τ. In this study, we use the method of harmonic balance to explore characteristics of energy flow in vibrations of beam for the first vibrational mode.

3. Equation of energy flow

Multiplying both sides of Eq. (11) by the function η', we obtain the following equation of energy flow (see [1])

$$K' + Q' + U' = e_{in} \quad (12)$$

where K is the kinetic energy, Q is the dissipation, U is the energy potential of the single-mode system, e_{in} is the input energy flow of the system; K', Q', U' denote as the corresponding derivatives of K, Q, U with respect to time τ; and

$$\begin{aligned} &K' = \eta''\eta',\ Q' = 2n\zeta\Omega_0\eta'^2, \\ &U' = n^2\left(\Omega_0^2 + \alpha^2\right)\eta\eta' + 6n^2\Omega_0^2\eta^3\eta', \\ &e_{in} = n^2 p_0\eta'\sin\left(n\tau\right) \end{aligned} \quad (13)$$

The energy flow quantities K', Q', U' show the rate of change of energies in time τ. The input energy flow e_{in} depends on the rate η' and periodic function $\sin\left(n\tau\right)$ at time τ. From Eq. (13), we get expressions for the kinetic energy and energy potential as follows

$$K = \frac{1}{2}\eta'^2, U = \frac{1}{2}n^2\left(\Omega_0^2 + \alpha^2\right)\eta^2 + \frac{3}{2}n^2\Omega_0^2\eta^4 \quad (14)$$

It is seen that the kinetic energy is a quadratic function of the slope η' whereas the energy potential is a quartic function of dimensionless deflection η. The expression of energy potential contains two parts: a part of linear vibration, $\frac{1}{2}n^2\left(\Omega_0^2 + \alpha^2\right)\eta^2$, and other of nonlinear vibration due to the effect of axial force of beam vibration, $\frac{3}{2}n^2\Omega_0^2\eta^4$.

The average dissipation in a time interval $\left[\tau_1, \tau_2\right]$ is given by

$$\bar{Q} = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} Q'(\tau)\,d\tau = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} 2n\zeta\Omega_0\eta'^2\,d\tau \qquad (15)$$

Similarly, the average input energy in a time interval $[\tau_1, \tau_2]$ is

$$\bar{E}_{in} = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} e_{in}\,d\tau = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} n^2 p_0\eta'\sin(n\tau)\,d\tau \qquad (16)$$

Energy expressions (13-16) will be explored after determining system responses in Eq. (11) using the harmonic balance method as presented below.

4. Method of harmonic balance

Because Eq. (11) represents for a vibrating system with damping, the part of free vibration of the system will vanish after a finite time interval. We here will investigate properties of vibration stage in which system responses are contributed by only periodic external loading [2]. In the framework of this study, for simplicity, we consider the first approximation of response η in which its frequency and the frequency of external loading are the same:

$$\eta = \hat{\eta}_1\cos(n\tau) + \hat{\eta}_2\sin(n\tau) \qquad (17)$$

where $\hat{\eta}_1, \hat{\eta}_2$ are unknown constants. The period of $\eta(\tau)$ in Eq. (17) is $2\pi/n$.

Substituting Eq. (17) into Eq. (11) and equating coefficients of the first harmonic terms $\cos(n\tau), \sin(n\tau)$ from obtained result, we get a nonlinear algebraic system of two equations for two unknowns $\hat{\eta}_1, \hat{\eta}_2$:

$$\begin{aligned}
&\left(\Omega_0^2 + \alpha^2 - 1\right)\hat{\eta}_1 + 2\zeta\Omega_0\hat{\eta}_2 + \frac{9}{2}\Omega_0^2\hat{\eta}_1\left(\hat{\eta}_1^2 + \hat{\eta}_2^2\right) = 0 \\
&-2\zeta\Omega_0\hat{\eta}_1 + \left(\Omega_0^2 + \alpha^2 - 1\right)\hat{\eta}_2 + \frac{9}{2}\Omega_0^2\hat{\eta}_2\left(\hat{\eta}_1^2 + \hat{\eta}_2^2\right) = p_0
\end{aligned} \qquad (18)$$

