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EARTH SCIENCES AND NATURAL RESOURCES FOR SUSTAINABLE DEVELOPMENT

SQUEEZING FUNCTION AND FRIDMAN INVARIANT IN SOME C^n-_DOMAINS

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ABSTRACT

The study of biholomorphic invariants has been attracted much attention in the complex differential geometry to enhance the comprehension and application of biholomorphic classification of complex domains. The squeezing function, the Fridman invariant, and the quotient invariant by using the Caratheodory and Kobayashi-Eisenman volume elements, have received increasing interest as biholomorphic invariants in recent years. We particularly consider both the squeezing function and the Fridman invariant associated to a certain class of pseudoconvex domains in C^n in this paper.

Introduction

The first, we show that any non-degenerate C-convex domain in C^n is uniformly squeezing; the second, we show that if a bounded domain Ω is exhausted by a bounded strickly pseudoconvex domain D with C^2 boundary, then Ω is holomorphically equivalent to D or the unit ball, and show that a bounded domain has to be holomorphically equivalent to the unit ball if its Fridman's invariant has certain growth condition near the boundary; and the last, we show that if the squeezing function $s_\Omega(\eta_j)$ tends to 1 or the Fridman invariant $h_\Omega(\eta_j)$ tends to 0 for some sequence $\{\eta_j\} \subset \Omega$ converging to ξ_0, then this point must be strongly pseudoconvex.

MAIN RESULTS

The purpose of this paper is to sum up problems of the squeezing function in some domains

Theorem 1. There exists a constant $c_n > 0$ such that $s_D \geq c_n$ for any non-degenerate C-convex domain D in C^n.

Theorem 2. Let D be a bounded strictly pseudoconvex domain in C^n with C^2 boundary. If a bounded domain $\Omega \subset C^n$ can be exhausted by D, then Ω must be biholomorphic to D or the unit ball B^n.

Theorem 3. If a bounded domain Ω can be exhausted by a homogenous regular domain, then Ω is homogenous regular.

Theorem 4. Let $D \subset C^n$ be a bounded domain and $p \in \partial D$ be a C^2 boundary point of D. If there is a sequence $z_j \in D$ $(j \geq 1)$ converging to p and a sequence of positive numbers ε_j $(j \geq 1)$ converges to 0 such that $e_D(z_j) > 1 - \varepsilon_j \delta(z_j)$ for all j, then D is biholomorphic to the unit ball, where $\delta(z)$ denotes the distance between z and ∂D.

Theorem 5. Let Ω be a bounded domain in Cn with smooth pseudoconvex boundary. If ξ_0 is a boundary point of Ω of D'Angelo finite type such that the Levi form has corank at most 1 at ξ_0 and if there exists a sequence $\{\eta_j\} \subset \Omega$ such that $\lim_{j\to\infty} \eta_j = \xi_0$ and $\lim_{j\to\infty} s_\Omega(\eta_j) = 1$ or $\lim_{j\to\infty} h_\Omega(\eta_j) = 0$, then $\partial\Omega$ is strongly pseudoconvex at ξ_0.

Corollary 6. Let Ω be a bounded domain in $\mathbb{C}^n$ with smooth pseudoconvex boundary. If ξ_0 is a boundary point of Ω of D'Angelo finite type such that the Levi form has corank at most 1 at ξ_0 and if $\lim_{z\in\Omega\to\xi_0} s_\Omega(z) = 1$ or $\lim_{z\in\Omega\to\xi_0} h_\Omega(z) = 0$, then $\partial\Omega$ is strongly pseudoconvex at ξ_0.

DEFINITIONS

Definition 1. The *squeezing function* $s_\Omega : \Omega \to \mathbb{R}$ is defined as $s_\Omega(p) := \sup_f \{s_{\Omega,f}(p)\}$.

Definition 2. The *Fridman invariant* is defined by $h_\Omega(p) = \inf_{r\in\Re} \frac{1}{r}$.

Definition 3. Let $\{\Omega_j\}_{j=1}^{\infty}$ be a sequence of open sets in $\mathbb{C}^n$ and Ω_0 be an open set of $\mathbb{C}^n$. The sequence $\{\Omega_j\}_{j=1}^{\infty}$ is said to converge to Ω_0 (written $\lim \Omega_j = \Omega_0$) if and only if it satisfies two conditions

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