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PAPER

Excitonic properties in a double-layer graphene

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Abstract

This paper investigates theoretically the excitonic condensation state at zero temperature in a double-layer graphene structure. In the framework of the unrestricted Hartree–Fock approximation, the electron-hole system in the structure described in the two-band electronic model is analyzed and one finds a set of self-consistent equations determining the excitonic order parameter. The optical properties of the excitonic condensation state then are examined in the Kubo linear optical response theory. Our results indicate that in the case of sufficiently large Coulomb interaction, the BEC excitonic condensation state might occur at low electronic excitation density. By turning the external electric field, the superfluid state stabilizes in the BCS-type excitonic condensate. The optical conductivity spectrum also provides us more insight into the excitonic condensation states.

1. Introduction

Even proposed for more than half of a century [1, 2], excitonic condensation stability still remains one of the most challenging and controversial problems in condensed matter physics. In a semiconducting or a semimetal material, an electron might couple to a hole to originate a bosonic quasi-particle namely exciton. At low temperatures, a macroscopic coherent state might be established by the condensation of these excitons if their density is sufficiently large, following the Bose–Einstein condensation (BEC) theory [3, 4]. In some senses, the coherent bound state of the excitons is similar to the superfluid state of the Cooper pairs described in the microscopic Bardeen-Cooper-Schrieffer (BCS) theory [5]. However, if the condensation of the Cooper pairs is the superconducting state, i.e., the electric resistance is completely zero, the excitonic condensation state is the insulating or non-conducting state. The excitonic condensate is thus sometimes called an excitonic insulator state [3]. Even predicted for a long time, excitonic condensate is still rarely observed experimentally so far.

To be observable of the excitonic condensation state experimentally, a sufficiently large number of long-live excitons is required. In a real material, an exciton is unstable against the recombination of the close proximity electron and hole. Excitons in a bulk semiconductor or semimetal have thus a very short lifetime. That is a reason the excitonic condensate is rarely observed as mentioned above. However, by placing the electron and hole spatially separated by an insulating barrier, the exciton might live longer [6, 7]. The excitonic condensation state, therefore, is effectually observed in a double-layer system (DLS). One of the most interesting DLSs is the double-layer graphene (DLG) where the two monolayers are graphene sheets [8, 9]. If an external electric field is applied to the two layers, excitons might be originated due to a couple of electrons in one layer and holes in the opposite layer by the Coulomb interaction. The excitonic condensation state in the DLG structure thus might appear if the exciton density is sufficiently large and the temperature is low enough [10–13]. In the studies, the excitonic condensation state is triggered only in the BCS-type [11, 14–16]. The BEC-type and BCS-BEC crossover of the excitonic condensation state thus have not been examined yet. However, in the unbiased case, the electron conduction band and hole valance band in DLG meet each other at the Dirac points. In this case, the DLG displays a zero-gap semiconductor character similar to a one-dimensional TMD Ta_2NiSe_5 in which the BCS-BEC crossover of the excitonic condensation has been experimentally observed [17, 18]. Studying the BEC-type

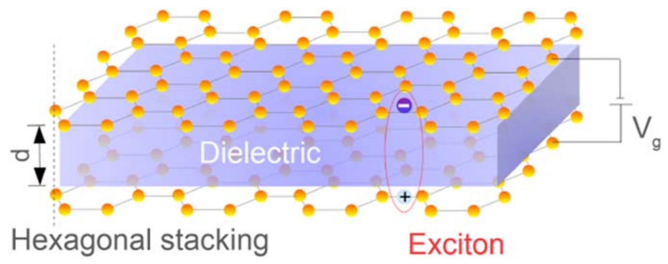

Figure 1. Schematic structure of double-layer graphene in hexagonal stacking system separated by a dielectric thickness d with electron and hole carriers induced by external gate V_g forming excitonic bound states due to their Coulomb attraction.

and also BCS-BEC crossover of the excitonic condensation state in the DLG is thus extremely important. In the present work, the tasks are considered by the use of an unrestricted Hartree–Fock (UHF) approximation to examine the general two-band electronic model applied for the structure of the two-graphene sheets. In the framework of the UHF approach, we find a set of self-consistent equations determining the excitonic condensate order parameter. The excitonic condensation state in the system thus would be explicitly inspected.