4.1 Case of linear vibration

In case of linear vibration, Eq. (18) is reduced to the following system for $\hat{\eta}_1, \hat{\eta}_2$

$$\begin{aligned}
&\left(\Omega_0^2 + \alpha^2 - 1\right)\hat{\eta}_1 + 2\zeta\Omega_0\hat{\eta}_2 = 0 \\
&-2\zeta\Omega_0\hat{\eta}_1 + \left(\Omega_0^2 + \alpha^2 - 1\right)\hat{\eta}_2 = p_0
\end{aligned} \qquad (19)$$

Solving Eq. (19) gives

$$\hat{\eta}_1 = -\frac{2\zeta\Omega_0}{\left(\Omega_0^2+\alpha^2-1\right)^2+\left(2\zeta\Omega_0\right)^2}p_0$$
$$\hat{\eta}_2 = \frac{\Omega_0^2+\alpha^2-1}{\left(\Omega_0^2+\alpha^2-1\right)^2+\left(2\zeta\Omega_0\right)^2}p_0 \tag{20}$$

The amplitude of linear vibration is

$$\chi_L = \sqrt{\hat{\eta}_1^2+\hat{\eta}_2^2} = \frac{1}{\sqrt{\left(\Omega_0^2+\alpha^2-1\right)^2+\left(2\zeta\Omega_0\right)^2}}p_0 \tag{21}$$

It is seen that the amplitude of linear vibration of beam is proportional to the p_0 amplitude of external periodic loading. If value of p_0 is fixed, characteristics of χ_L depend on parameters of linear system corresponding to Eq. (11), i.e. ζ and Ω_0. The properties of vibrational amplitude of response η of linear system will change if nonlinearity is included. We now consider this nonlinear situation in Eq. (18).

4.2 Case of nonlinear vibration

After several manipulation steps, we obtain two equations as follows:

$$\left(\Omega_0^2+\alpha^2-1\right)\chi^2+\frac{9}{2}\Omega_0^2\chi^4 = p_0\hat{\eta}_2$$
$$2\zeta\Omega_0\chi^2 = -p_0\hat{\eta}_1 \tag{22}$$

In Eq. (22), the first equation shows that value of $\hat{\eta}_2$ is positive whereas the second equation shows that value of $\hat{\eta}_1$ is negative. Eliminating $\hat{\eta}_1$ and $\hat{\eta}_2$ from (22) yields the following equation for determining amplitude of nonlinear vibration

$$\chi^2\left[\left(2\zeta\Omega_0\right)^2+\left(\Omega_0^2+\alpha^2-1+\frac{9}{2}\Omega_0^2\chi^2\right)^2\right]-p_0^2=0 \tag{23}$$

It is seen that Eq. (23) is a cubic equation in variable $y=\chi^2>0$. This equation may have one, two or three positive solutions y depending on parameters ζ, Ω_0, α and p_0. Solution results are presented in Section 5. We have an expression for the amplitude χ_{NL} in case of nonlinear vibration of beam after determining χ^2 from Eq. (23):

$$\chi_{NL} = \frac{p_0}{\sqrt{\left(2\zeta\Omega_0\right)^2+\left(\Omega_0^2+\alpha^2-1+\frac{9}{2}\Omega_0^2\chi^2\right)^2}} \tag{24}$$

The difference between amplitude expressions (21) and (24) lies on the term $\frac{9}{2}\Omega_0^2\chi^2$ generated by the nonlinearity of beam vibration. The expression of χ_L is explicit whereas that of χ_{NL} is implicit in χ^2.

