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Determining the size and shape of a dimension stone block under consideration on the spatial relationship of joint sets

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ABSTRACT

Dimension stone not only represents the quality, including strength, color, and polish indexes but also shows the size and shape to obtain standard requirements for the processing plant. One of the issues affecting the size and block recovery ratio is discontinuities inside the stone, dividing into specific sizes and shapes. Therefore, the paper shows a relationship among three main joint sets, existing in the quarries, influencing the size and shape of a stone block generated by intersections of these joint sets. Each joint set is characterized by dip, dip direction, and spacings. Modelling discontinuities from three main joint sets generated a stone block with a specific size and shape. The paper carried out at stone quarries in Phu Yen, Binh Dinh, and Khanh Hoa Provinces. The results showed that when changing one of the geometry parameters of these joint sets, the size, and shape will be correspondingly changed. These sizes and shapes depend on the spatial intersection of these joint sets. In addition, the recovery ratio of each block for processing was calculated regarding its shape to assess which quarries have a good relationship with joint sets. From the minimum size for the processing plant is a rectangular parallelepiped of 0.4 m^3, the minimum spacing of joints in a set was defined to satisfy such requirements. This contributes to showing which quarries have a favorable condition on stone size and shape the plant requires. From this, the spacings are equal or more than the minimum spacing in each joint set for quarries determined to calculate the reverse of the rectangular parallelepiped of equal or more than 0.4 m^3.

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1. Introduction

Dimensional stone is a natural stone, including intact rock groups of magma, sedimentation, and metamorphism without discontinuities quarried and processed to various sizes, shapes, colours, and polishes. Dimensional stone has many applications in practice such as slabs, blocks, and tiles. The stone must ensure lithology, mineral, and mechanics components, especially without discontinuities in stone (Taboada et al., 1999; Mosch et al., 2011; Morales Demarco et al., 2013).

In the world, the demand for dimensional stone has nowadays increased significantly due to building activities using natural stone blocks from groups of granite, quarts, crystal limestone, and shale. Particularly, the volume for tile, slab, kitchen, other applications, art statues, and building materials are 35%, 25%, 20%, 16%, and 9%, respectively. The quarries for dimensional stone, such as gabbro, marble, and granite have been developed in scale and output, and are distributed in some countries such as China, India, Turkey, Brazil, Italy, Iran, Spain, and Egypt. The world building association predicted the world would use slabs by more than 30 percent of dimensional stones, increasing 2.5 times throughout 15 years ago, and projecting there will develop dramatically due to high demand and modernly-invested technologies, more price competition than ceramic (Stonedeal, 2021).

Through document assessment, it is clearly indicated that factors affecting to recovery ability of dimensional stone from a quarry include fractures, strength, polish, and size. One of them, fracture, is a major factor in reducing dimensional stone recovery as shown in Figure 1 and some authors have shown (Palmström, 2001; Taboada et al., 2008; Sousa et al., 2010; Sousa et al., 2016).

In Vietnam, dimensional stone recovery was expressed by distributing spatial joint sets measured in scanline or window samples at the

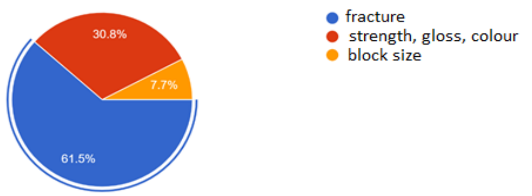


Figure 1. Reasons affecting the recovery ratio of dimensional stone.

surface, collected in drill holes, and checked in experiment pits with block size divided by them. Block sizes of more than 0.4 m^3 will be assigned to the reserve. The recovery ratio was different when calculating with scanline measure or window measure at the surface and with log length in the drill hole. The difference is due to fractures. The recovery in the geological document is usually derived from the data of experiment pits, less than 50 m^3, not characterizing the entire quarries. There has not been a study on fracture analysis to the recovery (An, 2017).