To investigate in more detail the excitonic condensation state, in the present study, we also consider the optical response once the system stabilizes in the condensation state. The optical response is examined in the meaning of the optical conductivity based on the Kubo linear response theory [19]. With the help of the UHFA, the real part of the optical conductivity is explicitly evaluated. Analyzing the optical conductivity spectrum also gives us a signature of the excitonic bound state, the hybridization features driven by the Coulomb interaction between electrons and holes in the different sheets in DLG [13].

We organize the paper as follows. In section 2, we present a microscopic Hamiltonian describing the electron-hole system in the DLG based on the low-energy electronic two-band model involving the Coulomb interaction. Section 3 briefly addresses the UHF approximation applied for the model mentioned above in section 2. The numerical results and discussions are left in section 4. Finally, section 5 ends the paper.

2. Microscopic Hamiltonian

In the present work, we consider a DLG structure fabricated by two graphene sheets separated by a dielectric thickness d (see figure 1). The two graphene sheets are hexagonal stacking in which each sublattice in one layer is on top of the corresponding sublattice in the other layer [20]. An external volgate V_g is applied between the two layers inducing the external electric field and then the potential difference.

To describe the electron-hole correlation in the DLG we use the following microscopic Hamiltonian written in momentum space

$$\begin{aligned}\mathcal{H} = &\sum\nolimits_{\mathbf{k}} (\varepsilon_{\mathbf{k}}^{+} e_{\mathbf{k}}^{\dagger} e_{\mathbf{k}} + \varepsilon_{\mathbf{k}}^{-} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}})\\ &- \frac{1}{N}\sum\nolimits_{\mathbf{k}_1\mathbf{k}_2,\mathbf{q}\neq 0} U_{\mathbf{k}_1\mathbf{k}_2\mathbf{q}}\, e_{\mathbf{k}_1+\mathbf{q}}^{\dagger} e_{\mathbf{k}_1} h_{\mathbf{k}_2-\mathbf{q}}^{\dagger} h_{\mathbf{k}_2},\end{aligned} \tag{1}$$

where $e_{\mathbf{k}}^{(\dagger)}$ and $h_{\mathbf{k}}^{(\dagger)}$ are the annihilation (creation) operators of electron and hole with momentum $\mathbf{k}$, respectively. The first term in the Hamiltonian expresses the non-interacting electron-hole system with respect to the band dispersions read

$$\varepsilon_{\mathbf{k}}^{+/-} = \gamma_0\left[1 + 4\cos^2\frac{k_y}{2} + 4\cos\frac{k_y}{2}\cos\frac{\sqrt{3}\,k_x}{2}\right]^{1/2} - \mu. \tag{2}$$

Here, $\gamma_0 \simeq 2.8$ eV is the nearest-neighbor hopping integral [21]. Note here that the electron-hole system in the DLG has been approximately described in the two-band models so filled valence and empty conduction bands of the upper and lower layers, respectively, have been neglected [20]. The simplification is applicable in the low-biased situation. In equation (2), μ is the chemical potential. Zero-chemical potential $\mu = 0$ indicates the unbiased case, so the two bands of conduction and valance electrons touch each other at the K points. The chemical potential can be tuned by the external electric field E_{ext} induced from the gate-voltage V_{g} $[E_{\text{ext}} = V_{\text{g}}/(ed)]$, $\mu = E_{\text{ext}}ed/2$ [22]. The last term in equation (1) indicates the Coulomb interaction between the conduction electrons in the upper layer and the valance-holes electrons in the lower layer. In the momentum space, it reads

$$U_{\mathbf{k}_1\mathbf{k}_2\mathbf{q}} = \kappa \frac{e^{-d|\mathbf{q}|}}{|\mathbf{q}|} \cos\frac{\phi_1}{2} \cos\frac{\phi_2}{2}, \tag{3}$$

