OPTIMAL CONTROL APPLICATIONS AND METHODS
Optim. Control Appl. Meth. (2015)
Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/oca.2216

New results on H_∞ filtering for nonlinear large-scale systems with interconnected time-varying delays

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SUMMARY

This paper proposes a new design method of H_∞ filtering for nonlinear large-scale systems with interconnected time-varying delays. The interaction terms with interval time-varying delays are bounded by nonlinear bounding functions including all states of the subsystems. A stable linear filter is designed to ensure that the filtering error system is exponentially stable with a prescribed convergence rate. By constructing a set of improved Lyapunov functions and using generalized Jensen inequality, new delay-dependent conditions for designing H_∞ filter are obtained in terms of linear matrix inequalities. Finally, an example is provided to illustrate the effectiveness of the proposed result. Copyright © 2015 John Wiley & Sons, Ltd.

Received 13 January 2015; Revised 13 July 2015; Accepted 5 October 2015

KEY WORDS: H_∞ filter; large-scale system; interconnected delay; Lyapunov function; linear matrix inequalities

1. INTRODUCTION

The H_∞ filtering problem has been extensively studied over the past decades because of its applications in a variety of areas such as signal processing, communications, power systems, pattern recognition, transportation networks, and telecommunication networks [1–3]. The H_∞ filtering problem consists of the design of a filter such that the filtering error system is asymptotically stable, and the induced $H-$norm from the perturbation signals to the filtering error remains bounded by a prescribed value. There has been substantial interest in the study of H_∞ filters for time-delay systems in the past decades because for many practical filtering applications, time-delays cannot be neglected in the procedure of filter design, and their existence usually results in a poor performance [4–7]. In the context of delayed large-scale systems [8, 9], the problem of H_∞ filtering has received lesser attention in the literature. When the problem of H_∞ filter design of large-scale delayed systems is considered, the conventional centralized H_∞ design approaches are neither robust nor scalable to interconnected subsystems, with their measurements distributed on a large geographical region. It is difficult to design an H_∞ filter for nonlinear interconnected systems when there are time-varying delay interactions among the subsystems. The reason is that the time-varying delay interconnected systems are of high dimension, and thus require extensive computations to implement the centralized procedure. It is worth pointing out that all the existing results on the H_∞ filtering problem in the continuous context were for point-wise, single, or constant time-delay systems. Recently, the study of large-scale time-delay interconnected systems has received much attention and many results have been achieved [10–12]. However, most of the existing results are based on the fact that the system states are available and/or the interconnected systems are

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linear or have linear nominal subsystems either with constant delays or differentiable time-varying delays. For nonlinear large-scale systems with unknown time-varying delayed interactions, however, the H_∞ filtering problem has not been fully investigated in the literature. In [13, 14], the fuzzy and stochastic H_∞ filter design method was proposed for nonlinear interconnected systems with constant time delays via the Takagi-Sugeno fuzzy model. Recently, the H_∞ control of large-scale interconnected systems with time-varying delays was discussed in [15–17] based on Lyapunov functional method.

In this paper, we will study the H_∞ filtering for nonlinear large-scale systems with interconnected time-varying delays. Compared with the existing methods, the main features of our method can be described as: (a) the nonlinear time-varying delayed interaction terms are bounded by nonlinear bounding functions including all state variables of subsystems. The time delays are assumed to be any continuous functions belonging to a given interval involved in both the state and observation output; (b) there has not been an effective method to design a decentralized filter for large-scale high-order nonlinear systems to ensure that the filtering error system is exponentially stable with a prescribed convergence rate. We propose a new design tool to solve the aforementioned decentralized H_∞ filtering problem for nonlinear large-scale systems with unknown time-varying delayed interconnections. We construct a set of improved Lyapunov–Krasovskii functionals based on the information of the lower and upper delay bounds and apply generalized Jensen inequality for lower bounding cross terms, which eliminates the need for overbounding and provides larger values of the delay bound. The filtering design solution is facilitated by introducing some additional instrumental matrix variables. These additional matrix variables decouple the Lyapunov and the system matrices, which make the filtering design feasible. In terms of linear matrix inequalities (LMIs), sufficient conditions for the solvability of this problem are obtained, and the desired H_∞ filter can be constructed by solving certain LMIs, which can be implemented by using standard numerical algorithms [18].

2. PROBLEM FORMULATION AND PRELIMINARIES

Throughout this paper, $\lambda(A)$ denotes all the eigenvalues of A; $\lambda_{max}(A) = max\{Re\lambda : \lambda \in \lambda(A)\}$; $\lambda_{min}(A) = min\{Re\lambda : \lambda \in \lambda(A)\}$; $\lambda_A = \lambda_{max}(A^T A)$; and $C([a,b], R^n)$ denotes the set of all R^n-valued continuous functions on $[a,b]$; $L_2([0,\infty], R^r)$ stands for the set of all square-integrable R^r- valued functions on $[0,\infty]$. The notation $i = \overline{1,N}$ means $i = 1,\ldots,N$; the symmetric terms in a matrix are denoted by $*$. Matrix A is semi-positive definite ($A \geqslant 0$) if $(Ax,x) \geqslant 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $(Ax,x) > 0$ for all $x \neq 0$; $A \geqslant B$ means $A - B \geqslant 0$. In this section, we first give the problem formulation, and then present some preliminaries, which will be used in the sequel.

Nonlinear large-scale systems with interconnected delays can be found in such diverse areas as electrical power systems, space structures, manufacturing processes, transportation, and communication. In this context, we consider the following nonlinear large-scale interconnected system with state delays composed of N subsystems as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + \sum\limits_{j=1}^{N} A_{ij} x_j(t - h_{ij}(t)) + D_i \omega_i(t) \\ \qquad\qquad + g_i(t, x_i(t), \{x_j(t - h_{ij}(t))\}_{j=1}^{N},\ \omega_i(t)), \\ z_i(t) = C_i x_i(t) + \sum\limits_{j=1}^{N} G_{ij} x_j(t - h_{ij}(t)), \\ y_i(t) = E_i x_i(t) + \sum\limits_{j=1}^{N} E_{ij} x_j(t - h_{ij}(t)), \quad \forall t \geqslant 0, \\ x_i(\theta) = \varphi_i(\theta), \quad \forall \theta \in [-h_2, 0], \quad i = \overline{1,N}, \end{cases} \tag{2.1}$$

where $x_i(t) \in R^{n_i}$ is the state vector, $z_i(t) \in R^{q_i}$ is the signal to be estimated, and $\omega_i \in L_2([0,\infty], R^{r_i})$ is the exogenous disturbance signal. The system matrices A_i, C_i, D_i, E_i,

DOI: 10.1002/oca

A_{ij}, G_{ij}, E_{ij} are of appropriate dimensions, the continuous time delays $h_{ij}(.)$ satisfy the following conditions:

$$0 < h_1 \leqslant h_{ij}(t) \leqslant h_2, \quad t \geqslant 0,\ \forall i, j = \overline{1, N},$$

the initial function $\varphi_i(t) \in C^1([-h_2, 0], R^{n_i})$, with the norm as follows:

$$\|\varphi_i\|_{C_1} = \sup_{-h_2 \leqslant t \leqslant 0} \|\varphi_i(t)\| + \sup_{-h_2 \leqslant t \leqslant 0} \|\dot{\varphi}_i(t)\|.$$

The nonlinear functions $g_i(.)$ satisfy the following growth conditions:

$$\exists\, a_i, d_i, a_{ij} > 0 : \|g_i(.)\| \leqslant a_i\|x_i(t)\| + d_i\|\omega_i(t)\| + \sum_{j=1}^{N} a_{ij}\|x_j(t - h_{ij}(t))\|. \tag{2.2}$$

We consider a decentralized filter of the form as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i^f \hat{x}_i(t) + D_i^f y_i(t),\ \hat{x}_i(0) = 0, \\ \hat{z}_i(t) = C_i^f \hat{x}_i(t) + G_i^f y_i(t), \quad i = \overline{1, N}, \end{cases} \tag{2.3}$$

where $A_i^f, D_i^f, C_i^f, G_i^f$ are the filter parameters to be designed.

