Modernization of Height System in Vietnam Using GNSS and Geoid Model

Abstract. Recently, Vietnam has modernized the national height system, including the development and establishment of new control points, completing measurement of the national height network figure, upgrading the national Geoid model. One of the most important contents to attain the goal of modernizing the height system is to connect the national grid lines of the grades I and II with GNSS CORS stations and the fundamental gravity points. Vietnam currently uses the 2010 Geoid model which was built on the basis of the Global EGM 2008 Geoid model with additional data of more than 30,000 detailed gravity points and over 800 GPS-TC points. The combined processing of GNSS-leveling data and the gravity geoid model to upgrade the local geoid model with high accuracy (size of 4-10 cm) can allow high satellite measurement technologies to replace the traditional leveling method in determining elevations to achieve accuracy level of grades III and IV. The article presents the research and results of a newly-developed method to combine the adjustments of normal, GNSS, and anomalies heights.

Keywords: GNSS · Height system · Levelling network · Geodetic computations · Geoid· Vietnam

1. Introduction

The height system plays a vital role in building spatial data infrastructure for sustainable development and exploitation of natural resources. The height is determined by using the point difference measurement method. Today, leveling technology is still applied with increasing accuracy of millimeters/km. However, since GNSS has been widely used, high satellite measurement technology has been studied and applied effectively. In many countries around the world, high satellite measurement methods are gradually replacing traditional leveling methods due to faster construction time, lower costs and potentially higher accuracy [13]. The trend of modernizing the height system is associated with the application of GNSS technology in combination with the building of a highly accurate Geoid model in each country to replace traditional leveling technology gradually.

In big cities of Vietnam, the process of urbanization has affected the environment, causing subsidence, surface deformation and damage to construction and transport works [13]. On the other hand, factors related to climate change make certain areas vulnerable to flooding and erosion of rivers. The modernization of the national height network to serve planning, construction, socio-economic development and climate change adaptation as well as the exploitation of natural resources and nature, is essential.

In this paper, we will present an overview of the process of the formation and development of height systems in Vietnam. We will also address some essentialcontents of the modernization of height systems using GNSS technology and the use of the Geoid model. The paper will focus on presenting issues related to processing geodetic network data with examples in the Central Highlands - South Central region of Vietnam.

1.1. The Formation and Development of Height Systems in Vietnam

The geodetic leveling network in the north of Vietnam was built from 1959 to 1964 with the starting point at the Hon Dau Island, Hai Phong City. In the South, before 1975, the height system with the fundamental point at the Ha Tien town, Kien Giang province was used. The Class I and II altitude network was built in 1959, following four national roads, Hanoi - Hai Phong, Hanoi - Dong Dang, Hanoi - Lao Cai and Hanoi Vinh Linh with a total length of 1175 km. The establishment of a unified height network for the entire territory of Vietnam was carried out from 1981 to 1991. The average sea level at Hon Dau Naval Station was computed with data from 43 years from 1950 to1992.

The national height network also has more than 12,000 coordinate points of the class III measured by GPS technology. This height is computed based on the geoid model, which was completed in 2012 with an accuracy equivalent to the leveling of IV-class.

In 2001, the Department of Surveying, Mapping and Geographic Information Vietnam completed the construction of the geoid model for the whole country, with standard gravity grid of 3 ' x 3 ' in the plains and midlands of Vietnam, based on more than 1038 GPS and leveling control points and nearly 30,000 detail gravity points, and EGM2008. This geoid has an accuracy of about 8 cm for the plain areas and about 20 cm for the midlands area of Vietnam. The geoid model was then used to determine the height for the whole territory of Vietnam using GNSS technology. Specifications of height networks is shown in Table 1.

Table 1. Characteristics of the national height networks of Vietnam.

Area	**Class**			
	I (m m)	II (m m)	III (m m)	IV (m m)
Flat topography	$\pm 2\sqrt{L}$	$\pm 4\sqrt{L}$	$\pm 10\sqrt{L}$	$\pm 20\sqrt{L}$
Mountain terrain	$\pm 3\sqrt{L}$	$\pm 5\sqrt{L}$	$\pm 12\sqrt{L}$	$\pm 25\sqrt{L}$

(L- is the linear length in units of Km)