where $\phi_i = \theta_{\mathbf{k}_i} - \theta_{\mathbf{k}_i+\mathbf{q}}$ is a scattering angle with $\theta_{\mathbf{k}} = \mathrm{atan}(k_y/k_x)$ [23]. Note here that the $\mathbf{q} = 0$ component comprising the jellium background has been excluded from our evaluation. Equation (3) clearly expresses that the long-ranged Coulomb interaction rapidly suppresses by a transferred momentum $\mathbf{q}$. That is completely different from the localized situation as assumed for TMD in the features of the extended Falicov-Kimball model [24–26]. The strength of the Coulomb interaction depends on the distance d between the two sheets and also on the embedding dielectric medium illustrated through a factor κ defined as

$$\kappa = g_s \frac{2\pi e^2}{\epsilon}, \tag{4}$$

with ϵ is the dielectric constant of the space embedding between the two graphene sheets. Changing the distance and also the dielectric constant might give us a complicated signature of the correlation picture in the DLG. In the present work, a ground state competition of the excitonic condensation stabilities will be examined in the influence of the Coulomb interaction through the factor κ and dielectric thickness d. In the Hamiltonian (1), the spin degeneracy has been neglected.

3. Unrestricted Hartree–Fock approximation

This section addresses the application of an unrestricted Hartree–Fock (UHF) approximation adapting to the microscopic Hamiltonian in equation (1) to investigate the excitonic condensation state in DLG. In the UHF approach, decoupling with respect to the off-diagonal expectation values is allowed, meaning that the hybridization between the conduction electron on one layer and valence hole on the opposite layer might be considered. Then a solution of the spontaneous symmetry breaking field is thus possibly achieved. By leaving out all fluctuation parts, an effective UHF Hamiltonian is delivered as

$$\mathcal{H}_{\mathrm{UHF}} = \sum_{\mathbf{k}} [\varepsilon_{\mathbf{k}}^{+} e_{\mathbf{k}}^{\dagger} e_{\mathbf{k}} + \varepsilon_{\mathbf{k}}^{-} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} + (\Delta_{\mathbf{k}} h_{\mathbf{k}} e_{\mathbf{k}} + \mathrm{H.c.})], \tag{5}$$

where all additional constants have been neglected. In the above equation, we have defined

$$\Delta_{\mathbf{k}} = -\frac{\kappa}{N} \sum_{\mathbf{q}} \frac{e^{-d|\mathbf{q}|}}{|\mathbf{q}|} \frac{(1 + \cos\phi)}{2} \delta_{\mathbf{k}+\mathbf{q}}, \tag{6}$$

due to the off-diagonal coupling driven by the Coulomb interaction, indicating a spontaneous symmetry breaking due to the formation of an electron-hole pair state. In equation (6), $\phi = \theta_{\mathbf{k}+\mathbf{q}} - \theta_{\mathbf{k}}$ and $\delta_{\mathbf{k}} = \langle e_{\mathbf{k}}^{\dagger} h_{\mathbf{k}}^{\dagger} \rangle$. In this sense, one can consider both $\Delta_{\mathbf{k}}$ and $\delta_{\mathbf{k}}$ as the excitonic condensate order parameters.

To proceed with our further calculation, we use a Bogoliubov transformation to diagonalize the Hamiltonian written in equation (5), which results

$$\mathcal{H}_{\mathrm{dia}} = \sum_{\alpha=\pm} \tilde{E}_{\mathbf{k}}^{\alpha} \tilde{c}_{\mathbf{k}\alpha}^{\dagger} \tilde{c}_{\mathbf{k}\alpha}, \tag{7}$$

where

$$\tilde{E}_{\mathbf{k}}^{\pm} = \mp \frac{\mathrm{sgn}(\varepsilon_{\mathbf{k}}^{-} + \varepsilon_{\mathbf{k}}^{+})}{2} W_{\mathbf{k}}, \tag{8}$$

indicate the electronic quasiparticle energies with respect to new fermionic operators

$$\tilde{c}_{\mathbf{k}\alpha}^{\dagger} = \sum_{\beta=\pm} \tau_{\mathbf{k}}^{\alpha\beta} \hat{c}_{\mathbf{k}\beta}^{\dagger}. \tag{9}$$

