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Decentralized stability for switched nonlinear large-scale systems with interval time-varying delays in interconnections

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ABSTRACT

In this paper, the problem of decentralized stability of switched nonlinear large-scale systems with time-varying delays in interconnections is studied. The time delays are assumed to be any continuous functions belonging to a given interval. By constructing a set of new Lyapunov–Krasovskii functionals, which are mainly based on the information of the lower and upper delay bounds, a new delay-dependent sufficient condition for designing switching law of exponential stability is established in terms of linear matrix inequalities (LMIs). The developed method using new inequalities for lower bounding cross terms eliminate the need for overbounding and provide larger values of the admissible delay bound. Numerical examples are given to illustrate the effectiveness of the new theory.

1. Introduction

Switching systems belong to an important class of hybrid systems, which are described by a family of differential equations together with specified rules to switch between them. A switching system can be represented by a differential equation of the form

$$\dot{x}(t) = f_\sigma(t, x(t)), \quad t \geq 0,$$

where $\{f_\sigma(.,.) : \sigma \in \mathcal{I}\}$ is a family of functions parameterized by some index set $\mathcal{I}$, which is typically a finite set, and $\sigma(.)$, which depends on the system state at each time, is the switching rule/signal determining a switching sequence for the given system. Switching systems arise in many practical processes that cannot be described by exclusively continuous or exclusively discrete models, such as manufacturing, communication networks, automotive engineering control, chemical processes [1–3]. During the last decades, the stability problem of switched linear time-delay systems has attracted a lot of attention [4–11].

On the other hand, there has been a considerable research interest in large-scale interconnected systems. A typical large-scale interconnected system such as a power grid consists of many subsystems and individual elements connected together to form a large, complex network capable of generating, transmitting and distributing electrical energy over a large geographical area. In general, a large-scale system can be characterized by a large number of variables representing the system, a strong interaction between subsystem variables, and a complex interaction between subsystems. The problem of decentralized control of large-scale interconnected dynamical systems has been receiving considerable attention, because there

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are a large number of large-scale interconnected dynamical systems in many practical control problems, e.g., transportation systems, power systems, communication systems, economic systems, social systems, and so on. [12–15]. The operation of large-scale interconnected systems requires the ability to monitor and stabilize in the face of uncertainties, disturbances, failures and attacks through the utilization of internal system states. However, even with the assumption that all the state variables are available for feedback control, the task of effective controlling a large-scale interconnected system using a global (centralized) state feedback controller is still not easy as there is a necessary requirement for information transfer between the subsystems [16–19].

The majority of the previous works treated asymptotic stability for switching linear time delay systems under arbitrary switching signal law. The exponential stability problem was considered in [20] for switching linear systems with impulsive effects by using the matrix measure concept, and in [21] for nonholonomic chained systems with strongly nonlinear input/state driven disturbances and drifts. Some extending results of [22–31] for switched systems with time-varying delays, however, the time delays are assumed to be differentiable and the switching rule was constructed on the solutions of a set of LMIs. To the best of our knowledge, there has been no investigation on the exponential stability of switched nonlinear large-scale systems with time-varying delays interacted between subsystems. In fact, this problem is difficult to solve; particularly, when the time-varying delays are interval, non-differentiable and the output is subjected to such time-varying delay functions. The time delay is assumed to be any continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is bounded but not necessary to be differentiable. This allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero. It is clear that the application of any memoryless feedback controller to such time-delay systems would lead to closed-loop systems with interval time-varying delays. The difficulties then arise when one attempts to derive exponential stability conditions. Indeed, existing Lyapunov–Krasovskii functionals and their associated results in [9,10,15,18,19,22,23] cannot be applied to solve the problem posed in this paper as they would either fail to cope with the non-differentiability aspects of the delays, or lead to very complex matrix inequality conditions and any technique such as matrix computation or transformation of variables fails to extract the parameters of the memoryless feedback controllers. This has motivated our research.

In this paper, we consider a class of large-scale nonlinear systems with interval time-varying delays in interconnections. Compared to the existing results, our result has its own advantages. (i) Stability analysis of previous papers reveals some restrictions: The time delay was proposed to be either time-invariant interconnected or the lower delay bound is restricted to being zero, or the time delay function should be differential and its derivative is bounded. In our result, the above restricted conditions are removed for the large-scale systems. In addition the time delay is assumed to be any continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is bounded but not necessary to be differentiable. This allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero. (ii) The developed method using new inequalities for lower bounding cross terms eliminate the need for over bounding and provide larger values of the admissible delay bound. We propose a set of new Lyapunov–Krasovskii functionals, which are mainly based on the information of the lower and upper delay bounds. (iii) The conditions will be presented in terms of the solution of LMIs, that can be solved numerically in an efficient manner by using standard computational algorithms [32]. (iv) A simple geometric design is employed to find the switching law and our approach allows to compute simultaneously the two bounds that characterize the exponential stability rate of the solution.