Substituting Eq. (17) into Eq. (15), the average dissipation in a period $2\pi/n$ can be derived as follows

$$\bar{Q} = \frac{n}{2\pi}\int_0^{2\pi/n} 2n\zeta\Omega_0\eta'^2 d\tau = n^3\zeta\Omega_0\chi^2 \tag{25}$$

Similarly, we have an expression for average input energy $\bar{E}_{in}$

$$\begin{aligned}\bar{E}_{in} &= \frac{n}{2\pi}\int_0^{2\pi/n} e_{in}(\tau)\,d\tau \\ &= \frac{n}{2\pi}\int_0^{2\pi/n} n^2 p_0\eta'\sin(n\tau)d\tau = -\frac{1}{2}n^3 p_0\hat{\eta}_1\end{aligned} \tag{26}$$

From (22) and (26), we get

$$\bar{E}_{in} = n^3\zeta\Omega_0\chi^2 \tag{27}$$

From Eqs. (25) and (27), it is observed that the average values of dissipation and input energy in a period are the same in the first approximation of single-mode system. This property may be changed if the higher approximations are considered.

Substitution of Eq. (17) into expression of kinetic energy in Eq. (14) gives

$$\begin{aligned}K &= \frac{1}{2}\left[-n\hat{\eta}_1\sin(n\tau) + n\hat{\eta}_2\cos(n\tau)\right]^2 \\ &\le \frac{1}{2}n^2\left(\hat{\eta}_1^2 + \hat{\eta}_2^2\right) = \frac{1}{2}n^2\chi^2 = K_{\max}\end{aligned} \tag{28}$$

where

$$K_{\max} = \frac{1}{2}n^2\chi^2 \tag{29}$$

is maximum value of kinetic energy. This value is proportional to the χ^2 amplitude of response $\eta(\tau)$. From (27) and (29), we have a relationship between the average input energy and maximum kinetic energy:

$$\bar{E}_{in} = 2n\zeta\Omega_0 K_{\max} \tag{30}$$

Substituting Eq. (24) into Eqs. (29) and (30), we obtain

$$K_{\max} = \frac{1}{2}\frac{n^2 p_0^2}{\left(2\zeta\Omega_0\right)^2 + \left(\Omega_0^2 + \alpha^2 - 1 + \frac{9}{2}\Omega_0^2\chi^2\right)^2} \tag{31}$$

$$\bar{E}_{in} = \frac{n^3\zeta\Omega_0 p_0^2}{\left(2\zeta\Omega_0\right)^2 + \left(\Omega_0^2 + \alpha^2 - 1 + \frac{9}{2}\Omega_0^2\chi^2\right)^2} \tag{32}$$

From expressions (31) and (32), we have upper evaluations $K_{\max}^u$ and $\bar{E}_{in}^u$ for the maximum kinetic energy and average input energy, respectively:

$$K_{\max} \le K_{\max}^u = \frac{1}{8}\frac{n^2 p_0^2}{\zeta^2\Omega_0^2} \tag{33}$$

$$\bar{E}_{in} \le \bar{E}_{in}^u = \frac{n^3 p_0^2}{4\zeta\Omega_0} \tag{34}$$

Several following characteristics of energy flows in beam vibration can be identified [see Eqs. (25) and (27); Eq. (30); Eqs. (33) and (34)]:

Characteristic 1: The average dissipation and average input energy are the same in a vibrational period if the first approximation of beam deflection is retained.

Characteristic 2: The maximum kinetic energy of beam vibration is proportional to the amplitude of vibration.

Characteristic 3: Upper evaluations of maximum kinetic energy and average input energy are independent of stiffness coefficient of elastic foundation.

5. Numerical results and discussion

Results of approximate responses of single-mode equation (11) are shown in Figs. 2-9 using the harmonic balance (HB) method as presented in Section 4. To evaluate the accuracy of HB method [see Eqs. (17) and (23)], results obtained from Runge-Kutta (RK) algorithm for Eq. (11) are also displayed. All calculations are performed for approximate responses in HB method with value $n=1$.