1.2. The Main Contents of the Modernization of the National Height System

- Determining the value "0" of the original height point through the calculation of the average sea level in Hon Dau using oceanographic data in 18.6 years. Determining the value "0" of the national height and determining the value "0" depth for specific waters through monitoring data at the Naval Station.
- Building and improving existing stable control points with high accuracy to ensure stable and long-term use. These control points were burried deep into the rock strata and located along the I - and II - classes lines.
- Completing the measurement of the new national height network connecting the lines of the I - and the II- classes networks, GNSS CORS stations, state gravity points, and navigational observation stations in the restored sea areas.
- Developing and repeating measurement of vertical shift monitoring network in Hai Phong, Hanoi and Da Nang cities by using leveling measurement; establishment and measurement of vertical displacement monitoring networks by combining hydro-leveling and satellite technologies in Ho Chi Minh City, Can Tho City, anld the Mekong Delta Long for analyzing, evaluating and determining the causes and trends of subsidence to supplement updated climate change scenarios.
- Measuring GNSS to achieve high accuracy at some I - and II- classes and at the basic gravity I-class control points, and at the points of the oceanographic monitoring stations in sea areas of Hon Dau (Hai Phong), Hoanh Son (Ha Tinh), Son Tra (Da Nang), Quy Nhon (Binh Dinh), Vung Tau (Ba Ria-Vung Tau) and origin point of Ha Tien height. The measurement sought to ensure a density of about 10 ÷ 20 km/ point for upgrading local Geoid model. At the end of 2019, Vietnam has 65 national satellite navigation stations, located mainly in Hanoi, Ho Chi Minh City and some provinces in Northern, Southern Delta and Highland areas. Covering the entire territory and the big islands need additional building about 75-80 stations.
- Detailing the gravity measurement of Vietnam's mountains. Upgrading the nationwide gravity geoid model by updating the 2011 gravity geoid model with new gravity data and numerical elevation models, and combining with the global geodetic systems WGS-84, Global Geoid EGM2008 or EGM
- Processing combined GNSS-Leveling data and gravity geoid model to upgrade the mixed geoid model on the territory of Vietnam in the mainland and coastal areas to achieve accuracy of 5 cm in the delta area and 10 cm in the mountains area. Employing the high accuracy GNSS method to replace the traditional leveling technology of the III- and IV- classes.

- Calculating adjustment of the national height network and announcing the new national height system. Building a national height database.

2. Development of Algorithms for the Adjustment and Analysis of the Height Networks

2.1. Theoretical background of Height Adjustment Networks

2.1.1. Adjustment of the leveling networks

In Vietnam, along with the development of Information and Communication Technologies (ICT), modern theories have also been employed to solve the problem of the geodetic adjustment. In 1999, the adjustment of the plane network combined with GPS data was completed, and in 2000, the VN-2000 coordinate system was announced. Meanwhile, the I- and II- classes leveling networks were adjusted in 1996, and then, after completing the national height network (2001-2008), the adjustment was revised. Herein, the adjustment software is based on the theory of the least square method was presented in [5,6,8,9,10].

It is noteworthy that the theory of the free network adjustment has been developed and widely applied to solve the vital problem of monitoring deformation of construction works due to the advantages that it is not being affected by the errors of the original data. Since the theory of freedom network adjustment has been presented in various studies, including books [5,8,9,10], salient features of calculating the network leveling are summarized in steps as follows:

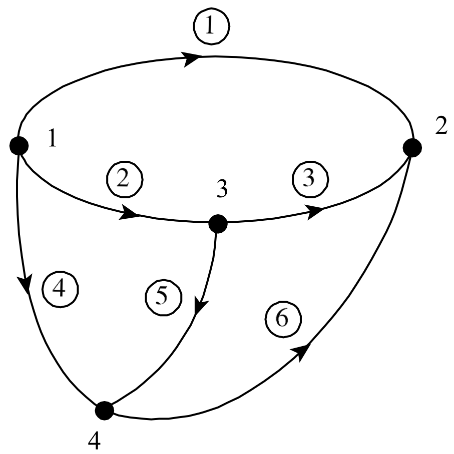

Figure 1. Free Levelling Network

Normal Equation System

$R\Delta x + b = 0$

Where: $R = A^{T}PA$;$b=A^{T}PL$.

$$b = \begin{pmatrix} 0.5 \\ 0.5 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Matrix } C = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Matrix R of Normal Equation System:

$$R = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \quad ; \quad R\Delta x + b = 0 \quad ; \quad \Delta x = -R^{+}b$$

Establish the matrix R as follows:

$$R_c = \begin{pmatrix} R & C \\ C^T & 0 \end{pmatrix}; \qquad R^{-1}{}_c = \begin{pmatrix} R & C \\ C^T & O \end{pmatrix}^{-1} = \begin{pmatrix} R^{\sim} & T \\ T^T & O \end{pmatrix}$$

$R^{\sim}$ calculated by the formula:

$$R^{\sim} = (R + CC^T)^{-1} - TT^T$$

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$$CC^T = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T = B(C^T.B)^{-1} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}; T.T^T = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}$$

Matrix symbol: $M = (R + CC^T)$.