The paper is organized as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Mail result for designing switching rule of exponential stability of the system is presented in Section 3. Numerical examples showing the effectiveness of the obtained results are given in Section 4. The paper ends with conclusions and cited references.

2. Preliminaries

The following notations will be used throughout this paper, R^+ denotes the set of all real-negative numbers; R^n denotes the n-dimensional space with the scalar product $(.,.)$ and the vector norm $\|\cdot\|$; $R^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimension. A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$; $\lambda_A = \lambda_{\max}(A^TA)$; $C^1([a,b], R^n)$ denotes the set of all R^n-valued differentiable functions on $[a,b]$; $L_2([0,\infty], R^r)$ stands for the set of all square-integrable R^r-valued functions on $[0,\infty]$. The symmetric terms in a matrix are denoted by $*$. Matrix A is semi-positive definite ($A \geq 0$) if $(Ax, x) \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $(Ax, x) > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. The segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in [-\tau, 0]\}$ with its norm

$$\|x_t\| = \sup_{s\in[-\tau,0]} \|x(t+s)\|.$$

Consider a class of nonlinear switched large-scale systems with time-varying delays composed of N interconnected subsystems $\Sigma_i,\ i = 1, 2, \ldots, N$ of the form:

$$\begin{cases} \dot{x}_i(t) = A_i^{\sigma_i} x_i(t) + \displaystyle\sum_{j\neq i, j=1}^{N} A_{ij}^{\sigma_i} x_j(t - h_{ij}(t)) + f_i^{\sigma_i}(t, x_i(t), \{x_j(t - h_{ij}(t))\}_{j=1, j\neq i}^{N}), \\ x_i(t) = \varphi_i(t), \quad \forall t \in [-h_2, 0], \end{cases} \tag{2.1}$$

where the function $\sigma_i : R^{n_i} \to \{1, \ldots, s\}$ is a switching rule within each subsystem which takes its values in the finite set of modes $\{1, \ldots, s\}$. This rule is to be selected for all i such that $\sigma_i(x_i(t)) = l$ implies that the lth switching mode is activated for the ith subsystem of the interconnected system. More precisely, $\sigma(x(t)) = (\sigma_1(x_1(t)), \ldots, \sigma_N(x_N(t))) = (l_1, \ldots, l_N)$ means that the l_ith switching mode is activated for the ith subsystem.

The system matrices $(A_i^{\sigma_i}, \{A_{ij}^{\sigma_i}\}_{j=1,j\neq i}^N)$ take values, at arbitrary discrete instants, in the finite set of

$$(A_i^l, \{A_{ij}^l\}_{j=1,j\neq i}^N), \quad \forall i = 1, \ldots, N, \ l = 1, \ldots, s.$$

Thus the matrices $(A_i^l, \{A_{ij}^l\}_{j=1,j\neq i}^N)$ denote the lth model of local subsystem i corresponding to the operational mode l. $x(t)^T = [x_1(t)^T, \ldots, x_N(t)^T]$, $x_i(t) \in R^{n_i}$, is the state vector with the norm

$$\|x(t)\| = \sqrt{\sum_{i=1}^N \|x_i(t)\|^2},$$

the delay functions $h_{ij}(.)$ are continuous and satisfy the following condition:

$$0 \le h_1 \le h_{ij}(t) < h_2, \quad t \ge 0, \ \forall i, j = 1, \ldots, N,$$

and the initial function $\varphi(t) = [\varphi_1(t), \ldots, \varphi_N(t)^T]$, $\varphi_i(t) \in C^1([-h_2, 0], R^{n_i})$, with the norm

$$\|\varphi_i\| = \sup_{-\bar{h} \le t \le 0} \{\|\varphi_i(t)\|, \|\dot{\varphi}_i(t)\|\}, \quad \|\varphi\| = \sqrt{\sum_{i=1}^N \|\varphi_i\|^2}.$$

Let $x_j^{h_{ij}}(t) := x_j(t - h_{ij}(t))$, $i \neq j$, the nonlinear function f_i^l satisfies the following growth condition

$$\exists\, a_i^l, a_{ij}^l > 0 : \|f_i^l\| \le a_i^l \|x_i(t)\| + \sum_{j\neq i, j=1}^N a_{ij}^l \|x_j^{h_{ij}}(t)\|, \quad i, j = 1, \ldots, N; l = 1, \ldots, s. \tag{2.2}$$

Before presenting the main result, we recall some well-known facts and propositions which will be used in the proof.