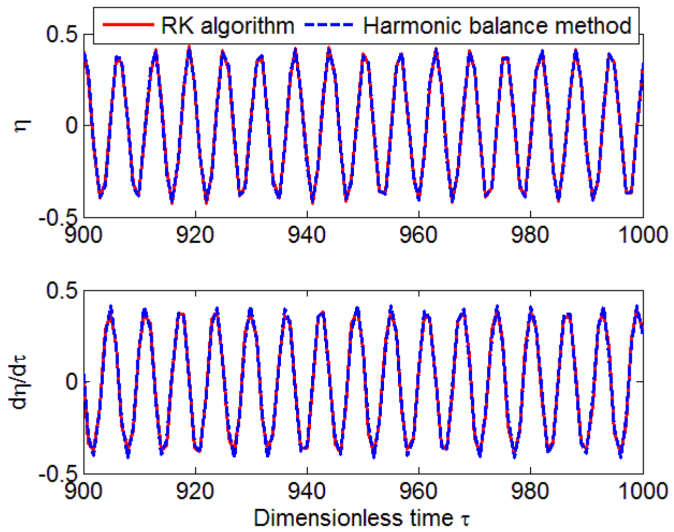

Fig. 2 – Evolutions in dimensionless time of η and η' by RK algorithm and HB method

Fig. 2 plots evolutions of dimensionless defection $\eta(\tau)$ and dimensionless slope $\eta'(\tau)$ by RK and HB methods with input parameters $\Omega_0=0.8333$, $\zeta=0.0044$, $\alpha=0$ and $p_0=0.1$. The figure displays graphs on time domain [900, 1000] on which the steady-state regime of vibration can be reached. It is observed that the evolution curves of HB method are very close to those of the RK numerical solutions. This shows the effectiveness of HB method although only the first approximation of solution responses is calculated. The value of $\eta(\tau)$ represents for the change of the deflection w_1 in comparison with the thickness h of the considered rectangular beam. In Fig. 2, at steady-state regime, the maximum value of η is smaller than 0.5 as the beam is subjected to periodic loading with amplitude $p_0=0.1$. In our computation, the nonlinear amplitude χ_{NL} in this case is 0.4176. Because approximate solution form (17) is periodic, the slope $\eta'(\tau)$ is also periodic and has corresponding amplitude $\kappa_{NL}=n\chi_{NL}=0.4176$.

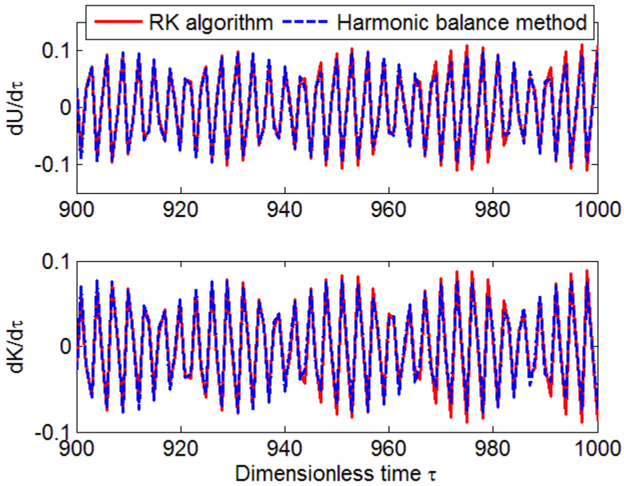

Fig. 3 – Evolutions in dimensionless time of energy flow quantities U' and K' by RK algorithm and HB method

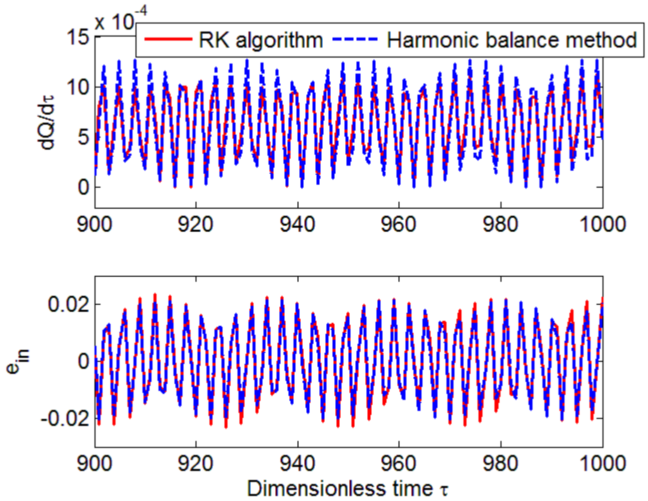

Fig. 4 – Evolutions in dimensionless time of dissipation rate Q' and input energy flow e_{in} by RK algorithm and HB method