$$M^{-1} = \frac{1}{64} \begin{pmatrix} 24 & 8 & 16 & 16 \\ 8 & 24 & 16 & 16 \\ 16 & 16 & 40 & 24 \\ 16 & 16 & 24 & 40 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 2 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 2 & 5 & 3 \\ 2 & 2 & 3 & 5 \end{pmatrix}$$

$$TT^T = \frac{1}{8} \begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

$R^{\sim}$ is calculated as follows:

$$R^{\sim} = (R + CC^T)^{-1} - TT^T = \frac{1}{8}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Vector Δx is determined as follows:

$$\Delta x = \frac{1}{8}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 1 \\ 1 \end{pmatrix} ;$$

$$\text{Vector } \Delta x = \begin{pmatrix} 0 \\ 0 \\ -0.5 \\ -0.5 \end{pmatrix}$$

2.1.2. Combined adjustment of the Height Networks with GNSS and Levelling Data

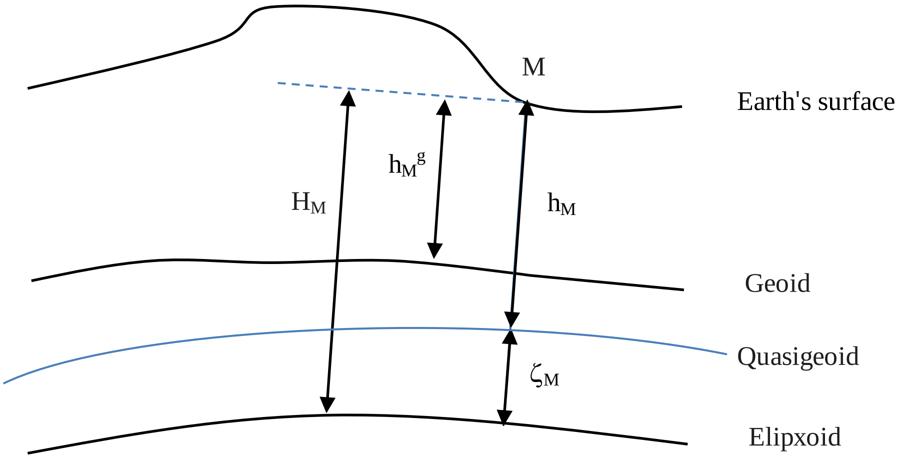

Figure 2.The normal height h and height anomaly ζ

Figure 2 depicts the relationship between normal height and ellipsoid height as follows:

$$h = H - \zeta \qquad (1)$$

where,

h: normal height

H: ellipsoid height (geodetic height)

ζ: height anomaly

Kotsakis and Sideris [7] discussed the adjustment problems, methods for combining GNSS, leveling and geoid heights, and suggested a method for evaluating the accuracy of geoid models. Fotopoulos [2] considered least-squares variance component estimation (VCE) in the combined adjustment of ellipsoid, orthometric and geoid height data. In this paper, we propose an algorithm based on the method of adjustment of the conditions with unknowns. According to theory of combined adjustment of leveling, GNSS networks continue to develop in many research projects, such as [2,4,3,1]

From equation (1) we have:

$$H - \zeta - h = 0 \qquad \textbf{(2)}$$

where,

$$H = H^{(0)} + v_H$$

$$h = h^{(0)} + v_h$$

$$\zeta = \zeta^{(0)} + f + v_\zeta$$

where $H^{(0)}$ and $h^{(0)}$ are determined by the adjustment of the GNSS and levelling networks, respectively

For each of the common points of the GNSS and the height networks we can form the following equation:

$$v_H - v_h - v_\zeta - f + w = 0 \qquad \textbf{(3)}$$

$$w = -(H^{(0)} - h^{(0)} - \zeta^{(0)})$$

In practice, f function in the formula (3) may be approximated by different kinds of functions in order to fit the quasigeoid. In Vietnam, the problem of choosing f = f (x, y) is presented in research works such as [8], [12].

a) Linear Model

$$f(x_i, y_i) = a.x_i + b.y_i + c \qquad \textbf{(4)}$$

xi, yi: is the coordinates of point i;

b) Polynomial Model of Degree 2

$$f(xi, yi) = a.x_i + b.y_i + c.x_i^2 + d.x_iy_i + e.y_i^2 + f \qquad \textbf{(5)}$$

c) Spline Function Model

$$f(x,y), = \sum_{i=1}^{n} a_i r_{PP_i}^2 \ln(r_{PP_i}) + \tau_1 + \tau_2 x + \tau_3 y \qquad \textbf{(6)}$$