Defining $\tilde{z}_i(t) \triangleq z_i(t) - \hat{z}_i(t)$ and $\xi_i(t) = [x_i(t)\, \hat{x}_i(t)]^T$ from (2.1) and (2.3), we have the filtering error system as follows:

$$\begin{cases} \dot{\xi}_i(t) = \bar{A}_i \xi_i(t) + \sum\limits_{j=1}^{N} \bar{A}_{ij} \xi_j(t - h_{ij}(t)) + \bar{D}_i \omega_i(t) + \bar{g}_i(\cdot), \\ \tilde{z}_i(t) = \bar{C}_i \xi_i(t) + \sum\limits_{j=1}^{N} \bar{G}_{ij} \xi_j(t - h_{ij}(t)), \quad i = \overline{1, N}, \\ \xi_i(t) = \bar{\varphi}_i(t) = [\varphi_i(t)\, 0]^T,\ \forall t \in [-h_2, 0], \end{cases} \tag{2.4}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ D_i^f E_i & A_i^f \end{bmatrix}, \bar{A}_{ij} = \begin{bmatrix} A_{ij} & 0 \\ D_i^f E_{ij} & 0 \end{bmatrix}, \bar{D}_i = \begin{bmatrix} D_i \\ 0 \end{bmatrix},\ \bar{g}_i(\cdot) = \begin{bmatrix} g_i(\cdot) \\ 0 \end{bmatrix},$$

$$\bar{C}_i = \begin{bmatrix} C_i - G_i^f E_i & -C_i^f \end{bmatrix}, \bar{G}_{ij} = \begin{bmatrix} G_{ij} - G_i^f E_{ij} & 0 \end{bmatrix}.$$

H_∞ filtering problem: given a scalar $\gamma > 0$, design a full order filter of the form (2.3) such that the filtering error system (2.4) has a prescribed H_∞ performance γ, that is as follows:

(i) the error system (2.4) with $\omega_i(t) = 0,\ i = \overline{1, N}$, is exponentially stable; and
(ii)

$$\sup \frac{\int\limits_0^\infty \sum\limits_{i=1}^{N} ||\tilde{z}_i(t)||^2 dt}{c_0 \sum\limits_{i=1}^{N} ||\varphi_i||_{C_1}^2 + \int\limits_0^\infty \sum\limits_{i=1}^{N} ||\omega_i(t)||^2 dt} \leqslant \gamma, \tag{2.5}$$

where the supremum is taken over all $\varphi_i \in C^1([-h_2, 0], R^{n_i})$ and the non-zero uncertainty $\omega_i(t) \in L_2([0, \infty], R^{r_i})$.

To end this section, we introduce the following lemmas needed for H_∞ performance analysis of the error system.

DOI: 10.1002/oca

Lemma 1 (Schur complement lemma [19]*)*
Given matrices X, Y, Z, where $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0.$$

Lemma 2 (Generalized Jensen inequality [20]*)*
For given symmtric matrix $R > 0$ and differentiable function $\phi : [a, b] \to R^n$, then the following inequality holds:

$$\int_a^b \dot{\phi}^T(t) R \dot{\phi}(t) dt \geqslant \frac{1}{b-a}(\phi(b) - \phi(a))^T R(\phi(b) - \phi(a)) + \frac{12}{b-a}\Omega^T R \Omega,$$

where $\Omega = \frac{\phi(b)+\phi(a)}{2} - \frac{1}{b-a}\int_a^b \phi(t)dt$.

3. MAIN RESULT

In this section, we give a design of H_∞ filter for large-scale system (2.1), such that the error system (2.4) is exponentially stable. Before introducing the main result, the following notations of several matrix variables are defined for simplicity as follows:

$$\begin{aligned}
x(t)^T &= \left[x_1(t)^T, \ \ldots, \ x_N(t)^T\right], \xi(t)^T = \left[\xi_1(t)^T, \ \ldots, \ \xi_N(t)^T\right], \\
\varphi(t)^T &= \left[\varphi_1(t)^T, \ \ldots, \ \varphi_N(t)^T\right], \\
||\varphi||_{C_1} &= \sqrt{\sum_{i=1}^N ||\varphi_i||^2_{C_1}}, \varepsilon_i = a_i + \frac{4d_i^2}{\gamma} + \sum_{j=1}^N a_{ij}, \quad \bar{P}_i = \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i2} \end{bmatrix}, \\
\alpha_1 &= \min_{i=\overline{1,N}} \lambda_{min}(\bar{P}_i), \quad \alpha_2 = \max_{i=\overline{1,N}} \left\{\lambda_{max}(\bar{P}_i) + h_1^3 \lambda_{max}(\bar{R}_i) + h_2^3 \lambda_{max}(\bar{R}_i)\right\}, \\
H^i_{jj} &= -\frac{1}{N+1} e^{-2\beta h_2} \bar{R}_i + (2 + 2a_{ji})I, \ j = \overline{1, N}, \\
H^i_{j(N+1)} &= \frac{1}{N+1} e^{-2\beta h_2} \bar{R}_i, \ j = \overline{1, N}, \\
H^i_{(N+1)(N+1)} &= \bar{P}_i \bar{A}_i + \bar{A}_i^T \bar{P}_i + 2\beta \bar{P}_i - 4e^{-2\beta h_1} \bar{R}_i - \frac{N+4}{N+1} e^{-2\beta h_2} \bar{R}_i + 2a_i I \\
&= \begin{bmatrix} P_{i1} A_i + A_i^T P_{i1} & E_i^T Z_i^T \\ Z_i E_i & Y_i + Y_i^T \end{bmatrix} + 2\beta \bar{P}_i - 4e^{-2\beta h_1} \bar{R}_i - \frac{N+4}{N+1} e^{-2\beta h_2} \bar{R}_i + 2a_i I, \\
H^i_{(N+1)(N+2)} &= -2e^{-2\beta h_1} \bar{R}_i, \ M^i_{(N+1)(N+3)} = -\frac{2}{N+1} e^{-2\beta h_2} \bar{R}_i, \\
H^i_{(N+1)(N+4)} &= \bar{A}_i^T \bar{P}_i = \begin{bmatrix} A_i^T P_{i1} & E_i^T Z_i^T \\ 0 & Y_i^T \end{bmatrix}, \ H^i_{(N+1)(N+5)} = \frac{6}{h_1} e^{-2\beta h_1} \bar{R}_i, \\
H^i_{(N+1)(N+6)} &= \frac{6}{(N+1)h_2} e^{-2\beta h_2} \bar{R}_i, \ H^i_{(N+2),(N+2)} = -4e^{-2\beta h_1} \bar{R}_i, \\
H^i_{(N+2),(N+5)} &= \frac{6}{h_1} e^{-2\beta h_1} \bar{R}_i, \\
H^i_{(N+3),(N+3)} &= -\frac{4}{N+1} e^{-2\beta h_2} \bar{R}_i, \ H^i_{(N+3),(N+6)} = \frac{6}{(N+1)h_2} e^{-2\beta h_2} \bar{R}_i, \\
H^i_{(N+4),(N+4)} &= \left(h_1^2 + h_2^2\right) \bar{R}_i - 2\bar{P}_i, H^i_{(N+5),(N+5)} = -12 \frac{e^{-2\beta h_1}}{h_1^2} \bar{R}_i, \\
H^i_{(N+6),(N+6)} &= -12 \frac{e^{-2\beta h_2}}{(N+1)h_2^2} \bar{R}_i, H^i_{N+6+j,N+6+j} = -I,
\end{aligned}$$