In Fig. 3, evolutions of energy flow quantities U' and K' are portrayed by the RK and HB methods. For HB method, periodic properties of U' and K' are different from $\eta(\tau)$ and $\eta'(\tau)$ because they are nonlinear functions of $\eta(\tau)$, $\eta'(\tau)$, and therefore higher-order harmonics can be observed at steady-state regime in Fig. 3. Similarly, properties of harmonic behavior of the dissipation rate Q' and input energy flow e_{in} obtained from the HB method can be seen in Fig. 4.

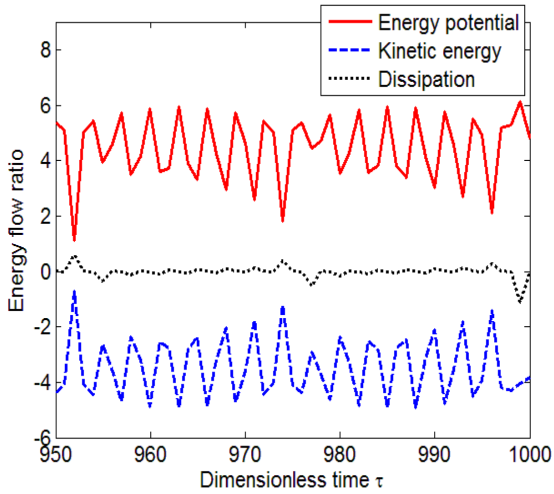

Fig. 5 – Ratio of energy flow quantities to input energy flow

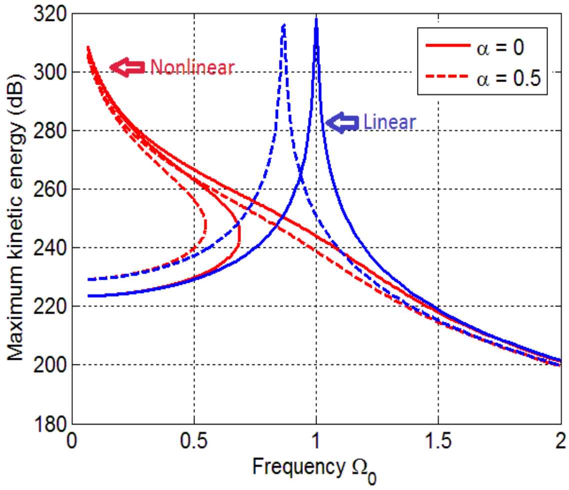

Fig. 6 – Plots of maximum kinetic energy with two different values of stiffness coefficient α in two cases of linear and nonlinear vibrations

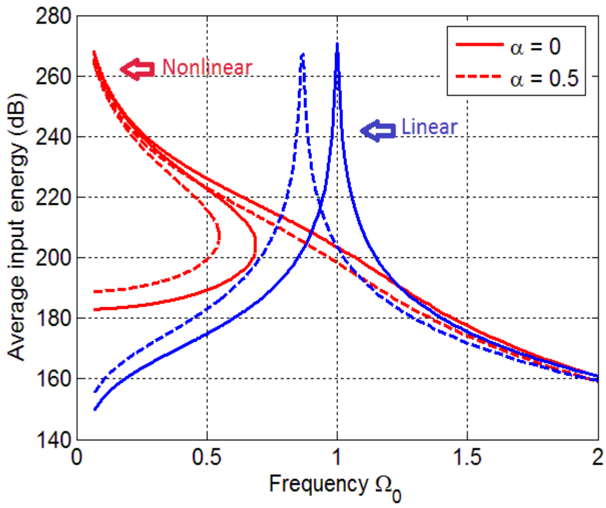

Fig. 7 – Plots of average input energy with two different values of stiffness coefficient α in two cases of linear and nonlinear vibrations