Where,

$$r_{PP_i} = \sqrt{(x - x_i)^2 + (y - y_i)^2};$$

τ_1, τ_2, τ_3 - is

the solution of the following system of equations:

$$\begin{bmatrix} 0 & g_{1,2} & \cdots & g_{1,n} & 1 & x_1 & y_1 \\ g_{2,1} & 0 & \cdots & g_{2,n} & 1 & x_2 & y_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{n,1} & g_{n,2} & \vdots & 0 & 1 & x_n & y_n \\ 1 & 1 & \vdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_n & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \cdots \\ \zeta_n \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$g_{i,i} = g_{j,j} = \begin{cases} r_{p_i p_j}^2 \ln\left(r_{p_i p_j}\right) & \text{with } i \neq j \\ 0 & \text{with } i = j \end{cases}$$

d) Several models can be used ranging from simple linear regression to more complicated seven parameter similarity transformation model, Kotsakis and Sideris (1999).In our study, the model selected for calculation will be a four-parameter model according to Heiskanen and Moritz (1967).

$$f_i = f(B_i, L_i) = a_i x = x_1 + x_2(\cos B_i \cos L_i) + x_3(\cos B_i \sin L_i) + x_4(\sin B_i) \qquad \textbf{(7)}$$

B,L - latitude, longitude of the network point

Suppose $f = f(B, L) = a^T x$. From the n point, GNSS has a leveling height h, anomaly height ζ and geodetic height H, we have a system of conditional equations with -unknowns:

$$\mathbf{BV} + \mathbf{A\Delta x} + \mathbf{W} = 0 \qquad \textbf{(10)}$$

$\mathbf{B}_{nx3n} = (\mathbf{E}_{nxn} - \mathbf{E}_{nxn} - \mathbf{E}_{nxn})$; $\mathbf{V}^T = (\mathbf{V}_{Hnx1} \quad \mathbf{V\zeta}_{nx1} \quad \mathbf{V}_{hnx1})$;

E – Matrix unit; $\mathbf{A}_{nxk}$ – Coefficient matrix.

$$\mathbf{W = -(H^{(0)} - h^{(0)} - \zeta^{(0)})} \qquad \textbf{(11)}$$

Matrix A has the form

$$A = \begin{pmatrix} 1 & \cos B_1 \cos L_1 & \cos B_1 \sin L_1 & \sin B_1 \\ \cdots & . & & . \\ . & & & \\ . & & . & \\ 1 & \cos B_k \cos L_k & \cos B_k \sin L_k & \sin B_k \end{pmatrix}$$

(8)

Also perform calculations to compare the Model 5 parameters:

$f=f(B_i,L_i)=a_i x= x_1 + x_2(\cos B_i \cos L_i)+ x_3(\cos B_i \sin L_i) + x_4 (\sin B_i) + x_5 \sin^2 B_i)$ **(9)**

Then Matrix A will be:

$$A=\begin{pmatrix} 1 & \cos B_1 \cos L_1 & \cos B_1 \sin L_1 & \sin B_1 & \sin^2 B_1 \\ & . & & . & \\ & . & & . & \\ & . & & . & \\ 1 & \cos B_k \cos L_k & \cos B_k \sin L_k & \sin B_k & \sin^2 B_k \end{pmatrix}$$

$\mathbf{A}^T = (a_1^T \quad a_2^T \quad \dots \quad a_n^T)$

$$Q_V = \begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix} \quad \textbf{(12)}$$

Δx- vector of unknowns.

- **Q_H, $Q\zeta$, Q_h**- covariance matrixes of vectors **H, ζ , h.**

Quantity

$$y = H-\zeta-h = (E-E-E)\begin{pmatrix} H \\ \zeta \\ h \end{pmatrix} \quad \textbf{(13)}$$

$$Q_y = BQB^T = (E-E-E)\begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix}(E-E-E)^T = (Q_H + Q\zeta + Q_h) \quad (10)$$

System of equations (15) is solved with the following conditions:

$$\Phi = V^T Q_V^{-1} V = V_H^T Q_H^{-1} V_H + V_\zeta^T Q_\zeta^{-1} V_\zeta + V_h^T Q_h^{-1} V_h = \text{Min} \quad \textbf{(14)}$$

We set up the Lagrange function

$$\Phi = V^T PV + 2K^T (BV + A\Delta x + W) = \min \quad \textbf{(15)}$$

Calculate derivatives in vectors:

$$\frac{\partial \phi}{\partial V} = 2\mathbf{V}^T\mathbf{P} - 2K^T \mathbf{A} = 0 \quad \textbf{(16)}$$

$$\frac{\partial \phi}{\partial \Delta x} = \mathbf{-}\ 2\mathbf{K}^T\mathbf{A} = 0 \quad \textbf{(17)}$$