Nowadays, there have been more studies on the formation of stone blocks generated from joint sets. According to the Institute for Research on non-metal mineral Geology in Russia, stone size was divided into sizes of 0.01 m^3, 0.4 m^3, 0.7 m^3, 1 m^3, 2.0 m^3, 2.5 m^3, 4.5 m^3, 5 m^3, 6 m^3, and 8 m^3. The recovery ratios for the sizes were derived from finding in appendix tables calculated instantly following three joint sets and the spacing of joints in a set. The study has not mentioned joint orientation and the relationship among joint sets. Blocks with sizes of more than 0.4 m^3 were assigned to the geological reserve, but their shapes were not interesting (Institution for scientific research of the Russian Federation on non-metal mineral Geology, 1985). Palmström (2001) calculated the sizes and shapes of blocks following three joint sets crossing at the right angle. The shapes could contain flat, flat and long, long and equal-dimensional, depending on shape factor (β), which could be defined with spacings of joint sets generating blocks. The study just showed the size and shape of the blocks generated from three joint sets crossing at the right angle, but in reality, these joint sets usually are not perpendicular to each other (Palmström, 2001). Tuan et al. (2019) were interested in joint sets to recover valuable blocks of more than 0.4 m^3 based on the modelling of a fracture network in rock mass but block sizes of more than 0.4 m^3 also were not considered their shapes. Mutluturk (2007) showed that besides the quality, dimensional stone also depended on the desired size. This would be done by blocks generated from joints in the rock mass. The block was put with market blocks (rectangular blocks inside and their sizes of 3x2x1 m or 1.5x1x1 m) to show how many market blocks. However, the author just showed the way to do it but did not give a result of the method because of lacking fracture modelling ability. Mosch (2011) showed the size and shape of blocks governed by the dip direction of joints. The paper showed spatial joint distribution in rock mass navigated three coordination points from the data of joints with window sample and scanline. From calculating pixels in the model to show the volume of blocks. The author just established a fracture network for the whole quarries in a simple way with three face boundaries of the model. Fernandez-de Arriba et al. (2013) contributed an optimization algorithm on the recovery ratio of dimensional stone based on blocks formatted by three joint sets with dip direction angle, dip angle, and spacing parameters to divide the blocks into smaller sizes of 1.5x2x1.5 m. Basing on the mining direction defined from minimum dip direction angle to maximum dip direction angle and mining direction increments determined a mining direction with the maximum recovery ratio, but the paper has not yet shown a change in the volume and the shape of stone blocks with the spatial relationship of joint sets. Yarahmadi et al. (2017) also approached various quarrying directions to optimize the recovery ratio, but the paper showed the intersection of three major joint sets to the actual cutting pattern to generate stone blocks with their specific shapes. From each of the shapes, the recovery ratio would be solved by comparing with rectangular blocks having the same volume as the ones. The recovery ratio of each stone block was calculated by comparing a rectangular area having the same volume as the stone block with a total of the surrounding area of the block, and the ratio changes from 0 to 1. When the ratio reaches 1, the shape of the block will be the best. However, the paper has not assessed the change in the volume of the block due to the intersection of three major joint sets and the recovery ratio has not been calculated with the volume and shape the plants need.

In general, all the research above has just studied fracture network modelling to produce stone blocks and analyzed the volume, but the research has not expressed the change in volume and shape of the stone block due to the generation of major joint sets, which change in dip angle, dip direction angle, spacing and spatial relationship of the major joint sets. Therefore, to have more knowledge on the change in three major joint sets to the volume and shape of each stone block generated by the joint sets, the paper developed further a discontinuity modelling method to show the volume and shape of each block when changing in dip angles, dip direction angles, spacings and spatial relationship of three major joint sets. Moreover, the paper also proved that the size equal to or more than 0.4 m^3, assigned to the reserve, has not ensured the size and shape according to the demand of the plant where the shape for cutting into slabs is rectangular parallelepiped. This could be demonstrated that the minimum size of 0.4 m^3 for three quarries has different recovery values corresponding to the relationship of joint sets in the quarries. From this, a minimum spacing of each joint set in each quarry will also calculate for collecting the rectangular parallelepiped of 0.4 m^3.

2. Method

The spatial position of fractures is defined by parameters navigating them in space. The indexes include strike, an intersection line between the fracture plane and horizontal plane; strike angle, an angle between the strike of fracture and the North; dip direction angle, a line being perpendicular to the strike; dip angle (β), an angle between dip direction line and its projection on the horizontal plane; dip direction angle (α), an angle between the orthogonal projection of dip on the horizontal plane and the North. In practice, to navigate the position of fracture, it usually defines dip direction angle (α) and dip angle (β). The figures can be collected using the compass in the field.

(a) Orientation of fracture: dip β and dip direction α; (b) presentation of fracture (stereographic projection Wulff or Schmidt); (c) line orientation (plunge and trend).

Figure 2. Terminology on geometric parameters of fracture (source: https://seismicconsolidation.com)

A group of parallel joints or nearly parallel joints is called a joint set. A network consisting of joint sets creates a fracture network separating rock mass into individual stone blocks (Figure 3). Spacing (S) is the true distance between the two adjacent joints in a joint set.

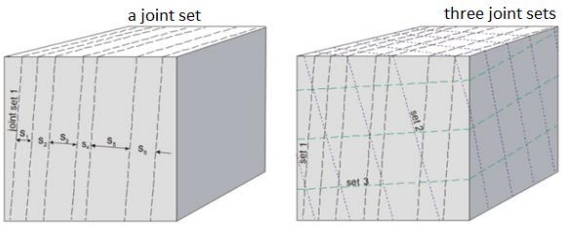


Figure 3. Joint sets and spacing (Palmström, 2001).

Characteristic parameters of the joint set are dip direction (α_n) and dip (β_n). The values are defined as given in Equation 1.

$$\alpha_n=\arctan(r_{xn}/r_{yn})+Q$$
$$\beta_n=\arctan(r_{zn}/(r_{xn}+r_{yn})^{0,5} \qquad (1)$$

In which: r_n- general normal vector with $r_{xn}=\sum n_{xi}$, $r_{yn}=\sum n_{yi}$ và $r_{zn}=\sum n_{zi}$

$Q=0^0$ if $r_{xn}\geq 0$ and $r_{yn}\geq 0$;

$Q=180^0$ if $r_{xn}<0$ and $r_{yn}\geq 0$ or $r_{xn}<0$ and $r_{yn}<0$;

$Q=360^0$ if $r_{xn}\geq 0$ and $r_{yn}<0$;

The determination of the stone block to be extracted requires the following input data of three joint sets: dip direction (α, in degree, with values between 0^0 and 360^0), dip (β, in degree, with values between 0^0 and 90^0) and spacing (S, in meters).