DOI: 10.1002/oca

$$H^i_{j,N+6+j} = \sqrt{N+1}\left[G_{ji} - G^f_j E_{ji} \quad 0\right]^T$$

$$H^i_{2N+6+j,2N+6+j} = -I,\ H^i_{N+1,2N+6+j} = \bar{P}_i\bar{A}_{ij} = \begin{bmatrix} P_{i1}A_{ij} & 0 \\ Z_i E_{ij} & 0 \end{bmatrix}, H^i_{3N+6+j,3N+6+j} = -I,$$

$$H^i_{N+4,2N+6+j} = \bar{P}_i\bar{A}_{ij} = \begin{bmatrix} P_{i1}A_{ij} & 0 \\ Z_i E_{ij} & 0 \end{bmatrix}, H^i_{4N+7,4N+7} = -I, H^i_{N+1,4N+7} = \sqrt{\varepsilon_i}\bar{P}_i,$$

$$H^i_{4N+8,4N+8} = -I,\ H^i_{N+1,4N+8} = \sqrt{N+1}\bar{C}_i^T = \sqrt{N+1}\left[C_i - G^f_i E_i \quad -C^f_i\right]^T,$$

$$H^i_{4N+9,4N+9} = -I,\ H^i_{N+4,4N+9} = \sqrt{\varepsilon_i}\bar{P}_i, H^i_{(4N+10),(4N+10)} = -\frac{\gamma}{4},$$

$$H^i_{(N+1),(4N+10)} = \bar{P}_i\bar{D}_i = \begin{bmatrix} P_{i1}D_i \\ 0 \end{bmatrix}, H^i_{(4N+11),(4N+11)} = -\frac{\gamma}{4},$$

$$H^i_{(N+4),(4N+11)} = \bar{P}_i\bar{D}_i = \begin{bmatrix} P_{i1}D_i \\ 0 \end{bmatrix}.$$

The following is the main result of the paper, which gives sufficient conditions for the H_∞ filtering design for system (2.1). Essentially, the idea of this proof is based on the construction of improved Lyapunov–Krasovskii functionals satisfying Lyapunov stability theorem for the time-delay system [21].

Theorem 1

For given $\gamma > 0, \beta > 0$, the filtering error system (2.4) has a prescribed H_∞ performance γ if there exist symmetric positive definite matrices $\bar{P}_i = \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i2} \end{bmatrix}$, $\bar{R}_i$ and matrices $Y_i, Z_i, C^f_i, G^f_i, i = \overline{1,N}$, such that the following LMIs hold:

$$\begin{bmatrix} H^i_{11} & H^i_{12} & \dots & H^i_{1(4N+11)} \\ * & H^i_{22} & \dots & H^i_{2(4N+11)} \\ . & . & \dots & . \\ * & * & \dots & H^i_{(4N+11)(4N+11)} \end{bmatrix} < 0, \quad i = \overline{1,N}. \tag{3.1}$$

Moreover, the filters are defined by the following:

$$A^f_i = P^{-1}_{i2}Y_i,\ D^f_i = P^{-1}_{i2}Z_i,\ C^f_i,\ G^f_i, \quad i = \overline{1,N},$$

and the solution of the filtering error system (2.4) satisfies the following:

$$||\xi(t)|| \leqslant \sqrt{\frac{\alpha_2}{\alpha_1}}||\varphi||_{C_1}e^{-\beta t}, \quad \forall t \geqslant 0.$$

Proof

The proof is rather long and technical, so for clarity, we divide it into two steps. The first step is to prove the exponential stability of the error system (2.4) by using Lyapunov fucntional method and LMI technique. The second step will focus on getting the H_∞ performance level condition (2.5).

Step 1. Exponential stability of system (2.4): consider the following Lyapunov–Krasovskii functional for the system (2.4): $V(t,x_t) = \sum_{i=1}^{N}\left[V_{i1}(t,x_t) + V_{i2}(t,x_t) + V_{i3}(t,x_t)\right]$, where

$$V_{i1}(t,x_t) = \xi_i(t)^T\bar{P}_i\xi_i(t), \quad V_{i2}(t,x_t) = h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\beta(\tau-t)}\dot{\xi}^T_i(\tau)\bar{R}_i\dot{\xi}_i(\tau)d\tau ds,$$

$$V_{i3}(t,x_t) = h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\beta(\tau-t)}\dot{\xi}^T_i(\tau)\bar{R}_i\dot{\xi}_i(\tau)d\tau ds.$$

DOI: 10.1002/oca

Firstly, we estimate the $\dot{V}_{i1}(t, x_t)$ as follows. From the condition (2.2), we obtain the following:

$$\begin{aligned}
2\xi_i(t)^T \bar{P}_i \bar{g}_i(.) &\leqslant 2||\xi_i(t)^T \bar{P}_i||.||\bar{g}_i(.)|| = 2||\xi_i(t)^T \bar{P}_i||.||g_i(.)|| \\
&\leqslant 2||\xi_i(t)^T \bar{P}_i||. \left[a_i \|\xi_i(t)\| + d_i \|\omega_i(t)\| + \sum_{j=1}^{N} a_{ij} \|\xi_j(t - h_{ij}(t))\| \right] \\
&\leqslant \varepsilon_i ||\xi_i(t)^T \bar{P}_i||^2 + a_i ||\xi_i(t)||^2 + 0.25\gamma ||\omega_i(t)||^2 + \sum_{j=1}^{N} a_{ij} \|\xi_j(t - h_{ij}(t))\|^2.
\end{aligned}$$

Using the Cauchy matrix inequality gives the following:

$$2\xi_i(t)^T \bar{P}_i \left[\sum_{j=1}^{N} \bar{A}_{ij} \xi_j(t - h_{ij}(t)) \right] \leqslant \sum_{j=1}^{N} \xi_i(t)^T \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i \xi_i(t) + \sum_{j=1}^{N} \xi_j(t - h_{ij}(t))^T \xi_j(t - h_{ij}(t)),$$

$$2\xi_i(t)^T \bar{P}_i \bar{D}_i \omega_i(t) \leqslant \frac{4}{\gamma} \xi_i(t)^T \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i \xi_i(t) + 0.25\gamma \omega_i(t)^T \omega_i(t).$$

Therefore, we have as follows:

$$\begin{aligned}
\dot{V}_{i1}(t, x_t) &= 2\xi_i(t)^T \bar{P}_i \left[\bar{A}_i \xi_i(t) + \sum_{j=1}^{N} \bar{A}_{ij} \xi_j(t - h_{ij}(t)) + \bar{D}_i \omega_i(t) + \bar{g}_i(.) \right] \\
&\leqslant \xi_i(t)^T \left[\bar{P}_i \bar{A}_i + \bar{A}_i^T \bar{P}_i \right] \xi_i(t) + \sum_{j=1}^{N} \xi_i(t)^T \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i \xi_i(t) \\
&\quad + \sum_{j=1}^{N} \xi_j(t - h_{ij}(t))^T \xi_j(t - h_{ij}(t)) + \frac{4}{\gamma} \xi_i(t)^T \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i \xi_i(t) + 0.25\gamma \omega_i(t)^T \omega_i(t) \\
&\quad + \varepsilon_i ||\xi_i(t)^T \bar{P}_i||^2 + a_i ||\xi_i(t)||^2 + 0.25\gamma ||\omega_i(t)||^2 + \sum_{j=1}^{N} a_{ij} \|\xi_j(t - h_{ij}(t))\|^2.
\end{aligned} \tag{3.2}$$