We have the formula:

$$V = P^{-1} B^T K \quad \textbf{(18)}$$

$$A^T K = 0 \quad \textbf{(19)}$$

Substituting the expressions (18) and (19) into (10) we have:

$$\left.\begin{aligned} & NK + A\Delta x + W = 0 \\ & A^T k = 0 \\ & N = BQB^T = Q_H + Q_\zeta + Q_h \end{aligned}\right\} \quad \textbf{(20)}$$

Where $N = B\,Q\,B^T$ ·

· $Q = P^{-1}$

Expression **(16)** can be rewritten as:

$$\begin{pmatrix} N & A \\ A^T & O \end{pmatrix} \begin{pmatrix} K \\ \Delta x \end{pmatrix} + \begin{pmatrix} W \\ O \end{pmatrix} = 0 \quad \textbf{(21)}$$

The solution vector of system of equations (21) will be

$$\begin{pmatrix} K \\ \Delta x \end{pmatrix} = -\ N_\beta^{-1} \begin{pmatrix} W \\ O \end{pmatrix} \quad \textbf{(22)}$$

Here the matrix

$$N_A = \begin{pmatrix} N & A \\ A^T & O \end{pmatrix}.$$

From the first equation of the system (20), we have the expression

$$K = -N^{-1}\mathbf{A\Delta x} - \mathbf{N}^{-1}\mathbf{W} = -\mathbf{N}^{-1}(\mathbf{A\Delta x} + \mathbf{W}) = -\mathbf{N}^{-1}\mathbf{W}_1 \qquad \mathbf{(23)}$$

Here vector

$$\mathbf{W}_1 = \mathbf{A\Delta x} + \mathbf{W} \qquad \mathbf{(24)}$$

Substituting the expression **(23)** into the second equation of the system **(20)** we get

$$\mathbf{A}^{\mathbf{T}}\mathbf{N}^{-1}\,\mathbf{A\Delta x} + \mathbf{A}^{T}\mathbf{N}^{-1}\mathbf{W} = 0$$

Or

$$\mathbf{W}_1 = \mathbf{A\Delta x} + \mathbf{W} \qquad \mathbf{(24)}$$

$$\mathbf{\Delta x} = -\,(\mathbf{A}^{\mathbf{T}}\mathbf{N}^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{N}^{-1}\mathbf{W} \qquad \mathbf{(25)}$$

$$= -\,[\mathbf{A}^{\mathbf{T}}(\mathbf{Q_H}+\mathbf{Q_\zeta}+\mathbf{Q_h})^{-1}\mathbf{A}\mathbf{A}^{\mathbf{T}}(\mathbf{Q_H}+\mathbf{Q_\zeta}+\mathbf{Q_h})^{-1}\mathbf{W}$$

Vector V is calculated using the expressions **(18)** and **(23)** as follows

$$V = \begin{pmatrix} V_H \\ V_\zeta \\ V_h \end{pmatrix} = QB^T K = -\begin{pmatrix} Q_H & & \\ & Q_\zeta & \\ & & Q_h \end{pmatrix}\begin{pmatrix} E \\ -E \\ -E \end{pmatrix} N^{-1} W_1$$

$$= -\begin{pmatrix} Q_H \\ Q_\zeta \\ Q_h \end{pmatrix}\left(Q_H + Q_\zeta + Q_h\right)^{-1} W_1 \qquad \mathbf{(26)}$$

Or is:

V= T W_1

Where, the matrix

$$\mathbf{T} = \begin{pmatrix} Q_H \\ Q_\zeta \\ Q_h \end{pmatrix} \mathbf{N}^{-1} \qquad \mathbf{(27)}$$

To evaluate the accuracy after adjustment, we need to calculate the covariance matrix of vector V after adjustment as follows:

$$\mathbf{Q_V} = \mathbf{T\,Qw1\,T^T} \qquad \textbf{(28)}$$

The covariance matrix of vector x determined from the formula (27)

$$\mathbf{Q_{\Delta x}} = (\mathbf{A^T N^{-1} A})^{-1}$$

Symbol matrix R= $\mathbf{A}^T\mathbf{N}^{-1}\,\mathbf{A}$ substitute formula (25) into formula (24) we have

$$\mathbf{W}_1 = \mathbf{A\Delta x} + \mathbf{W} = (\mathbf{-A\ R^{-1}\,A^T N^{-1} + E})\ \mathbf{W}$$

E is the unit matrix.