With the data from these sets, the first thing to do is calculate the normal vector related to each plane ($\vec{p_i}$) defined by: (Fernández-de Arriba et al., 2013).

$$\vec{p_i}=[\sin(\alpha_i).\sin(\beta_i), \cos(\alpha_i).\sin(\beta_i), \cos(\beta_i)] \qquad (2)$$

The next step needs to determine for each set after achieving each normal vector $\vec{p_i}$, the equation of the plane that crosses the origin of the coordinates: (Fernández-de Arriba et al., 2013)

$$p_{ix}.x+p_{iy}.y+p_{iz}.z = 0 \qquad (3)$$

Intersecting the three planes in pairs yields the axes ($\vec{u}, \vec{v}, \vec{w}$) that determine the sets:

$$\vec{u}=\frac{\overrightarrow{p_3}x\overrightarrow{p_1}}{|\overrightarrow{p_3}x\overrightarrow{p_1}|}; \vec{v}=\frac{\overrightarrow{p_1}x\overrightarrow{p_2}}{|\overrightarrow{p_1}x\overrightarrow{p_2}|}; \vec{w}=\frac{\overrightarrow{p_2}x\overrightarrow{p_3}}{|\overrightarrow{p_2}x\overrightarrow{p_3}|} \qquad (4)$$

The parallelepiped formed by the three sets (considering their corresponding spacings S_1, S_2, and S_3) is obtained by measuring the distances (d_u, d_v, d_w) from the origin of the coordinate to each direction that defines the axis formed by the sets: (Fernández-de Arriba et al., 2013).

$$d_u=\frac{S_2}{|\overrightarrow{p_2}.\vec{u}|}; d_v=\frac{S_3}{|\overrightarrow{p_3}.\vec{v}|}; d_w=\frac{S_1}{|\overrightarrow{p_1}.\vec{w}|} \qquad (5)$$

Finally, the volume of the parallelepiped (V_p) is determined by the modulus of the scalar triple product: (Fernández-de Arriba et al., 2013).

$$V_p = \left|\left[(d_u.\vec{u})x\,(d_v.\vec{v})\right].(d_w.\vec{w})\right| \qquad (6)$$

From the equations above, it is easy to calculate the volume of the parallelepiped formed by three joint sets but it is difficult to visualize the change in shapes and sizes of stone blocks. Therefore, input data to implement to model and visualize stone blocks consists of joint set parameters: dip direction (α), dip (β), and spacing (S). To visualize the shape and size of stone blocks produced from three major joint sets, the paper employed the software 3DEC, based on Distinct Element Method (DEM) to model the stone blocks. From the output data of the software, we could visualize the model in three dimensions and measure all their size (Figure 4).

In the practice of stone bock cutting, stone blocks, assigned to the reverse to obtain a mining license, is more than 0.4 m^3. However, the blocks of

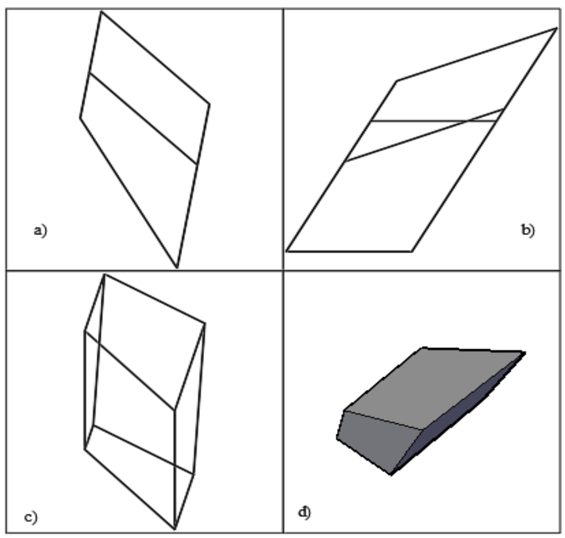


Figure 4. Stone block modelling in three dimensions generated by three major joint sets. a) Top view; b) Side view; c) Plan view; d) Perspective view.

0.4 m^3 for the plants for cutting the stone block into slabs that could be used must be rectangular parallelepiped with a minimum size of 1.2x0.6x0.6 m (0.4 m^3). Due to changes in joint set parameters, the shape of stone blocks is always parallelepipeds, not ensuring the standard size for the plant. Therefore, it is significantly necessary to convert the shape of the actual stone block to a rectangular parallelepiped. The parameters of the rectangular parallelepiped in 3D and its volume could be given in the Equations from 7 to 15 and Figure 5.