Siminarly,

$$\begin{aligned}
0 &= -2\dot{\xi}_i(t)^T \bar{P}_i \left[\dot{\xi}_i(t) - \bar{A}_i \xi_i(t) - \sum_{j=1}^{N} \bar{A}_{ij} \xi_j(t - h_{ij}(t)) - \bar{D}_i \omega_i(t) - \bar{g}_i(.) \right] \\
&\leqslant -2\dot{\xi}_i(t)^T \bar{P}_i \left[\dot{\xi}_i(t) - \bar{A}_i \xi_i(t) \right] + \sum_{j=1}^{N} \dot{\xi}_i(t)^T \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i \dot{\xi}_i(t) \\
&\quad + \sum_{j=1}^{N} \xi_j(t - h_{ij}(t))^T \xi_j(t - h_{ij}(t)) + \frac{4}{\gamma} \dot{\xi}_i(t)^T \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i \dot{\xi}_i(t) + 0.25\gamma \omega_i(t)^T \omega_i(t) \\
&\quad + \varepsilon_i ||\dot{\xi}_i(t)^T \bar{P}_i||^2 + a_i ||\xi_i(t)||^2 + 0.25\gamma ||\omega_i(t)||^2 + \sum_{j=1}^{N} a_{ij} \|\xi_j(t - h_{ij}(t))\|^2
\end{aligned} \tag{3.3}$$

because of the following inequalities:

$$2\dot{\xi}_i(t)^T\bar{P}_i\left[\sum_{j=1}^{N}\bar{A}_{ij}\xi_j(t-h_{ij}(t))\right]\leqslant\sum_{j=1}^{N}\dot{\xi}_i(t)^T\bar{P}_i\bar{A}_{ij}\bar{A}_{ij}^T\bar{P}_i\dot{\xi}_i(t)+\sum_{j=1}^{N}\xi_j(t-h_{ij}(t))^T\xi_j(t-h_{ij}(t)),$$

$$2\dot{\xi}_i(t)^T\bar{P}_i\bar{D}_i\omega_i(t)\leqslant\frac{4}{\gamma}\dot{\xi}_i(t)^T\bar{P}_i\bar{D}_i\bar{D}_i^T\bar{P}_i\dot{\xi}_i(t)+0.25\gamma\omega_i(t)^T\omega_i(t),$$

$$\begin{aligned}2\dot{\xi}_i(t)^T\bar{P}_i\bar{g}_i(.)&\leqslant 2||\dot{\xi}_i(t)^T\bar{P}_i||.||\bar{g}_i(.)||=2||\dot{\xi}_i(t)^T\bar{P}_i||.||g_i(.)||\\&\leqslant\varepsilon_i||\dot{\xi}_i(t)^T\bar{P}_i||^2+a_i||\xi_i(t)||^2+0.25\gamma||\omega_i(t)||^2\\&\quad+\sum_{j=1}^{N}a_{ij}\|\xi_j(t-h_{ij}(t))\|^2.\end{aligned}$$

Secondly, we estimate $\dot{V}_{i2}(t,x_t)$ as follows. Applying the Lemma 2, we have the following:

$$\begin{aligned}-h_1\int_{t-h_1}^{t}\dot{\xi}_i(s)^T\bar{R}_i\dot{\xi}_i(s)ds&\leqslant-[\xi_i(t)-\xi_i(t-h_1)]^T\bar{R}_i[\xi_i(t)-\xi_i(t-h_1)]\\&-12\left[\frac{\xi_i(t)+\xi_i(t-h_1)}{2}-\frac{1}{h_1}\int_{t-h_1}^{t}\xi_i(s)ds\right]^T\bar{R}_i\left[\frac{\xi_i(t)+\xi_i(t-h_1)}{2}-\frac{1}{h_1}\int_{t-h_1}^{t}\xi_i(s)ds\right].\end{aligned}$$

Therefore, we obtain the following:

$$\begin{aligned}\dot{V}_{i2}(t,x_t)&\leqslant h_1^2\dot{\xi}_i(t)^T\bar{R}_i\dot{\xi}_i(t)-2\beta V_{i2}(t,x_t)-h_1e^{-2\beta h_1}\int_{t-h_1}^{t}\dot{\xi}_i(s)^T\bar{R}_i\dot{\xi}_i(s)ds\\&\leqslant h_1^2\dot{\xi}_i(t)^T\bar{R}_i\dot{\xi}_i(t)-2\beta V_{i2}(t,x_t)-e^{-2\beta h_1}[\xi_i(t)-\xi_i(t-h_1)]^T\bar{R}_i[\xi_i(t)-\xi_i(t-h_1)]\\&\quad-12e^{-2\beta h_1}\left[\frac{\xi_i(t)+\xi_i(t-h_1)}{2}-\frac{1}{h_1}\int_{t-h_1}^{t}\xi_i(s)ds\right]^T\\&\quad\times\bar{R}_i\left[\frac{\xi_i(t)+\xi_i(t-h_1)}{2}-\frac{1}{h_1}\int_{t-h_1}^{t}\xi_i(s)ds\right]\end{aligned}\tag{3.4}$$

Thirdly, we evaluate $\dot{V}_{i3}(t,x_t)$ as follows. To do that, we need some the following inequalities:

$$\begin{aligned}-h_2\int_{t-h_2}^{t}\dot{\xi}_i(s)^T\bar{R}_i\dot{\xi}_i(s)ds&\leqslant-[\xi_i(t)-\xi_i(t-h_2)]^T\bar{R}_i[\xi_i(t)-\xi_i(t-h_2)]\\&-12\left[\frac{\xi_i(t)+\xi_i(t-h_2)}{2}-\frac{1}{h_2}\int_{t-h_2}^{t}\xi_i(s)ds\right]^T\bar{R}_i\left[\frac{\xi_i(t)+\xi_i(t-h_2)}{2}-\frac{1}{h_2}\int_{t-h_2}^{t}\xi_i(s)ds\right],\end{aligned}$$

and

DOI: 10.1002/oca

$$-h_2 \int_{t-h_2}^{t} \dot{\xi}_i(s)^T \bar{R}_i \dot{\xi}_i(s) ds \leqslant -h_{ji}(t) \int_{t-h_{ji}(t)}^{t} \dot{\xi}_i(s)^T \bar{R}_i \dot{\xi}_i(s) ds$$
$$\leqslant -[\xi_i(t) - \xi_i(t-h_{ji}(t))]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_{ji}(t))]$$

because of applying Lemma 2 and the condition $0 \leqslant h_{ji}(t) \leqslant h_2$ for $j = \overline{1, N}$.

Consequently,

$$\begin{aligned}
-h_2 \int_{t-h_2}^{t} \dot{\xi}_i(s)^T \bar{R}_i \dot{\xi}_i(s) ds &\leqslant -\sum_{j=1}^{N} \frac{1}{N+1} [\xi_i(t) - \xi_i(t-h_{ji}(t))]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_{ji}(t))] \\
&- \frac{1}{N+1} [\xi_i(t) - \xi_i(t-h_2)]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_2)] \\
&- \frac{12}{N+1} \left[\frac{\xi_i(t)+\xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds \right]^T \bar{R}_i \left[\frac{\xi_i(t)+\xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds \right].
\end{aligned}$$

Hence, we obtain the following:

$$\begin{aligned}
\dot{V}_{i3}(t, x_t) &\leqslant h_2^2 \dot{\xi}_i(t)^T \bar{R}_i \dot{\xi}_i(t) - 2\beta V_{i3}(t, x_t) - h_2 e^{-2\beta h_2} \int_{t-h_2}^{t} \dot{\xi}_i(s)^T \bar{R}_i \dot{\xi}_i(s) ds \\
&\leqslant h_2^2 \dot{\xi}_i(t)^T \bar{R}_i \dot{\xi}_i(t) - 2\beta V_{i3}(t, x_t) \\
&\quad - \sum_{j=1}^{N} \frac{1}{N+1} e^{-2\beta h_2} [\xi_i(t) - \xi_i(t-h_{ji}(t))]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_{ji}(t))] \\
&\quad - \frac{1}{N+1} e^{-2\beta h_2} [\xi_i(t) - \xi_i(t-h_2)]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_2)] \\
&\quad - \frac{12}{N+1} e^{-2\beta h_2} \left[\frac{\xi_i(t) + \xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds \right]^T \bar{R}_i \times \\
&\quad \times \left[\frac{\xi_i(t) + \xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds \right].
\end{aligned} \tag{3.5}$$