Follow the formula for calculating covariance matrix of the function we have

$$\mathbf{Q_{w1}} = (\mathbf{-A\ R^{-1}\,A^T N^{-1} + E) N\,(-N^{-1} A\,R^{-1} A^T + E) = (-A\,R^{-1} A^T + N)\,(-N^{-1} A\,R^{-1} A^T + E)}$$

$$= \mathbf{(A\,R^{-1} A^{T1} A\,R^{-1} A^T - N^{-1} A\,R^{-1} A^T + {}^1 A\,R^{-1} A^T + N = N - \ A\,R^{-1} A^T}$$

So we have the expression

$$\mathbf{Q_{w1} = N - AQ_{\Delta x}A^T} \qquad \textbf{(29)}$$

$V_{Hi}, V_{\zeta}, V_{hi}$ used to calibrate H_i, ζ_i, h_i at the GNSS common points and levelling network.

It should be noted that, in the adjustment of mixed quantities of different properties, we should use the matrix defined as follows:

$$\mathbf{C} = \sigma^2\mathbf{Q} \qquad \textbf{(30)}$$

$$C_V = \begin{pmatrix} C_H & & \\ & C_\zeta & \\ & & C_h \end{pmatrix}$$

Here σ- standard deviation.The Q matrix is determined from the results of separate adjustment of geodetic networks. The calculation formulas from (15) to (25), the matrices $\mathbf{Q_H}$, $\mathbf{Q\zeta}$, $\mathbf{Q_h}$ are replaced with the matrix $\mathbf{C_H}$, $\mathbf{C\zeta}$, $\mathbf{C_h}$.

2.2. Methodology for the Combined adjustment and Analysis of the Height networks

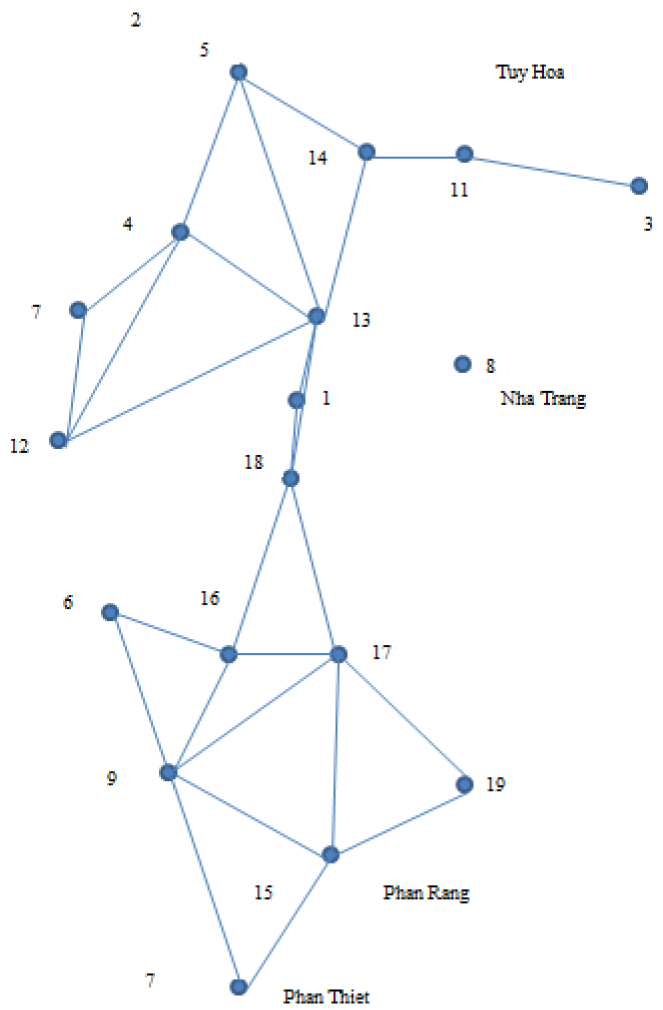

Figure 3. The Common Points of the GNSS and Levelling Network at the Central Highlands of Vietnam

Step 1:Data preprocessing

- Input:
  - Leveling observations.
  - GNSS observations.
  - Geoid model EGM 2008
- Output:
  - Normal, Ellipsoid height and height anomalis ζ
  - The matrices QH, Qζ, Qh

To illustrate the theory, we proceed to adjust the GNSS and regional leveling network of the I - class and the II - class in the Central Highlands with the grid diagram shown in the Figure 3. In this figure, there are 19 points in the GNSS network, measured and adjusted by the Natural Resources and Environment of Vietnam Corporation. The calculation and adjustment of the GNSS network with one origin point with coordinates B, L, and H were based on the VN- 2000 coordinate]. The height anomaly ζ is calculated from the harmonic coefficients with data taken from the Geoid model EGM 2008. GM2008 altitude anomaly is determined in the WGS-84 coordinate system and calculated from the harmonic coefficients using the HARMONIC_SYM-WGS-84 program with EGM2008_to_2190_TideFee and Zeta_To_N_2160_EGM_2008. From the calculation results, we have vector h, geodetic height vector H, and $\mathbf{C}_h$ and $\mathbf{C}_H$ matrices. The data included in the calculation are given in Table 2 [8]. To illustrate the method, we choose models 4 and 5 parameters.