1. Calculating according to spacing S_1 of the first joint set.

$$S_{20} = \frac{S_2}{sin(\beta_{12})} - \frac{S_1}{tg(\beta_{12})}, m \quad (7)$$

$$S_{30} = \frac{S_3}{sin(\beta_{13})} - \frac{S_1}{tg(\beta_{13})}, m \quad (8)$$

$$V_1 = S_1 x S_{20} x S_{30},\ m^3 \quad (9)$$

2. Calculating according to spacing S_2 of the second joint set.

$$S_{10} = \frac{S_1}{sin(\beta_{12})} - \frac{S_2}{tg(\beta_{12})}, m \quad (10)$$

$$S_{30} = \frac{S_3}{sin(\beta_{23})} - \frac{S_2}{tg(\beta_{23})}, m \quad (11)$$

$$V_2 = S_{10} x S_2 x S_{30},\ m^3 \quad (12)$$

3. Calculating according to spacing S_3 of the third joint set.

$$S_{10} = \frac{S_1}{sin(\beta_{13})} - \frac{S_3}{tg(\beta_{13})}, m \quad (13)$$

$$S_{20} = \frac{S_2}{sin(\beta_{23})} - \frac{S_3}{tg(\beta_{23})}, m \quad (14)$$

$$V_3 = S_{10} x S_{20} x S_3,\ m^3 \quad (15)$$

In which: S_1, S_2, S_3 are spacing of joint set 1, 2, and 3, respectively, m; S_{10}, S_{20}, S_{30} are sized in three dimensions of a rectangular parallelepiped in the parallelepiped, respectively, m; β_{12}, β_{23}, β_{13} are angles between pairs of joint-set planes 1 and 2, joint-set planes 2 and 3, joint-set planes 3 and 1, respectively, degree; V_1, V_2, V_3 are volume of rectangular parallelpiped according to spacing S_1, S_2 and S_3, respectively, m^3.

The volume of the rectangular parallelepiped that could be recovered is the maximum value in the three volumes V_1, V_2, and V_3 suggested above. From the volume and size of the parallelepiped and internal rectangular parallelepiped, the recovery ratio could be determined by a ratio between the volume of the internal rectangular parallelepiped and one of the parallelepipeds outside. Through the ratio, we could assess joint sets with which parameters on the dip, dip direction, and spacing the quarry bring back more effectiveness.

Moreover, when knowing the size of the rectangular parallelepiped following the demand of the processing plant or that of the minimum rectangular parallelepiped (0.4 m^3), the limit spacing of joint in a set could be given as Equations below where stone with the spacing of equal or more than the limit will be estimated as dimension stone reserve.

1. Calculating according to spacing $S_1 = S_{10}$ of the first joint set.

$$S_2 = (S_{20} tg(\beta_{12}) + S_1) cos(\beta_{12}), m \quad (16)$$

$$S_3 = (S_{30} tg(\beta_{13}) + S_1) cos(\beta_{13}), m \quad (17)$$

2. Calculating according to spacing $S_2 = S_{20}$ of the second joint set.

$$S_1 = (S_{10} tg(\beta_{12}) + S_2) cos(\beta_{12}), m \quad (18)$$

$$S_3 = (S_{30} tg(\beta_{23}) + S_2) cos(\beta_{23}), m \quad (19)$$

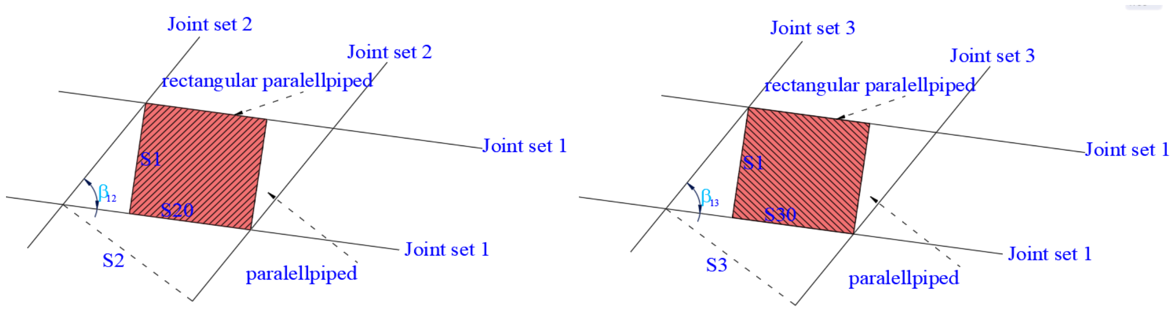


Figure 5. Drawing to define size and volume of rectangular paralellpiped from three joint sets.

3. Calculating according to spacing S_3=S_{30} of the second joint set.

$$S_1 = (S_{10}tg(\beta_{13}) + S_3)cos(\beta_{13}), m \qquad (20)$$

$$S_2 = (S_{20}tg(\beta_{23}) + S_2)cos(\beta_{23}), m \qquad (21)$$

Calculation following spacing S_1 or S_2 or S_3 depends on the recovery ratio of rectangular parallelepiped reaching the maximum value regarding which spacing is chosen to calculate for rectangular parallelepiped of equal or more than 0.4 m^3.

3. Case Study

The study was carried out at three stone quarries in the South Central Coast area of Vietnam, including Phu Yen, Binh Dinh, and Khanh Hoa Provinces. The area is a place with big dimensional stone quarries, and most of the stone in the area is listed in the granite group. The quality was described through lithology components (quartz, plagioclase, K, Mica), chemical components (SiO_2, Fe_2O_3, Al_2O_3, SO_2), mechanics properties (unit weight, saturated uniaxial compressive strength, saturated uniaxial tensile strength, coefficient of variation), gloss.