Here, we have denoted $\hat{c}_{\mathbf{k}+}^{(\dagger)} = e_{\mathbf{k}}^{(\dagger)}$ and $\hat{c}_{\mathbf{k}-}^{(\dagger)} = h_{\mathbf{k}}^{(\dagger)}$ for the original annihilation (creation) operators of the conduction and valance electrons. The prefactors $\tau_{\mathbf{k}}^{\alpha\beta}$ are

$$\tau_{\mathbf{k}}^{++} = -\tau_{\mathbf{k}}^{--} = \frac{1}{2}\sqrt{1 + \Gamma_{\mathbf{k}}}$$
$$\tau_{\mathbf{k}}^{+-} = \tau_{\mathbf{k}}^{-+} = \frac{1}{2}\sqrt{1 - \Gamma_{\mathbf{k}}}, \tag{10}$$

where

$$\Gamma_{\mathbf{k}} = \mathrm{sgn}(\varepsilon_{\mathbf{k}}^{-} + \varepsilon_{\mathbf{k}}^{+}) \frac{\varepsilon_{\mathbf{k}}^{-} + \varepsilon_{\mathbf{k}}^{+}}{W_{\mathbf{k}}} \tag{11}$$

and

$$W_{\mathbf{k}} = \sqrt{(\varepsilon_{\mathbf{k}}^{+} + \varepsilon_{\mathbf{k}}^{-})^2 + 4|\Delta_{\mathbf{k}}|^2}\,. \tag{12}$$

From the diagonal form of equation (7), one can easily evaluate the expectation value $\delta_{\mathbf{k}}$ in equation (6), which reads

$$\delta_{\mathbf{k}} = -[f^F(\tilde{E}_{\mathbf{k}}^{+}) - f^F(\tilde{E}_{\mathbf{k}}^{-})]\,\mathrm{sgn}(\varepsilon_{\mathbf{k}}^{-} + \varepsilon_{\mathbf{k}}^{+})\frac{\Delta_{\mathbf{k}}}{W_{\mathbf{k}}}. \tag{13}$$

Here $f(\tilde{E}_{\mathbf{k}}^{\alpha}) = 1/(1 + e^{\tilde{E}_{\mathbf{k}}^{\alpha}/T})$ has been used to define the Fermi function for temperature T. From equations (6) and (13) one finds a self-consistent equation so the excitonic condensate order parameters can be evaluated self-consistently.

With the results of the UHF approximation addressed above, the optical properties in the system might be discussed by evaluating the optical conductivity. The real part of the optical conductivity in the framework of the Kubo linear optical response theory can be defined as [19]

$$\sigma(\omega) = \mathrm{Re}\frac{1}{\omega}\int_0^{\infty} dt e^{i\omega t}\langle[\hat{j}(t), \hat{j}(0)]\rangle. \tag{14}$$

$\hat{j}(t)$ in equation (14) is the current operator. With respect to the Hamiltonian written in equation (1), one has

$$\hat{j}(t) = \sum_{\mathbf{k},\alpha} v_{\mathbf{k}}^{\alpha}\hat{c}_{\mathbf{k}\alpha}^{\dagger}(t)\hat{c}_{\mathbf{k}\alpha}(t), \tag{15}$$

with $v_{\mathbf{k}}^{\alpha} = \nabla\varepsilon_{\mathbf{k}}^{\alpha}$. The average in equation (14) is formed with the original Hamiltonian, however, in the UHF approximation, it can be taken following the diagonal Hamiltonian in equation (7), then one easily arrives

$$\sigma(\omega) = \frac{1}{\omega}\Sigma_{\mathbf{k}\beta\beta',\,\alpha\alpha'}\, \mathrm{v}_{\mathbf{k}}^{\alpha}\mathrm{v}_{\mathbf{k}}^{\alpha'} u_{\mathbf{k}}^{\alpha\beta} u_{\mathbf{k}}^{\alpha\beta'} u_{\mathbf{k}}^{\alpha'\beta'} u_{\mathbf{k}}^{\alpha'\beta} \times [f(\tilde{E}_{\mathbf{k}}^{\beta}) - f(\tilde{E}_{\mathbf{k}}^{\beta'})]\delta(\omega + \tilde{E}_{\mathbf{k}}^{\beta} - \tilde{E}_{\mathbf{k}}^{\beta'}). \tag{16}$$

Here, the renormalized dispersions $\tilde{E}_{\mathbf{k}}^{\beta}$ and $\tilde{E}_{\mathbf{k}}^{\beta'}$ have been defined in equation (8) with β or β' is either $+$ or $-$.