Noting that the following:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \xi_j(t-h_{ij}(t))^T \xi_j(t-h_{ij}(t)) = \sum_{i=1}^{N} \left[\sum_{j=1}^{N} \xi_i(t-h_{ji}(t))^T \xi_i(t-h_{ji}(t)) \right],$$
$$\sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \xi_j(t-h_{ij}(t))^T \xi_j(t-h_{ij}(t)) = \sum_{i=1}^{N} \left[\sum_{j=1}^{N} a_{ji} \xi_i(t-h_{ji}(t))^T \xi_i(t-h_{ji}(t)) \right],$$

because of the following identity:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ji}.$$

Therefore, combining the earlier identities and the inequalities (3.2)–(3.3)–(3.4)–(3.5), we obtain that the following:

DOI: 10.1002/oca

$$\begin{aligned}
\dot{V}(t,x_t) + 2\beta V(t,x_t) \leqslant & \sum_{i=1}^{N} \left[\xi_i(t)^T \left[\bar{P}_i \bar{A}_i + \bar{A}_i^T \bar{P}_i\right] \xi_i(t) + 2\beta \xi_i(t)^T \bar{P}_i \xi_i(t)\right. \\
& + \sum_{j=1}^{N} \xi_i(t)^T \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i \xi_i(t) + \sum_{j=1}^{N} \xi_i(t-h_{ji}(t))^T \xi_i(t-h_{ji}(t)) \\
& + \frac{4}{\gamma} \xi_i(t)^T \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i \xi_i(t) + 0.25\gamma \omega_i(t)^T \omega_i(t) \\
& + \varepsilon_i ||\xi_i(t)^T \bar{P}_i||^2 + a_i ||\xi_i(t)||^2 + 0.25\gamma ||\omega_i(t)||^2 + \sum_{j=1}^{N} a_{ji} \|\xi_i(t-h_{ji}(t))\|^2 \\
& + h_1^2 \dot{\xi}_i(t)^T \bar{R}_i \dot{\xi}_i(t) - e^{-2\beta h_1} [\xi_i(t) - \xi_i(t-h_1)]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_1)] \\
& - 12 e^{-2\beta h_1} \left[\frac{\xi_i(t) + \xi_i(t-h_1)}{2} - \frac{1}{h_1} \int_{t-h_1}^{t} \xi_i(s) ds\right]^T \\
& \times \bar{R}_i \left[\frac{\xi_i(t) + \xi_i(t-h_1)}{2} - \frac{1}{h_1} \int_{t-h_1}^{t} \xi_i(s) ds\right] \\
& + h_2^2 \dot{\xi}_i(t)^T \bar{R}_i \dot{\xi}_i(t) - \sum_{j=1}^{N} \frac{1}{N+1} e^{-2\beta h_2} [\xi_i(t) - \xi_i(t-h_{ji}(t))]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_{ji}(t))] \\
& - \frac{1}{N+1} e^{-2\beta h_2} [\xi_i(t) - \xi_i(t-h_2)]^T \bar{R}_i [\xi_i(t) - \xi_i(t-h_2)] \\
& - \frac{12}{N+1} e^{-2\beta h_2} \left[\frac{\xi_i(t) + \xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds\right]^T \\
& \times \bar{R}_i \left[\frac{\xi_i(t) + \xi_i(t-h_2)}{2} - \frac{1}{h_2} \int_{t-h_2}^{t} \xi_i(s) ds\right] \\
& - 2\dot{\xi}_i(t)^T \bar{P}_i \left[\dot{\xi}_i(t) - \bar{A}_i \xi_i(t)\right] + \sum_{j=1}^{N} \dot{\xi}_i(t)^T \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i \dot{\xi}_i(t) \\
& + \sum_{j=1}^{N} \xi_i(t-h_{ji}(t))^T \xi_i(t-h_{ji}(t)) + \frac{4}{\gamma} \dot{\xi}_i(t)^T \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i \dot{\xi}_i(t) + 0.25\gamma \omega_i(t)^T \omega_i(t) \\
& + \varepsilon_i ||\dot{\xi}_i(t)^T \bar{P}_i||^2 + a_i ||\xi_i(t)||^2 + 0.25\gamma ||\omega_i(t)||^2 + \sum_{j=1}^{N} a_{ji} \|\xi_i(t-h_{ji}(t))\|^2.
\end{aligned}$$

Hence,

$$\begin{aligned}
\dot{V}(t,x_t) + 2\beta V(t,x_t) \leqslant \gamma \sum_{i=1}^{N} ||\omega_i(t)||^2 + \sum_{i=1}^{N} \Xi_i(t)^T M^i \Xi_i(t) \\
- (N+1) \sum_{i=1}^{N} \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ji} \xi_i(t-h_{ji}(t))||^2 \right],
\end{aligned} \tag{3.6}$$

where $v_{ij} = \xi_i(t-h_{ji}(t))^T,\ j = \overline{1,N}$, and

DOI: 10.1002/oca

$$\Xi_i(t) = \left[v_{i1} \ \ldots \ v_{iN} \ \xi_i(t)^T \ \xi_i(t-h_1)^T \ \xi_i(t-h_2)^T \ \dot{\xi}_i(t)^T \ \int\limits_{t-h_1}^{t} \xi_i(s)^T ds \ \int\limits_{t-h_2}^{t} \xi_i(s)^T ds \right]^T,$$

$$M^i = \begin{bmatrix} M^i_{11} & M^i_{12} & \ldots & M^i_{1(N+6)} \\ * & M^i_{22} & \ldots & M^i_{2(N+6)} \\ . & . & \ldots & . \\ * & * & \ldots & M^i_{(N+6)(N+6)} \end{bmatrix}, \quad i = \overline{1,N},$$

$$\begin{aligned}
M^i_{jj} &= -\frac{1}{N+1} e^{-2\beta h_2} \bar{R}_i + (2 + 2a_{ji})I + (N+1)\bar{G}^T_{ji}\bar{G}_{ji}, \ j = \overline{1,N}, \\
M^i_{j(N+1)} &= \frac{1}{N+1} e^{-2\beta h_2} \bar{R}_i, \ j = \overline{1,N}, \\
M^i_{(N+1)(N+1)} &= \bar{P}_i \bar{A}_i + \bar{A}_i^T \bar{P}_i + 2\beta \bar{P}_i - 4e^{-2\beta h_1} \bar{R}_i - \frac{N+4}{N+1} e^{-2\beta h_2} \bar{R}_i \\
&\quad + \sum_{j=1}^{N} \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i + \frac{4}{\gamma} \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i + 2a_i I + \varepsilon_i \bar{P}_i^2 + (N+1)\bar{C}_i^T \bar{C}_i, \\
M^i_{(N+1)(N+2)} &= -2e^{-2\beta h_1} \bar{R}_i, \ M^i_{(N+1)(N+3)} = -\frac{2}{N+1} e^{-2\beta h_2} \bar{R}_i, \\
M^i_{(N+1)(N+4)} &= \bar{A}_i^T \bar{P}_i, M^i_{(N+1)(N+5)} = \frac{6}{h_1} e^{-2\beta h_1} \bar{R}_i, \ M^i_{(N+1)(N+6)} = \frac{6}{(N+1)h_2} e^{-2\beta h_2} \bar{R}_i, \\
M^i_{(N+2),(N+2)} &= -4e^{-2\beta h_1} \bar{R}_i, \ M^i_{(N+2),(N+5)} = \frac{6}{h_1} e^{-2\beta h_1} \bar{R}_i, \\
M^i_{(N+3),(N+3)} &= -\frac{4}{N+1} e^{-2\beta h_2} \bar{R}_i, M^i_{(N+3),(N+6)} = \frac{6}{(N+1)h_2} e^{-2\beta h_2} \bar{R}_i, \\
M^i_{(N+4),(N+4)} &= (h_1^2 + h_2^2)\bar{R}_i - 2\bar{P}_i + \sum_{j=1}^{N} \bar{P}_i \bar{A}_{ij} \bar{A}_{ij}^T \bar{P}_i + \frac{4}{\gamma} \bar{P}_i \bar{D}_i \bar{D}_i^T \bar{P}_i + \varepsilon_i \bar{P}_i^2, \\
M^i_{(N+5),(N+5)} &= -12 \frac{e^{-2\beta h_1}}{h_1^2} \bar{R}_i, \ M^i_{(N+6),(N+6)} = -12 \frac{e^{-2\beta h_2}}{(N+1)h_2^2} \bar{R}_i.
\end{aligned}$$