3.1. Quarry location

Cay Sung 4 quarry is extracted in the Northwestern sidehill of the summit of 347 m, belonging to Dien Tan commune, Dien Khanh district, Khanh Hoa province. The quarry is far about 10km from the central Dien Khanh district in the West - Southwest direction and far about 20 km from western Nha Trang City.

Tan Long quarry is situated at Trai Mountain, Cat Hung commune, Phu Cat district, Binh Dinh province. The quarry is far about 35 km from Northern Quy Nhon City, and far about 12 km from Eastern Ngo May town, and about 1 km from the Northeastern headquarter of Cat Hung's committee.

Hoa Quang Bac quarry is situated at Hoa Quang Bac commune, Phu Hoa district, Phu Yen province. The quarry is far about 16 km Northwestern Tuy Hoa City, about 17 km Western Hoa Da T-intersection of High Way No1.

3.2. Joint description

Cay Sung 4 quarry: Through fracture measures at the field, the quarry existed three major joint sets with their dip directions and dips of $220^0\angle 80^0$, $160^0\angle 70^0$, and $260^0\angle 60^0$. The spacings of the sets change from 0.4÷7.6 m. The analyzed results of the sets are implemented with the software Dips and shown in Figure 6.

Tan Long quarry: Through fracture measures at the field, the quarry existed three major joint sets with their dip directions and dip of $70^0\angle 80^0$, $190^0\angle 80^0$ and $35^0\angle 80^0$. The spacings of the sets changed from 0.4÷3 m. The analyzed results of the sets are implemented with the software Dips and shown in Figure 7.

Hoa Quang Bac quarry: Through fracture measures at the field, the quarry existed three major joint sets with their dip directions and dip of $20^0\angle 70^0$, $250^0\angle 60^0$, and $280^0\angle 53^0$. The spacings of the sets changed from 0.4÷4 m. The analyzed results of the sets are implemented with the software Dips (Rocscience Inc, 2016) and shown in Figure 8.

3.3. Discontinuity Modelling

Discontinuity modelling was established for stone blocks generated from three major joint sets

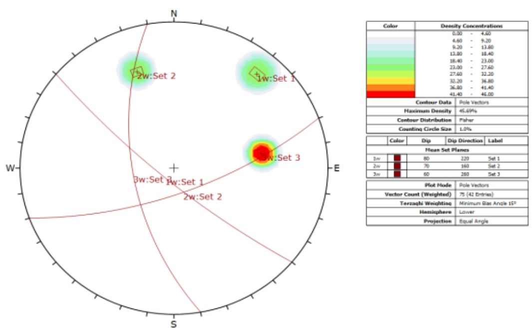


Figure 6. Analysis and representation of joint sets at Cay Sung 4 quarry.

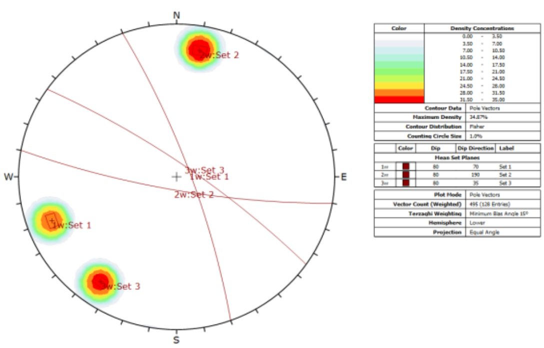


Figure 7. Analysis and representation of joint sets at Tan Long quarry.

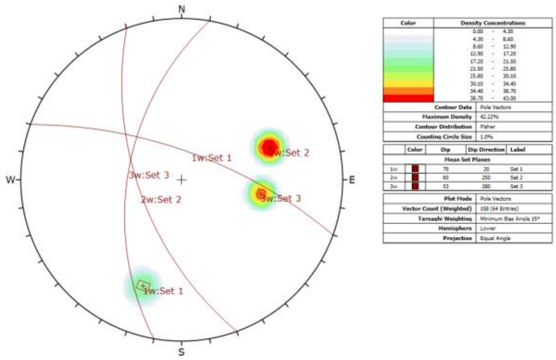
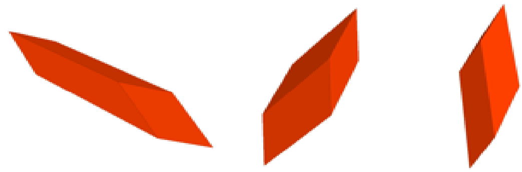


Figure 8. Analysis and representation of joint sets at Hoa Quang Bac quarry.

at the three quarries, and the modelling was built with input data including dips, dip directions and spacings of joint sets, with the software 3DEC (Itasca, 2019). To show the minimum rectangular parallelepiped of 0.4 m^3 and its size of 1.2x0.6x0.6 m the plant of dimensional stone could suitably process, the paper selected spacings S_1, S_2, S_3 for respective joint sets 1, 2, 3. For each modelling, the spacing for each joint set changes in values of 1.2 m, 0.6 m, and 0.6 m to show a model with a specific size and shape of stone block, shown in Figures 9, 10, and 11. After a completed modelling, geometric parameters of the stone block were applied to calculate the volume of the rectangular parallelepiped inside.