4. Numerical results

From equations (6) and (13), a solution of the excitonic condensate order parameters can be found by a numerical method. Starting from an initial value of $\delta_{\mathbf{k}}$, the hybridization gap $\Delta_{\mathbf{k}}$ is evaluated via equation (6). The eigenenergies of the diagonalized Hamiltonian then are determined so the expectation value of $\delta_{\mathbf{k}}$ is recalculated. The iterative process can be stopped if one finds an achieved convergence. The ground state of the system is evaluated for zero temperature, i.e., at $T = 0$. In the present work, the numerical results are evaluated in the momentum space specified instead of the hexagonal Brillouin zone but by an equivalent triangular Brillouin given by vectors $\mathbf{b}_1 = (2\pi/3)(1, \sqrt{3})$ and $\mathbf{b}_2 = (2\pi/3)(1, -\sqrt{3})$ [13, 21].

To analyze the excitonic condensation state in the DLG structure, first of all, we show in figure 2 the band dispersions once the system stabilizes in the excitonic condensate due to the Coulomb interaction in comparison to the non-interacting case. In the whole first Brillouin zone, the band structures are illustrated for the unbiased ($E_{\mathrm{ext}} = 0$) and biased ($E_{\mathrm{ext}} = 1$) cases. For the unbiased case, the conduction electron and the valance hole bands touch each other at the Dirac points with zero density of states at the Fermi level. The band dispersion looks like that of the monolayer graphene [see figure 2(a)]. In that case, due to large Coulomb interaction, $\kappa = 12$ for instance, the hybridization between a small number of the conduction electrons in the upper layer and the valance holes in the lower layer sufficiently induces a gap opened around the K and K'-points of the band structure. The excitonic condensation state thus occurs even in an unbiased situation [see figure 2(b)]. The excitonic bound state in this case is completely driven by the strong Coulomb coupling for a very small density of states around the Fermi level. When the external electric field is finite due to applying the external gate-voltage, the chemical potential is non-zero and two non-interacting conduction and valance electron bands are overlapped [see figure 2(c)]. In this case, the density of electrons in the conduction band is enhanced and the same for the holes in the valance band. That develops the possibility of the coupling between the electrons and the holes to form an excitonic bound state. Due to the large Coulomb interaction, the hybridization is strong, and a large band gap is opened at the Fermi level [see figure 2(d)] [27]. In this situation, a large number of electrons and holes combine each other to form excitons around the Fermi level. At zero temperature, these excitons condense in the macroscopic coherent state called excitonic condensate.

The excitonic condensation state is also indicated by a non-zero value of $\delta_{\mathbf{k}} = \langle a_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}\rangle$ characterizing the density of the electron-hole pairs condensed in the coherent state. Apparently, it directly induces the energy gap $\Delta_{\mathbf{k}}$ in the band-dispersions due to the hybridization in equation (6). Figure 3 displays the momentum