Therefore, applying the Schur complement lemma, Lemma 1 and the condition (3.1) leads to $M^i < 0, \forall i = \overline{1,N}$, and hence, from the inequality (3.6), it follows that:

$$\dot{V}(t,x_t) + 2\beta V(t,x_t) \leqslant \sum_{i=1}^{N} \gamma ||\omega_i(t)||^2 - (N+1) \sum_{i=1}^{N} \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ji}\xi_i(t - h_{ji}(t))||^2 \right]. \tag{3.7}$$

Letting $\omega_i(t) = 0, \forall i = \overline{1,N}$, and because the following:

$$-(N+1) \sum_{i=1}^{N} \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ji}\xi_i(t - h_{ji}(t))||^2 \right] \leqslant 0,$$

we finally obtain from the inequality (3.7) that the following:

$$\dot{V}(t,x_t) + 2\beta V(t,x_t) \leqslant 0, \quad t \geqslant 0,$$

which implies

$$V(t,x_t) \leqslant V(0,x_0) e^{-2\beta t}, \quad t \geqslant 0.$$

Besides, it is easy to verify that the following:

$$\alpha_1 \sum_{i=1}^{N} ||\xi_i(t)||^2 \leqslant V(t, x_t), \quad V(0, x_0) \leqslant \alpha_2 \sum_{i=1}^{N} ||\bar{\varphi}_i||_{C_1}^2 = \alpha_2 \sum_{i=1}^{N} ||\varphi_i||_{C_1}^2. \tag{3.8}$$

Hence, we have as follows:

$$\alpha_1 \sum_{i=1}^{N} ||\xi_i(t)||^2 \leqslant V(t, x_t) \leqslant V(0, x_0)e^{-2\beta t} \leqslant \alpha_2 \sum_{i=1}^{N} ||\varphi_i||_{C_1}^2 e^{-2\beta t},$$

consequently,

$$||\xi(t)|| \leqslant \sqrt{\frac{\alpha_2}{\alpha_1}} ||\varphi||_{C_1} e^{-\beta t}, \quad t \geqslant 0.$$

Step 2. H_∞ performance level γ: consider the following relation:

$$\int_0^s \sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2\right] dt = \int_0^s \left(\sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2\right] + \dot{V}(t, x_t) \right) dt - \int_0^s \dot{V}(t, x_t) dt.$$

Because $V(t, x_t) \geqslant 0$, $t \geqslant 0$, we have as follows:

$$-\int_0^s \dot{V}(t, x_t) dt = V(0, x_0) - V(s, x_s) \leqslant V(0, x_0),$$

and hence,

$$\int_0^s \sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2\right] dt \leqslant \int_0^s \left(\sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2\right] + \dot{V}(t, x_t) \right) dt + V(0, x_0). \tag{3.9}$$

Combining (3.7) and the inequality $V(t, x_t) \geqslant \sum_{i=1}^{N} \xi_i(t)^T \bar{P}_i \xi_i(t)$, we obtain the following:

$$\begin{aligned} \dot{V}(t, x_t) \leqslant & \sum_{i=1}^{N} \gamma ||\omega_i(t)||^2 - 2\beta \sum_{i=1}^{N} \xi_i(t)^T \bar{P}_i \xi_i(t) \\ & - (N+1) \sum_{i=1}^{N} \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ji} \xi_i(t - h_{ji}(t))||^2 \right]. \end{aligned} \tag{3.10}$$

Observe that the value of $||\tilde{z}_i(t)||^2$ is estimated due to (2.4) as follows:

$$||\tilde{z}_i(t)||^2 = ||\bar{C}_i \xi_i(t) + \sum_{j=1}^{N} \bar{G}_{ij} \xi_j(t - h_{ij}(t))||^2 \leqslant (N+1) \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ij} \xi_j(t - h_{ij}(t))||^2 \right].$$

Then, from the expressions as follows:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} ||\bar{G}_{ij} \xi_j(t - h_{ij}(t))||^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} ||\bar{G}_{ji} \xi_i(t - h_{ji}(t))||^2,$$

DOI: 10.1002/oca

we have

$$\sum_{i=1}^{N} ||\tilde{z}_i(t)||^2 \leqslant \sum_{i=1}^{N} (N+1) \left[||\bar{C}_i \xi_i(t)||^2 + \sum_{j=1}^{N} ||\bar{G}_{ji} \xi_i(t - h_{ji}(t))||^2 \right]. \tag{3.11}$$

Submitting the estimation of $\dot{V}(t, x_t)$ and $||\tilde{z}_i(t)||^2$ defined by (3.10) and (3.11) into (3.9), respectively, we obtain the following:

$$\int_0^s \sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2 \right] dt \leqslant \int_0^s \left[-2\beta \sum_{i=1}^{N} \xi_i(t)^T \bar{P}_i \xi_i(t) \right] dt + V(0, x_0), \ \forall s \geqslant 0. \tag{3.12}$$

Hence, from (3.8) and (3.12) it follows that

$$\int_0^s \sum_{i=1}^{N} \left[||\tilde{z}_i(t)||^2 - \gamma ||\omega_i(t)||^2 \right] dt \leqslant V(0, x_0) \leqslant \sum_{i=1}^{N} \alpha_2 ||\varphi_i||^2_{C_1}, \ \forall s \geqslant 0.$$

Letting $s \to +\infty$, and setting $c_0 = \frac{\alpha_2}{\gamma} > 0$, we obtain the following:

$$\int_0^\infty \sum_{i=1}^{N} ||\tilde{z}_i(t)||^2 dt \leqslant \gamma \int_0^\infty \sum_{i=1}^{N} ||\omega_i(t)||^2 dt + \alpha_2 \sum_{i=1}^{N} ||\varphi_i||^2_{C_1},$$

implies

$$\frac{\int_0^\infty \sum_{i=1}^{N} ||\tilde{z}_i(t)||^2 dt}{c_0 \sum_{i=1}^{N} ||\varphi_i||^2_{C_1} + \int_0^\infty \sum_{i=1}^{N} ||\omega_i(t)||^2 dt} \leqslant \gamma.$$

This completes the proof of the theorem. □

Remark 3.1
Theorem 1 provides sufficient conditions for designing the decentralized H_∞ filter of the nonlinear large-scale system (2.1) in terms of the solutions of LMIs, which guarantee the filter error system to be exponentially stable with a prescribed decay rate β. Moreover, note that the time-varying delays are non-differentiable, therefore, the methods proposed in [5, 10, 13–15] are not applicable to system (2.1). The LMI conditions (3.1) depend on parameters of the system under consideration as well as the delay bounds. The feasibility of the LMIs can be tested by the reliable and efficient MATLAB LMI CONTROL TOOLBOX [18].