4. Results and discussions

Discontinuity modelling from three major joint sets is represented in Figures 9, 10, and 11. Output



(a) Case1: 1.2x0.6x0.6 m (b) Case2: 0.6x1.2x0.6 m (c) Case3: 0.6x0.6x1.2 m

Figure 9. Stone block modelling at Cay Sung 4 quarry.

a.Case1: 1.2x0.6x0.6 m b.Case2: 0.6x1.2x0.6 m c.Case 3: 0.6x0.6x1.2 m

Figure 10. Stone block modelling at Tan Long quarry.

a.Case1: 1.2x0.6x0.6 m b.Case2: 0.6x1.2x0.6 m c.Case3: 0.6x0.6x1.2 m

Figure 11. Stone block modelling at Hoa Quang Bac quarry.

from the modelling and calculations following Equations from 7 to 15 are shown in Table 1.

Table 1. Output from the modelling and calculations

No	Quarries	Dip, degree	Dip Direction, Degree	Spacing, m	Parallelpined volume, m^3	Angles between joint set planes, degree	Size of Rectangular parallelepiped, m	Rectangular parallelepiped volume, m^3	Recovery ratio,%
I	Cay Sung 4								
1	*Case 1*				0.96			0.40	41.67
-	Set 1	80	220	1.2		β_{12}=39.8(140.2)	S_{10}= 1.15		
-	Set 2	70	160	0.6		β_{23} = 88.30(91.70)			
-	Set 3	60	260	0.6		β_{13} =42.26(137.74)	S_{30}= 0.58		
2	*Case 2*				0.96			0.16	16.6
-	Set 1	80	220	0.6		β_{12}=39.8(140.2)			
-	Set 2	70	160	1.2		β_{23} = 88.30(91.70)	S_{20} =1.15		
-	Set 3	60	260	0.6		β_{13} =42.26(137.74)	S_{30} =0.23		
3	*Case 3*				0.96				
-	Set 1	80	220	0.6		β_{12}=39.8(140.2)	S_{10} No exist		
-	Set 2	70	160	0.6		β_{23} = 88.30(91.70)	S_{20} =0.56		
-	Set 3	60	260	1.2		β_{13}=42.26(137.74)			

II	Tan Long								
1	*Case 1*				2.5			0.107	4.3
-	Set 1	80	70	1.2		β_{12}=62.95(117.05)	S_{10}=1.04		
-	Set 2	80	190	0.6		β_{23}=31.92(148.08)			
-	Set 3	80	35	0.6		β_{31}=34.45(145.55)	S_{30}=0.17		
2	*Case 2*				2.5			0.145	5.8
-	Set 1	80	70	0.6		β_{12}=62.95(117.05)			
-	Set 2	80	190	1.2		β_{23}=31.92(148.08)	S_{20}=1.30		
-	Set 3	80	35	0.6		β_{31}=34.45(145.55)	S_{30}=0.186		
3	*Case 3*				2.5				
-	Set 1	80	70	0.6		β_{12}=62.95(117.05)	S_{10} No exist		
-	Set 2	80	190	0.6		β_{23}=31.92(148.08)	S_{20} No exist		
-	Set 3	80	35	1.2		β_{31}=34.45(145.55)			
III	Hoa Quang Bac								
1	*Case 1*				3.8			0.049	1.3
-	Set 1	70	20	1.2		β_{12}=69.4(110.6)	S_{10}=1.02		
-	Set 2	60	250	0.6		β_{23}=25.86(154.14)			
-	Set 3	53	280	0.6		β_{31}=85.67(94.33)	S_{30}=0.069		
2	*Case 2*				3.8			0.35	9.2
-	Set 1	70	20	0.6		β_{12}=69.4(110.6)			
-	Set 2	60	250	1.2		β_{23}=25.86(154.14)	S_{20}=1.056		
-	Set 3	53	280	0.6		β_{31}=85.67(94.33)	S_{20}=0.55		
3	*Case 3*				3.8				
-	Set 1	70	20	0.6		β_{12}=69.4(110.6)	S_{10}=0.51		
-	Set 2	60	250	0.6		β_{23}=25.86(154.14)	S_{20} No exist		
-	Set 3	53	280	1.2		β_{31}=85.67(94.33)			

Table 1 shows that with three joint sets of Cay Sung 4 quarry, the volume of the parallelepiped is 0.96 m^3, but the volume of the rectangular parallelepiped inside maximizes by 0.4 m^3. However, with three joint sets of Tan Long quarry, the volume of the parallelepiped is 2.5 m^3, but the volume of the rectangular parallelepiped inside maximizes 0.145 m^3. Moreover, with three joint sets of Hoa Quang Bac quarry, the volume of the parallelepiped reaches 3.8 m^3, but the volume of the rectangular parallelepiped inside only maximizes 0.35 m^3. When changing in spacings each other for joint sets in a quarry, the volume of stone blocks is similar. For example, the volumes for Cay Sung 4 quarry, Tan Long quarry, and Hoa Quang Bac quarry are 0.96 m^3, 2.5 m^3, and 3.8 m^3, respectively, and the volumes are more than 0.4 m^3, but the volumes of the rectangular parallelepipeds, the plant could process, are only 0.4 m^3, 0.145 m^3, and 0.35 m^3. Generally, the maximum recovery ratios for Cay Sung 4, Tan Long, and Hoa Quang Bac quarries are 41.67%, 5.8%, and 9.2%, respectively. This shows that joint-set orientation in Cay Sung 4 quarry creates more favorable compared with that in the rest quarries. Moreover, to recover the rectangular blocks of more than 0.4 m^3 with their minimum size of 1.2x0.6x0.6 m, the limit spacing in a set is also defined based on Equations from 16÷21 for Cay Sung 4, Tan Long, and Hoa Quang Bac quarries, shown in Table 2 and Figures 12÷14.