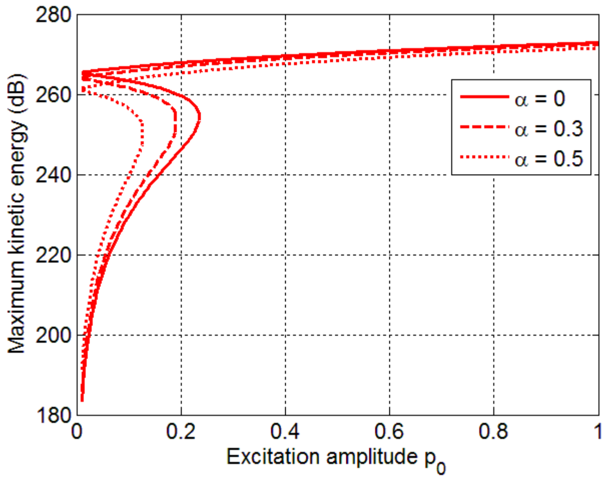

Fig. 8 – Comparisons of maximum kinetic energy versus excitation amplitude p_0 with various values of stiffness coefficient α.

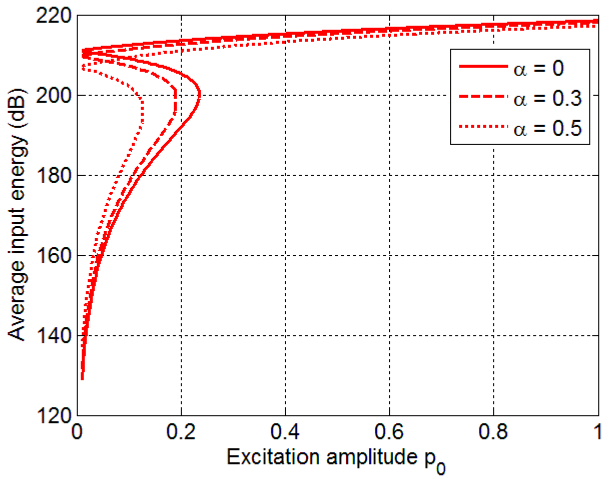

Fig. 9 – Comparisons of average input energy versus excitation amplitude p_0 with various values of stiffness coefficient α.

From the energy flow equation (12), summation of rates of energy changing is equal to input energy flow e_{in}. The energy flow e_{in} plays as a "driving force" of the first mode of beam vibration. It supplies a periodic input source to drive the changes of kinetic energy, energy potential and dissipation energy. To explore these changes, at instant τ at which the energy flow e_{in} does not vanish, dividing both sides of Eq. (12) by e_{in} to obtain

$$\theta_K + \theta_Q + \theta_U = 1 \tag{35}$$

where

$$\theta_K = \frac{K'}{e_{in}},\ \theta_Q = \frac{Q'}{e_{in}},\ \theta_U = \frac{U'}{e_{in}} \tag{36}$$

are ratios of kinetic energy, dissipation energy, and energy potential to input energy flow. Because $\theta_K + \theta_Q + \theta_U = 1$ is larger than zero, there exist two values of summation in which a value is larger than zero, and other is smaller than zero. Fig. 5 presents the plots of ratios $\theta_K, \theta_Q, \theta_U$ in dimensionless time τ obtained from the HB method. It is seen that value of ratio θ_Q is very small in comparison with two remaining ratios. Hence, from Eq. (35), we can use approximation as follows

$$\theta_K + \theta_U \approx 1 \tag{37}$$

If $\theta_K < 0$ then $\theta_U > 0$. At time τ, if $\theta_K < 0$, i.e. $\frac{K'}{e_{in}} < 0$, we have two cases:

Case 1: If $e_{in} > 0$ then $K' < 0$. This means the kinetic energy restored in the system is decreasing, and therefore the energy potential is increasing.

Case 2: If $e_{in} < 0$ then $K' > 0$. This shows that the change of kinetic energy is increasing, leading to the decrease of energy potential.