Remark 3.2
It is worth noting that the considered system (2.1) contains a single delay in interactions, that is, the differential equation describing x_i can only accept x_j with one delay. By using the same approach of Theorem 3.1, we can extend the result of this paper to the general case where the large-scale system contains multiple delays in the interactions, that is, the differential equation describing x_i can accept x_j with different delays. However, the derived stability condition will contain more complicated LMI conditions with additional free-weighting matrices and thus, it will increase the computational complexity. For example, for the system where the differential equation describing x_i accepts x_j with different two delays as follows:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t - h_{ij}(t)) + D_i \omega_i(t) + \sum_{j=1, j \neq i}^{N} A^\star_{ij} x_j(t - h^\star_{ij}(t)) + g(.),$$

DOI: 10.1002/oca

the number of interacted delays is increased to $N(N-1)$, the variable $\xi_j(t-h_{ij}^\star(t))$ makes the size of matrices M^i, under the (3.6), increase to $(2N+5)\times(2N+5)$, and hence, the size of matrices in LMI (3.1) is increased to $(8N+8)\times(8N+8)$.

Example 3.1
Consider a large-scale system of the form as follows:

$$\begin{cases} \dot{x}_1(t) = A_1x_1(t) + A_{11}x_1(t-h_{11}(t)) + A_{12}x_2(t-h_{12}(t)) + D_1\omega_1(t) \\ \quad +g_1(t, x_1(t), x_1(t-h_{11}(t)), x_2(t-h_{12}(t)),\ \omega_1(t)), \\ z_1(t) = C_1x_1(t) + G_{11}x_1(t-h_{11}(t)) + G_{12}x_2(t-h_{12}(t)), \\ y_1(t) = E_1x_1(t) + E_{11}x_1(t-h_{11}(t)) + E_{12}x_2(t-h_{12}(t)), \quad \forall t \geqslant 0, \\ x_1(\theta) = \varphi_1(\theta), \quad \forall\theta \in [-h_2, 0], \end{cases}$$

$$\begin{cases} \dot{x}_2(t) = A_2x_2(t) + A_{22}x_2(t-h_{22}(t)) + A_{21}x_1(t-h_{21}(t)) + D_2\omega_2(t) \\ \quad +g_2(t, x_2(t), x_2(t-h_{22}(t)), x_1(t-h_{21}(t)),\ \omega_2(t)), \\ z_2(t) = C_2x_2(t) + G_{22}x_2(t-h_{22}(t)) + G_{21}x_1(t-h_{21}(t)), \\ y_2(t) = E_2x_2(t) + E_{22}x_2(t-h_{22}(t)) + E_{21}x_1(t-h_{21}(t)), \quad \forall t \geqslant 0, \\ x_2(\theta) = \varphi_2(\theta), \quad \forall\theta \in [-h_2, 0], \end{cases}$$

where the absolute rotor angle and angular velocity of the machine in each subsystem are denoted by $x_1 = [x_{11},\ x_{12}]^T$, $x_2 = [x_{21},\ x_{22}]^T$, respectively; the ith system coefficient A_i, A_{ii}; the uncertain coefficients D_i; the ith system perturbations $g_i(.)$, and the modulus of the transfer admittance A_{12}, A_{21}; output observations $z_i = [z_{i1},\ z_{i2}]^T$; the initial input φ_i; the time-varying delays $h_{ij}(t)$ between the two machine in the subsystem as follows:

$$h_{11}(t) = \begin{cases} 1+0.2sin(t), & t \in H, \\ 1, & t \notin H, \end{cases} \qquad h_{12}(t) = \begin{cases} 1.1+0.1sin(t), & t \in H, \\ 1.1, & t \notin H, \end{cases}$$

$$h_{21}(t) = \begin{cases} 1.3+0.1sin(t), & t \in H, \\ 1.3, & t \notin H, \end{cases} \qquad h_{22}(t) = \begin{cases} 1.2+0.2sin(t), & t \in H, \\ 1.2, & t \notin H, \end{cases}$$

$$H = \cup_{k\in N}(2k\pi, (2k+1)\pi),$$

$$A_1 = \begin{bmatrix} -1 & 0.1 \\ 1 & -2 \end{bmatrix},\ A_{11} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0.001 & 0.002 \\ 0.003 & 0.004 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 0.1 & -1 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.001 & 0.003 \\ 0.003 & 0.001 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.004 & 0.003 \\ 0.002 & 0.001 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.004 & 0.003 \\ 0.003 & 0.004 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.01 & 0.02 \\ 0.03 & 0.04 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.04 & 0.03 \\ 0.02 & 0.01 \end{bmatrix},$$

$$G_{11} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},\ G_{22} = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix},\ G_{12} = \begin{bmatrix} 0.02 & 0.03 \\ 0.03 & 0.02 \end{bmatrix},\ G_{21} = \begin{bmatrix} 0.03 & 0.02 \\ 0.02 & 0.03 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},\ E_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},\ E_{12} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}, E_2 = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix},\ E_{21} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix},\ E_{22} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$g_1(.) = 0.0001 \begin{bmatrix} \sqrt{x_{11}(t)^2 + x_{11}(t-h_{11})^2 + x_{21}(t-h_{12}(t))^2} \\ \sqrt{x_{12}(t)^2 + x_{12}(t-h_{11})^2 + x_{22}(t-h_{12}(t))^2} \end{bmatrix},$$

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$$g_2(.) = 0.0001 \begin{bmatrix} \sqrt{x_{21}(t)^2 + x_{21}(t-h_{22})^2 + x_{11}(t-h_{21}(t))^2} \\ \sqrt{x_{22}(t)^2 + x_{22}(t-h_{22})^2 + x_{12}(t-h_{21}(t))^2} \end{bmatrix},$$

$$a_1 = a_2 = a_{21} = a_{12} = a_{11} = a_{22} = 0.0001, \quad d_1 = d_2 = 0.01$$

We see that the time delay functions $h_{ij}(t)$ are bounded but non-differentiable, and that the outputs are nonlinear observation outputs. Therefore, most existing design methods of [5, 10, 13–15] are not applicable to this system. For $\beta = 0.1,\ \gamma = 2,\ h_1 = 1,\ \text{and} h_2 = 1.4$, the LMIs (3.1) are feasible with the following:

$$\bar{P}_1 = \begin{bmatrix} 235.4865 & -22.1780 & 0 & 0 \\ -22.1780 & 158.0102 & 0 & 0 \\ 0 & 0 & 217.3651 & -0.1307 \\ 0 & 0 & -0.1307 & 217.6605 \end{bmatrix}, \bar{R}_1 = \begin{bmatrix} 57.1627 & 3.5050 & 0.5222 & -1.0482 \\ 3.5050 & 32.7948 & 0.0310 & -0.6471 \\ 0.5222 & 0.0310 & 59.6926 & -0.0181 \\ -1.0482 & -0.6471 & -0.0181 & 59.7441 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} -137.7520 & 0.4251 \\ 0.4222 & -138.6972 \end{bmatrix}, \bar{P}_2 = \begin{bmatrix} 136.8554 & -17.6992 & 0 & 0 \\ -17.6992 & 199.7307 & 0 & 0 \\ 0 & 0 & 203.1388 & -0.0258 \\ 0 & 0 & -0.0258 & 203.1053 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.0001 & 0.0273 \\ -0.0024 & -0.0818 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} 28.3050 & 3.2713 & -0.1654 & 0.0676 \\ 3.2713 & 48.8079 & -0.2188 & 0.1246 \\ -0.1654 & -0.2188 & 52.7605 & -0.0018 \\ 0.0676 & 0.1246 & -0.0018 & 52.7573 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -127.1231 & 0.0710 \\ 0.0699 & -127.0316 \end{bmatrix}, \ Z_2 = \begin{bmatrix} -0.0174 & 0.0022 \\ 0.0095 & -0.0015 \end{bmatrix},$$