Table 2. Spacings of joint sets for Cay Sung 4, Tan Long, and Hoa Quang Bac quarries.

No	Quarries	Dip, degree	Dip Direction, Degree	Size of Rectangular parallelepiped, m	Angles between joint set planes, degree	Spacing, m
I	Cay Sung 4					
1	*Case 1*					
-	Set 1	80	220	S_{10}= 1.2	β_{12}=39.8(140.2)	S_1= 1.23
-	Set 2	70	160	S_{20}= 0.6	β_{23} = 88.30(91.70)	S_2=0.6
-	Set 3	60	260	S_{20}= 0.6	β_{13}=42.26(137.74)	S_3= 0.63
II	Tan Long					

2	*Case 2*					
-	Set 1	80	70	S_{10}= 0.6	β_{12}=62.95(117.05)	S_1= 0.6
-	Set 2	80	190	S_{20}= 1.2	β_{23}=31.92(148.08)	S_2 =1.34
-	Set 3	80	35	S_{30}= 0.6	β_{31}=34.45(145.55)	S_3 =0.83
III	Hoa Quang Bac					
3	*Case 2*					
-	Set 1	70	20	S_{10}=0.6	β_{12}=69.4(110.6)	S_1= 0.6
-	Set 2	60	250	S_{20}=1.2	β_{23}=25.86(154.14)	S_2= 1.33
-	Set 3	53	280	S_{30}=0.6	β_{31}=85.67(94.33)	S_3= 0.64

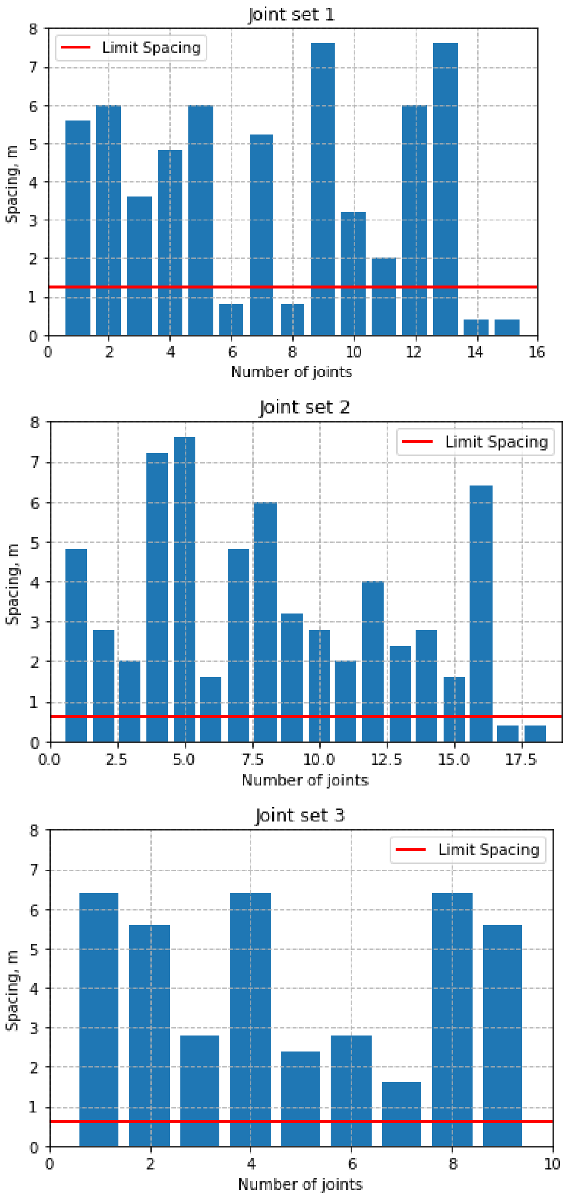


Figure 12. Limit spacing for dimension stone at Cay Sung 4 quarry.