Table 2. Data of Points in the Central Highlands - South Central Coast.

	B ° ' "	**L ° ' "**	**x (m)**	**y (m)**	**H (m)**	**h (m)**	$\zeta_{EGM2008}$
1	12 17 49.7278	108 8 13.1651	1361403.1	84126742	533.791	531.386	0.975
2	13 3 51.8607	108 13 3.5957	1446475.1	849003.67	762.892	760.658	-0.832
3	12 44 2.6369	108 45 8.5642	1410672.6	907606.89	423.331	420.94	1.917
4	12 39 33.7426	108 1 35.6883	1401372	828787.48	433.139	431.206	-0.998
5	12 54 47.3372	108 15 57.685	1429790.8	854468.8	683.955	681.622	-0.286
6	12 48 18.5108	108 32 19.266	1418219.7	884258.82	468.914	466.566	0.897
7	12 34 55.5563	107 50 29.777	1392591.3	808770.27	360.331	358.653	-1.641
8	12 29 42.5938	107 44 18.233	1382848.8	797647.78	582.95	581.077	-1.607
9	12 33 12.6619	108 10 6.5913	1389832.8	844360.07	479.26	476.956	0.117
10	12 48 36.0408	108 26 9.7056	1418608.3	873092.14	496.481	494.14	0.501
11	11 55 21.3842	107 44 56.576	1319480.6	799449.86	595.505	593.597	0.249
12	11 16 9.3570	108 6 11.131	1247546.9	838836.33	212.09	210.656	2.398
13	11 49 27.3762	108 34 10.514	1309607.8	889064.06	1019.755	1016.085	4.646
14	11 38 48.6369	108 1 9.264	1289255.2	829234.28	1034.089	1031.218	2.406
15	11 37 13.0575	108 13 53.458	1286570.5	852437.46	774.008	770.976	3.405
16	11 52 30.3191	107 55 23.186	1314414.1	818479.88	856.696	854.127	1.354
17	11 56 11/9678	108 9 36.538	1321514.4	844251.83	1115.27	1112.005	2.762
18	12 12 54.3090	108 7 23.684	1352299	839876.04	498.455	495.959	1.197
19	11 46 2.4338	108 21 39.369	1303020.4	866373.15	966.814	963.3346	3.993

Step 2: Combined network adjustment for corrector Surface Model (CSM)

Twelve common points of GNSS and leveling network were chosen to calculate the adjustment. Points 13 to 19 are control points. 4-parameter model according to formula (7) was selected, and the components of matrix A are defined in Table 3.

Table 3. Matrix Components A

x_1	x_2	x_3	x_4
1	-0.30415	0.9285117	0.2129815
1	-0.30453	0.9252903	0.226046
1	-0.31357	0.9236267	0.220426
1	-0.30193	0.9277971	0.2191542
1	-0.30550	0.9255964	0.2234736
1	-0.31004	0.9245298	0.2216358
1	-0.29903	0.9290476	0.2178381
1	-0.297453	0.9298984	0.216357
1	-0.304357	0.9274295	0.2173512
1	-0.308373	0.9250659	0.2217187
1	-0.298270	0.9318558	0.2065901
1	-0.304735	0.9321736	0.1954199

To calculate we have: $C_H= \sigma^2_H.E$; $C\zeta= \sigma^2\zeta$ E ;$C_h=\sigma^2_h$ E'

Where: E- matrix unit (12x12)

.σ_H=20mm, $\sigma\zeta$ =100 mm,σ_h=15mm.

Calculated b.y formula (25), vector x would be

$$x = \begin{pmatrix} -473.397 \\ 55.4673 \\ 473.631 \\ 244.289 \end{pmatrix}$$

Step 3: Conducting normal heights calculation for the test points according to the following formula

$$h_j = H_j - \zeta_j + a_j \mathbf{x} \qquad (31)$$

$(j= 13,14...19)$

where: x is the parameter vector of the corrector surface extimated via the combined adjustment.

As long as the GNSS observations and a geoid model are available, the normal heights of new points are achievable by using the above equation.

To calculate only the normal height of the GNSS measurement points without leveling, we will use the difference

$$\Delta h_{ji} = \Delta H_{ji} - \Delta\zeta_{ji} + (a_j - a_i)\ x \qquad (32)$$

Here, vector **x** is calculated in expression (25).