The filter can be obtained as follows:

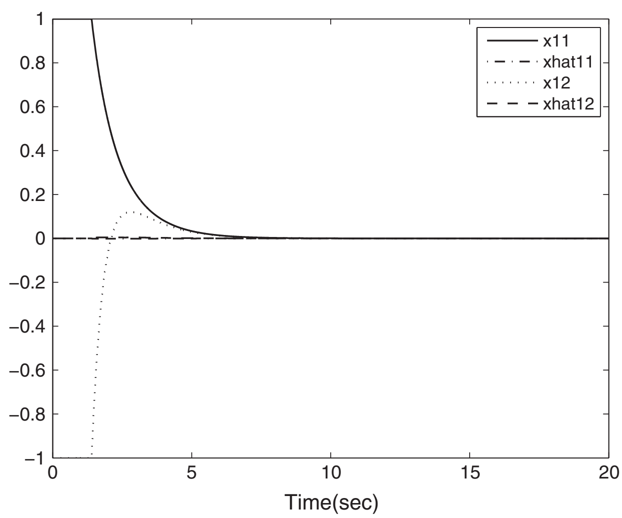

Figure 1. The state x_1 and the estimated state $\hat{x}_1$.

DOI: 10.1002/oca

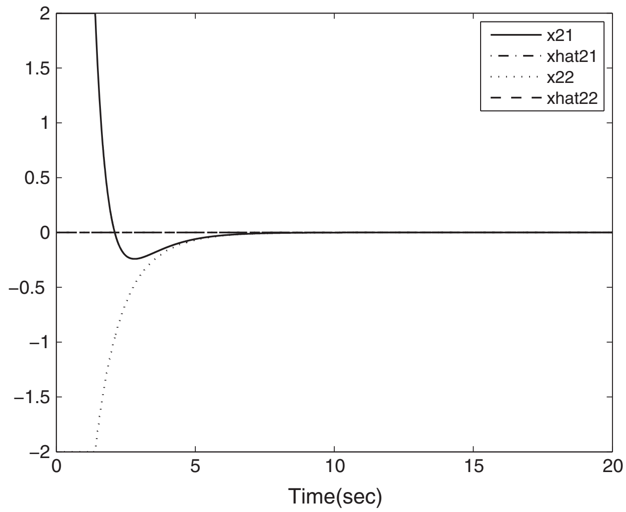

Figure 2. the state x_2 and the estimated state $\hat{x}_2$.

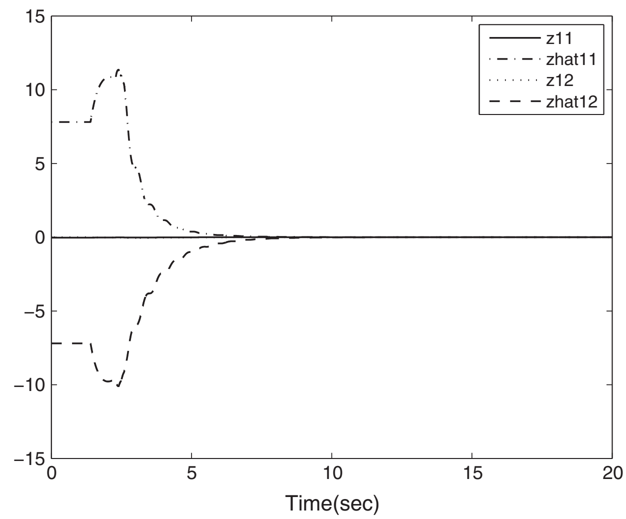

Figure 3. the signal z_1 and the estimated signal $\hat{z}_1$.

$$A_1^f = \begin{bmatrix} 68.8760 & 0.2125 \\ 0.2111 & -69.3486 \end{bmatrix},\ D_1^f = \begin{bmatrix} 0.0001 & 0.0137 \\ -0.0012 & -0.0409 \end{bmatrix}, C_1^f = \begin{bmatrix} 3.7945 & 0.0789 \\ 0.0789 & 3.5889 \end{bmatrix},$$

$$G_1^f = \begin{bmatrix} 1.2089 & -0.6973 \\ -0.6973 & 0.8509 \end{bmatrix}, A_2^f = \begin{bmatrix} -10.5936 & 0.0059 \\ 0.0058 & -10.5860 \end{bmatrix},$$

$$D_2^f = \begin{bmatrix} -0.0014 & 0.0002 \\ 0.0008 & -0.0001 \end{bmatrix}, C_2^f = \begin{bmatrix} 3.9165 & 0.0116 \\ 0.0116 & 3.9327 \end{bmatrix}, G_2^f = \begin{bmatrix} 0.2600 & -0.1763 \\ -0.1763 & 0.2107 \end{bmatrix}.$$

Moreover, the filtering error solution $\xi(t)$ of the system satisfies the following:

$$||\xi(t)|| \leqslant 1.8799.\, e^{-0.1t}.||\varphi||_{C_1}, \quad \forall t \geqslant 0.$$

DOI: 10.1002/oca

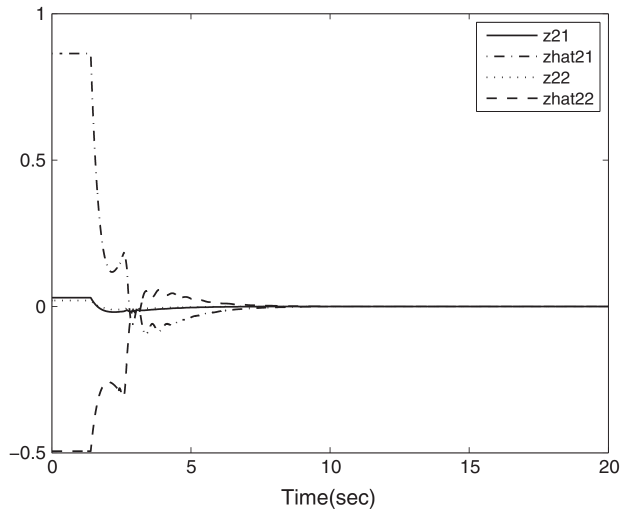

Figure 4. the signal z_2 and the estimated signal $\hat{z}_2$.

Figures 1–4 show the trajectories of the states, signals, and their estimations of the system with the initial conditions $\varphi_1 = [1, \ -1]^T$, $\varphi_2 = [2, \ -2]^T$, and the exogenous disturbance signals $\omega_i \equiv 0, \ i = 1, 2$.

4. CONCLUSION

The problem of H_∞ filtering for nonlinear large-scale systems with interval time-varying delays with time-varying delayed interactions has been investigated. We have posed a systematic way to study the H_∞ filtering problem for such a system, which combines the Lyapunov functional method and the pertubation approach. Moreover, it is important that the interaction terms with time-varying delays are bounded by nonlinear bounding functions including all states of subsystems. By introducing a set of augmented Lyapunov–Krasovskii functionals and using generalized Jensen inequality, sufficient conditions for designing decentralized H_∞ filter have been established in terms of LMIs. A system simulation example is presented to verify the effectiveness of the proposed result.

ACKNOWLEDGEMENTS

This work was supported by the the National Foundation for Science and Technology Development, Vietnam (grant 101.01-2014.35) and the Faculty Strategic Fund, Deakin University, Australia. The authors would like to thank anonymous reviewers for their valuable comments and suggestions, which allowed us to improve the paper.

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DOI: 10.1002/oca