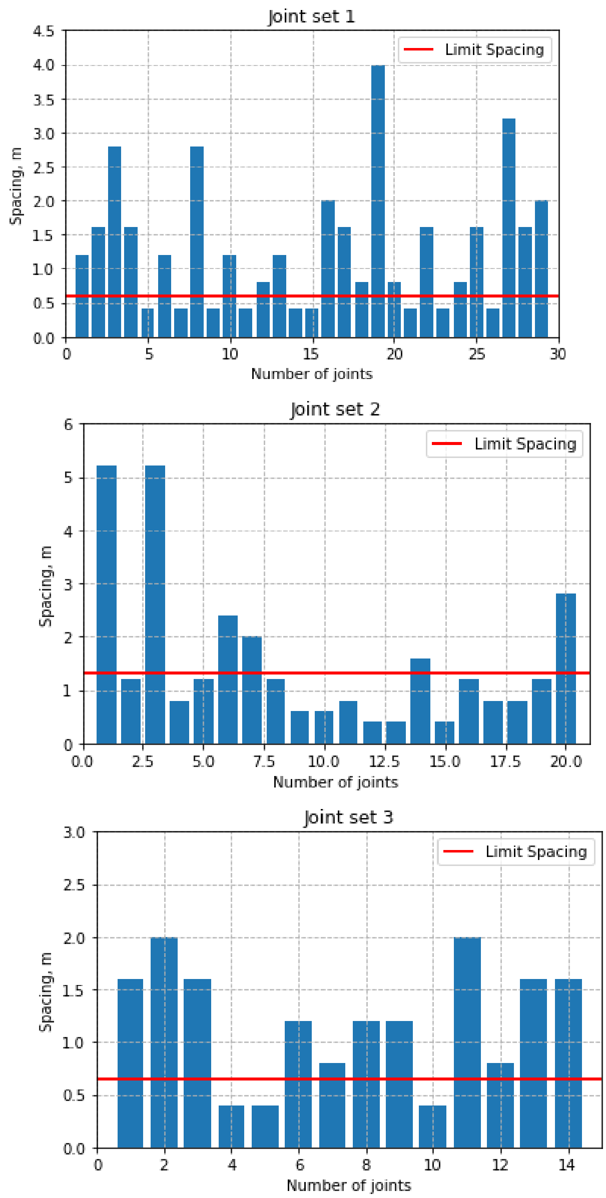


Figure 13. Limit spacing for dimension stone at Hoa Quang Bac quarry.

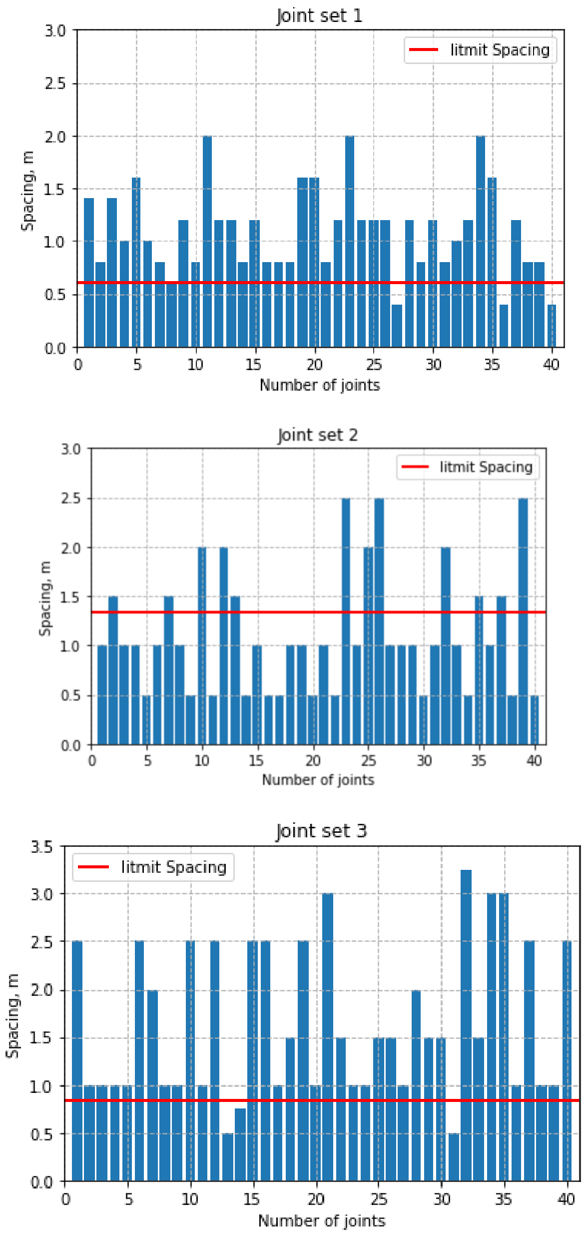


Figure 14. Limit spacing for dimension stone at Tan Long quarry.

From Table 2 and Figures 12÷14 among the quarries, most spacings in the Cay Sung quarry are longer than the limit spacing for rectangular blocks of 0.4 m^3. The vast number of joint set 2 with their spacings at Tan Long quarry is lower than the limit spacing. This shows Cay Sung quarry has the highest recovery ratio of a rectangular parallelepiped, while that of Tan Long quarry is the lowest. The results complement stone sizes assigned into the reserve when assessing all the geometry parameters of joint sets to the size of 0.4, m^3 the processing plant could use, while the reserve is being used for the size of equal or more than 0,4 m^3, which shape is just a parallelepiped.

5. Conclusions

By discontinuity modelling, we will define the volume and shape of the stone block formed by three major joint sets in quarries. The volume and shape depend on not only the spacing of the joint in a set but also the dip, dip direction of a joint set, and spatial relationship of joint sets in a quarry. When these parameters are applied, the recovery ratio following the required size of the processing plant will be calculated, and spacings longer than the limit spacing in each joint set at each quarry will be deployed to estimate dimension stone reserves. Therefore, determining the stone block volume of equal and more than 0.4 m^3 to estimate the reserves for quarries is not exact, and it is necessary to update all the geometry parameters of joint sets to estimate the reserves.

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Contribution of authors

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