Numerical calculations show that, at steady-state regime, in the interval [950, 1000] displayed in Fig. 5, the ratio θ_K is negative whereas the ratio θ_U is positive.

To understand further on the characteristics of energy flows, we illustrate graphs of maximum kinetic energy (MKE) and average input energy (AIE) (in dB unit, the reference value for MKE and AIE is set as 10^{-12}) versus parameters of single-mode system as presented in Figs. 6-9.

In Fig. 6, graphs of maximum kinetic energy versus the frequency Ω_0 are demonstrated with two different values of stiffness coefficient α in both linear and nonlinear vibrations. The solid line is corresponding to the case $\alpha = 0$, i.e. the beam system has not elastic foundation. In this case, the curve of maximum kinetic energy with respect to linear vibration occurs resonant phenomenon at frequency $\Omega_0 = 1$. For nonlinear vibration, the curve of MKE is very different from that of linear vibration. The MKE curve of nonlinear system is shifted to the left of the curve of linear system in the direction of low frequency Ω_0. Because the coefficient of nonlinear term in Eq. (11) is $6n^2\Omega_0^2$ that depends on the frequency Ω_0, this frequency also plays as a factor causing the nonlinear property of the system. The dashed line expresses curves of MKE in the case $\alpha = 0.5$. For both linear and nonlinear vibrations, the corresponding MKE curves are shifted to the left of the MKE curves in the case of $\alpha = 0$.

From Eq. (29), because of the proportional property of the maximum kinetic energy and response amplitude, the behavior of MKE is similar to that of amplitude versus the change of the frequency Ω_0. At the tail of MKE curves, i.e. the Ω_0 large value, the curves are nearly coincidental.

Fig. 7 portraits the behavior of average input energy with two different values of stiffness parameter α, $\alpha = 0$ and $\alpha = 0.5$. In the dB scale of vibration energy, the AIE curves are viewed clearly, especially for the range of low value of frequency Ω_0. Similar to MKE curves, AIE curves of nonlinear vibration are shifted to the left of those of linear vibration. There exists a frequency position from which the system gives value of AIE smaller if increasing frequency Ω_0. In range of very large Ω_0, the difference between the AIEs of linear and nonlinear systems are not considerable.

Figs. 8 and 9 present the results of MKE and AIE curves versus the excitation amplitude p_0 with three cases of the stiffness parameter α, $\alpha = 0$, $\alpha = 0.3$ and $\alpha = 0.5$. The effect of nonlinearity is also observed from the energy curves. The excitation amplitude is taken from 0.01 to 1.0 of the dimensionless system (11). If α increases, the MKE and AIE curves will move to the left of p_0-axis. At high value of p_0, for example, $p_0 = 0.8,\ p_0 = 0.9,\ p_0 = 1.0$, the change of AIE values is not considerable.

6. Conclusion

The research on characteristics of energy flow provides a new look for approaches of solving problems of mechanical systems. Because the energy flow quantities contain information of dynamical system, for example displacement, velocity, acceleration, one may find useful principle or characteristics for purposes of calculations, measurements, design, and so on in technical applications and technology. In this paper, we have investigated some featured characteristics of energy flow in vibration of a beam subjected to periodic loading using the single-mode model associated with the method of harmonic balance. For simplicity in analytical calculations, our approach based on the harmonic balance is studied for the case of the first approximation of system responses. The obtained analytical results show several fundamental features of energy flows such as relationship between the average dissipation and average input energy; the proportional property of the maximum kinetic energy and response amplitude of vibration; upper evaluations of maximum kinetic energy and average input energy. Evolutions of energy flow quantities obtained from the harmonic balance method are also compared with those obtained from Runge-Kutta algorithm to verify the accuracy of analytical method in the case of the first approximation of system responses. Our results exhibit that the characteristics of energy flow quantities such as maximum kinetic energy and average input energy are affected considerably by the geometrical nonlinearity of beam due to the effect of axial force.

For further understanding on the energy flow in vibration of beam, researches on the cases of multi-mode and higher approximation should be required. This direction of research will be carried out in our future papers.

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