The weight of these measurements is calculated by the following formula:

$$Q_{\Delta h_{ji}} = Q_{\Delta H_{ji}} + Q_{\Delta\zeta_j} + (a_j - a_i) Q_x (a_j - a_i)^T \qquad (33)$$

$$P_{\Delta h_{ji}} = Q^{-1}_{\Delta h_{ji}} \qquad (34)$$

2.3.Results and discussion

Executing the calculation by formula (28) we have matrices $\mathbf{Q_H}$, $\mathbf{Q\zeta}$, and $\mathbf{Q}$,. Diagonal components of these matrices are presented in Table 4.

Table 4. The Main Diagonal Components of the Matrices Q_H, $Q\zeta$, Q_h

Poins	diag Q_H	diag $Q\zeta$	diag Q_h
1	8.758	5474.069	2771
2	7.040	4400.026	2.227
3	8.689	5431.065	2.749
4	12.958	8099.079	4.100
5	12.348	7717.589	3.907
6	11.764	7352.73	3.722
7	11.771	7356.774	3.724
8	10.643	665.114	3.368
9	11748	7342.411	3.717
10	12.608	7880.295	3.989
11	10.549	6592.938	3.338

12	1.59204	995.025	0.504

Estimated Variance Component (Unit [mm^2]

σ^2=1.422289

Analysis of values of vectors V_H ,V_ζ ,V_h for model 4 and 5 parameters is presented in Table 5.

Table 5. Results of the empirical test for the 12 GNSS/Levelling points used in the combined adjustment. Unit [m]

	4 ParamtrsTransf Model			5 ParamtrsTransf Model		
	V_H	V_ζ	V_h	V_H	V_ζ	V_h
Mean	0.000	0.000	0.000	0.000	0.000	0.000
Max	0.008	0.121	0.005	0.005	0.086	0.002
Min	- 0.004	- 0.103	- 0.002	-0.008	- 0.187	- 0.004
RMS	0.004	0.094	0.002	0.004	0.092	0.002

The deviation of the differences between the heights calculated by the formula (31) and the standard height of the test points is shown in Table 6.

Table 6. Error and Deviation of the levelling lines related to 13,14,15,16, and 17 Test Points.

Levelling line	**L(km)**	**4 P M (mm)**	**5 P M (mm)**	**Deviation Limit (III-class) (mm)**	**Deviation Limit (IV-class) (mm)**
14_13	63.197	-15.94	2.699	95.396	198.741
13_12	79.840	-116.01	-138.37	107.224	223.383
13_18	65.13064	-278.01	-303.59	96.844	201.759
18_17	31.094	50.916	49.793	66.914	139.405
17_16	26.732	65.5215	82.904	62.0438	129.258
17_19	28.834	117.110	121.501	64.436	134.2426
15_17	35.88984	-201.435	-201.67	71.88975	149.7703

15_19	21.559	-84.324	-80.168	55.718	116.080
σ (1km)		14.873	15.683	12.000	25.000

From Table 4 and Table 5, for the twelve participating common points, the accuracy of the GNSS height, normal height, and anomaly height all increased after the computation. However, the combined adjustment result did not increase the quality of the network leveling.

From the calculation results in Table 7 for seven test points, it is evident the results of the adjustment allowed the determination of the parameters in the model (7) and (9). The results of the adjustment also enabled the determination the altitude of the GNSS that meet the III- and IV- classes standards

Summary and conclusion

This paper aims to present the development, modernization of the height system in Vietnam, and the algorithm for the adjustment and analysis of height systems. It included the proposed methodology for the combined adjustment of leveling and GNSS networks.

The modernization of the height system in Vietnam, as in many other countries, essentially develops a new modern technological method to process the combined data measured both from the GNSS method and traditional leveling technique. It uses the best modeling geoid in every country. These new technologies shorten the time to develop height data system from several decades to two to three years. Thanks to this modernization, data accuracy has been greatly improved, and data could be stored more conveniently for easy exploitation for diverse purposes. The application of new technology allows solving complex problems such as monitoring the subsidence of construction works as well as forecasting problems related to mining.

The proposed methodology for the combined adjustment of the height networks by experimental calculation has been conducted in the Central Highlands - South Central region of Vietnam. The content of the algorithm includes the following processes: (1) Preprocessing and separate adjustment of leveling and GNSS heights, (2) Combined adjustment with leveling, GNSS, and height anomalis, and (3) Evaluation of the control points.

This algorithm allows inputting covariance matrices of geodetic height, leveling height, and height anomaly. If the error of height anomaly is greater than 2 dm, it will not significantly affect the adjustment of GNSS and leveling height values. Correction numbers are primarily used to correct height anomalies.

The experimental results demonstrated the effectiveness of this algorithm. This algorithm can be applied to adjust the height networks in Vietnam.

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