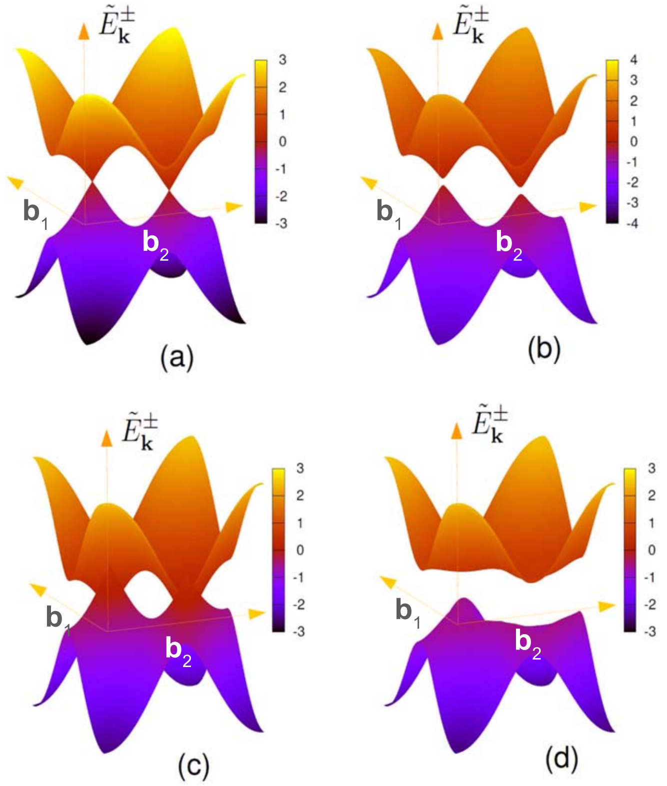

Figure 2. Band dispersions $\tilde{E}^{\pm}_{\mathbf{k}}$ in case of the non-interacting electron-hole system (left panels) and in the excitonic condensation state (right panels) in the whole first Brillouin zone specified by $\mathbf{b}_1$ and $\mathbf{b}_2$ of DLG for the unbiased case or the external electric field $E_{\mathrm{ext}} = 0$ [(a) & (b)] and for the biased situation with $E_{\mathrm{ext}} = 1$ [(c) & (d)]. Here, one chooses the dielectric thickness $d = 1$ and $\kappa = 12$.

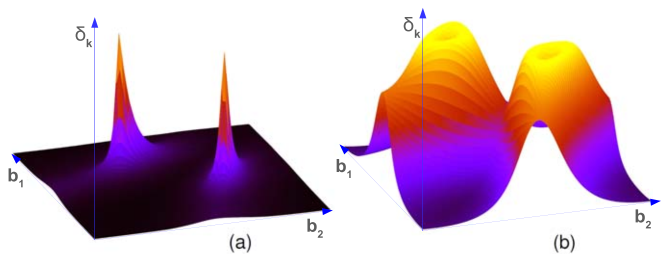

Figure 3. Excitonic condensate order parameter $\delta_{\mathbf{k}} = \langle a^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} \rangle$ in the whole first Brillouin zone specified by $\mathbf{b}_1$ and $\mathbf{b}_2$ of DLG with the dielectric thickness $d = 1$ and $\kappa = 12$ for $E_{\mathrm{ext}} = 0$ (a) and $E_{\mathrm{ext}} = 1$ (b).

distribution of $\delta_{\mathbf{k}}$ in the whole first Brillouin zone of DLG for the set of parameters set as in figure 2. For the unbiased case, figure 3(a) shows us that $\delta_{\mathbf{k}}$ is almost zero except at some momenta close to the K and K' points. In this case, the Fermi surface shrinks to one point. At the K and K' points, $\delta_{\mathbf{k}}$ get sharp peaks indicating that the excitonic condensation state typifies the condensation of a more local two-body BEC- like bound state. That is completely different from the biased case in figure 3(b) in which the $\delta_{\mathbf{k}}$ gets peaks at momenta deviating from the K and K' points. In the biased situation, two non-interacting bands of conduction and valance electrons overlap. The Fermi surface in this case plays an important role in establishing the formation of excitons like the Cooper pairs in the BCS theory for the superconducting state. In this situation, one specifies the excitonic condensate as the BCS type driven by the large Coulomb interaction. A detailed discussion about the excitonic condensate BCS-BEC crossover in DLG will be addressed below in figure 5.

To discuss the influence of the Coulomb interaction and external voltage on the excitonic condensation state in the DLG system, in figure 4 we show an external electric field dependence of $\delta = (1/N)\sum_{\mathbf{k}}\delta_{\mathbf{k}}$ for some values of κ once dielectric thickness $d = 1$. For a given value of κ, one always finds the excitonic condensate region expands by increasing the external electric field. For small and intermediate values of κ, $\kappa \leqslant 10$ for instance, the critical value of the external electric field is finite with respect to being biased. That makes sense because, at small or intermediate Coulomb interaction, the excitons might be formed only if the two non-interacting bands of

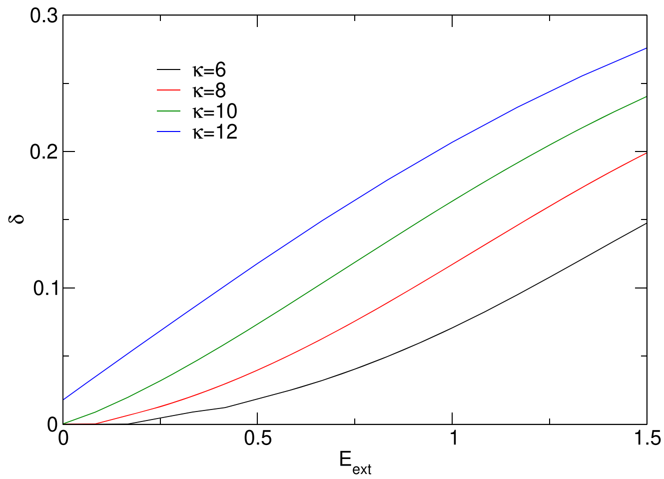

Figure 4. Excitonic condensate order parameter δ as a function of κ for some values of E_{ext} with $d = 1$.

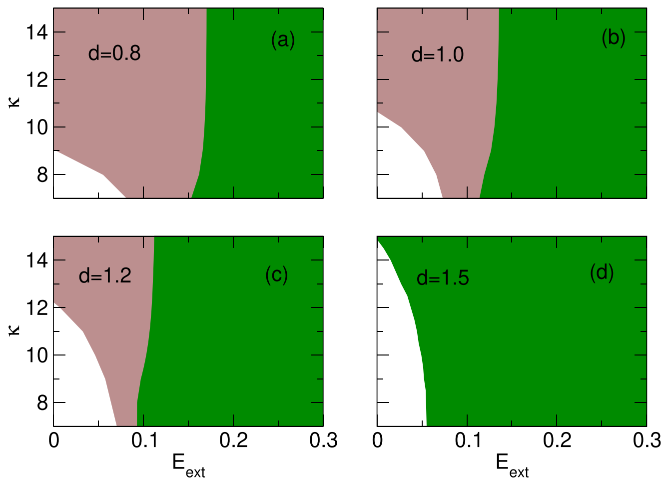

Figure 5. The excitonic condensate ground state phase diagram of the DLG in the (κ, E_{ext})-plane for some values d. The excitonic condensation state typifying either BCS- or BEC-type is indicated, respectively, by green or gray.

conduction and valance electrons overlap. Enlarging the overlap develops a probability of the excitons formation. If the Coulomb interaction is larger, $\kappa = 12$ for instance, one can find a non-zero value of δ even at zero external electric field $E_{\mathrm{ext}} = 0$ (see the blue line in figure 4). That indicates the excitonic condensate stability in the unbiased DLG as long as the Coulomb interaction is strong enough.

As mentioned before in equation (3) that the Coulomb interaction depends on both the factor κ (or the dielectric constant) and the distance d of a medium embedded between two sheets in DLG, considering the instability of the excitonic condensate as a function of κ and d thus is necessary. In figure 5, we show the excitonic condensate phase diagram at zero temperature in the (κ, E_{ext})-plane for some values of d. For a fixed d, one always finds a stabilized regime of the excitonic condensation state when the external electric field E_{ext} is large than a critical value E_{ext}^{c} that is suppressed by increasing the κ factor. If κ is large enough, the excitonic condensate can be found even at zero external electric field, or unbiased DLG system. For a given small dielectric thickness d, the excitonic condensation state typifies both the BCS (at large E_{ext}) and BEC (at small E_{ext}) types. The BEC regime spreads out both sides of E_{ext} axis by enlarging the factor κ or the Coulomb interaction. By increasing the Coulomb attraction, the excitonic BEC-BCS crossover thus shifts to the right. As in some semimetal-semiconductor transition materials, one finds the BEC excitonic condensation state in the ranges of small excitation charge density and large Coulomb interaction corresponding to small d, large κ, and low E_{ext}. Increasing d leads to the decrease of the Coulomb interaction, the BEC regime is thus suppressed, whereas, the BCS regime is expanded [see figure 5(a)–(c)]. Once the two sheets of the DLG are separated widely enough, i.e., d is sufficiently large, the Coulomb interaction would be small in the range of κ so one finds only the BCS excitonic condensation state [see figure 5(d)].

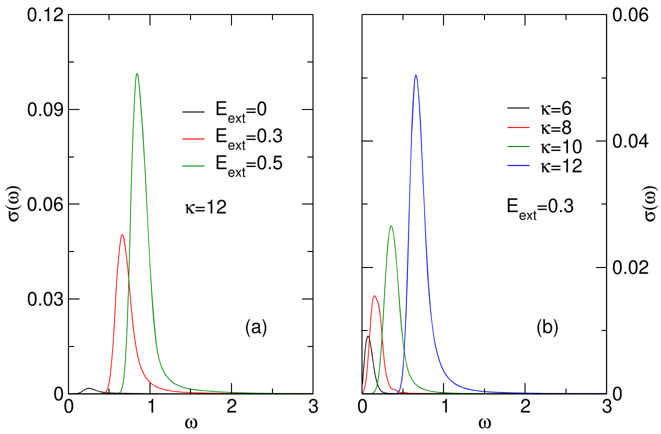

Figure 6. The real part of the optical conductivity $\sigma(\omega)$ in $d = 1$ DLG for some values E_{ext} with $\kappa = 12$ (left panel) and for some different values of κ with $E_{ext} = 0.3$ (right panel).

Last, in figure 6 we discuss the real part of the optical conductivity $\sigma(\omega)$ evaluated from equation (16) for the system with dielectric thickness $d = 1$. The left panel of figure 6 illustrates $\sigma(\omega)$ for $\kappa = 12$ with some different values of E_{ext}, whereas, the right panel gives that $\sigma(\omega)$ for $E_{ext} = 0.3$ by varying κ or strength of the Coulomb interaction. In all cases, one always finds that a peak appears in the optical conductivity spectrum, at a finite frequency ω_c. At a frequency $\omega < \omega_c$, the optical conductivity is almost zero. Otherwise, for $\omega > \omega_c$, it drops down as in the normal state. The peak appearance in the optical conductivity signature indicates the resonance state due to the strong hybridization of electrons and holes corresponding to the stability of the excitonic condensate. As discussed before in figure 4, one learns that, for a large fixed Coulomb interaction, increasing the external electric field E_{ext} strengthens the stability of the excitonic condensation state due to the development of the possibility for the formation of the bound electron-hole pairs. The peak in the optical spectrum thus shifts towards higher frequencies by increasing E_{ext} [see figure 6(a)]. For $\kappa = 12$, one can find the excitonic condensation state even at zero external electric fields. In the unbiased case, the system settles like a zero-gap semiconductor and due to the large Coulomb interaction, a small amount of electron-hole pairs slightly around Dirac points might be originated. At zero temperature, the excitons condense in the BEC type indicated by a small peak signature at low frequency. At a given large enough E_{ext}, $E_{ext} = 0.3$ for instance, increasing Coulomb interaction by increasing κ, the feature of the real part of optical conductivity is remained and the peak is shifted up to the higher frequency by reinforcing its spectral weight. That behavior indicates the enhancement of the electron-hole coherence by increasing the Coulomb interaction in the biased DLG system.

5. Concluding remarks

In summary, we have discussed the ground state properties of the excitonic condensation state in the DLG structure. In doing so, the electron-hole correlations in the system are described by an electronic two-band model involving the interlayer Coulomb interaction. In the framework of the unrestricted Hartree–Fock approximation, we derive equations that might help us to evaluate numerically the excitonic condensate order parameter once the model parameters are given. In the excitonic condensation state, the real part of the optical conductivity is also evaluated based on the Kubo linear response theory. For a large Coulomb interaction, we find that the ground state of the system stabilizes in the excitonic condensate, even in an unbiased situation. Turning a gated voltage reinforces the excitonic condensation stability. Depending on the Coulomb interaction and the external bias, the excitonic condensate BCS-BEC crossover in the structure is addressed. As increasing the Coulomb interaction, the BEC excitonic condensate region expands in the range of a small external electric field. In the present work, optical properties in the instability state of the electron-hole systems have been also discussed. Due to the Coulomb interaction, hybridization between electrons and holes in the two opposite sheets might originate in the excitonic bound state, which is indicated by a sharp peak raised in the real part of the optical conductivity spectrum. Our findings, in one way, show a possibility of the BEC-type and BCS-BEC crossover of the excitonic condensate stabilizing in a DLG system, other ways, discuss the properties of the optical signatures in the condensation state. The results thus are purposeful for inspecting experimentally the excitonic bound state stability by analyzing the optical response in a double-layer system.

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Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

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