Definition 2.1. Given $\beta > 0$. Switched system (2.1) is exponentially stable if there exists a switching rule $\sigma(\cdot)$ such that every solution $x(t, \varphi)$ of the system satisfies the conditions

$$\exists M > 0, \quad \beta > 0 : \|x(t, \varphi)\| \le M e^{-\beta t}, \quad \forall t \ge 0.$$

Definition 2.2. The system of matrices $\{L_i\}$, $i = 1, 2, \ldots, N$, is strictly complete if for every $x \in R^n \setminus \{0\}$ there is $i \in \{1, 2, \ldots, N\}$ such that

$$x^T L_i x < 0.$$

It is easy to see that system $\{L_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^N \Omega_i = R^n \setminus \{0\},$$

where

$$\Omega_i = \{x \in R^n : x^T L_i x < 0\}, \quad i = 1, 2, \ldots, N.$$

It is shown in [33] that system $\{L_i\}$, $i = 1, 2, \ldots, N$, is strictly complete if there exist numbers $\xi_i \ge 0$, $i = 1, 2, \ldots, N$, $\sum_{i=1}^N \xi_i > 0$, such that

$$\sum_{i=1}^N \xi_i L_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.1. *For any $x, y \in R^n$, and matrices P, E, F, H where $P > 0$, $F^T F \le I$, and scalar $\varepsilon > 0$, one has*

(a) $EFH + H^T F^T E^T \le \varepsilon^{-1} E E^T + \varepsilon H^T H$,
(b) $2x^T y \le x^T P x + y^T P y$.

Proposition 2.2 (*Schur Complement Lemma [34]*). *Given constant matrices X, Y, Z, where $Y = Y^T > 0$. Then*

$$X + Z^T Y^{-1} Z < 0 \quad \text{if and only if} \quad \begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0.$$

Proposition 2.3 ([35]). *For any constant matrix $Z = Z^T > 0$ and scalar $h, \bar{h},\ 0 < h < \bar{h}$ such that the following integrations are well defined, then*

$$-\int_{t-h}^{t} x(s)^T Z x(s) ds \le -\frac{1}{h}\left(\int_{t-h}^{t} x(s)ds\right)^T Z \left(\int_{t-h}^{t} x(s)ds\right).$$
$$-\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x(\tau)^T Z x(\tau) d\tau ds \le -\frac{2}{\bar{h}^2 - h^2}\left(\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x(\tau)d\tau ds\right)^T Z \left(\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x(\tau)d\tau ds\right).$$

Proposition 2.4 (*Lower Bounds Lemma [36]*). *Let $f_1, f_2, \dots, f_N : R^m \to R$ have positive values in an open subset D of R^m. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{r_i | r_i > 0, \sum_i r_i = 1\}} \sum_i \frac{1}{r_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \ne j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : R^m \to R, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0 \right\}.$$

3. Main result

In this section, we investigate the exponential stability of linear system (2.1) with interval time-varying delays. Before introducing the main result, the following notations of several matrix variables are defined for simplicity.

$$P_{i1} = P_i^{-1}, \quad Q_{i1} = P_i^{-1} Q_i P_i^{-1}, \quad R_{i1} = P_i^{-1} R_i P_i^{-1}, \quad U_{i1} = P_i^{-1} U_i P_i^{-1},$$
$$\Lambda_{i1} = P_i^{-1} \Lambda_i P_i^{-1}, \quad S_{i1}^l = P_i^{-1} S_i^l P_i^{-1}, \quad i = 1, 2, \dots, N,\ l = 1, 2, \dots, s$$
$$a_{ij} = \sup_{l \in \overline{1,s}} a_{ij}^l, \quad i, j \in \overline{1, N}, \qquad H_{11}^l(i) = -\frac{e^{-4\beta h_2}(h_2 - h_1)}{h_2 + h_1}\Lambda_i - e^{-2\beta h_1} R_i - e^{-2\beta h_2} R_i$$
$$H_{1k}^l(i) = 0, \quad \forall k = \overline{2, N}, \qquad H_{1(N+1)}^l(i) = e^{-2\beta h_1} R_i, \qquad H_{1(N+2)}^l(i) = e^{-2\beta h_2} R_i,$$
$$H_{1(N+3)}^l(i) = P_i A_i^{lT}, \qquad H_{1,(N+4)}^l(i) = 2\frac{e^{-2\beta h_2}}{h_2 + h_1}\Lambda_i, \qquad H_{km}^l(i) = 0, \quad \forall k \ne m,\ k, m = \overline{2, N},$$
$$H_{kk}^l(i) = -2\frac{e^{-2\beta h_2}}{N-1} U_i + \frac{e^{-2\beta h_2}}{N-1}(S_i^l + S_i^{lT}), \quad \forall k = \overline{2, N},$$
$$H_{k(N+1)}^l(i) = \frac{e^{-2\beta h_2}}{N-1} U_i - \frac{e^{-2\beta h_2}}{N-1} S_i^l; \qquad H_{k(N+2)}^l(i) = \frac{e^{-2\beta h_2}}{N-1} U_i - \frac{e^{-2\beta h_2}}{N-1} S_i^{lT},$$
$$H_{k(N+3)}^l(i) = H_{k(N+4)}^l(i) = 0, \quad k = \overline{2, N}, \qquad H_{(N+1)(N+1)}^l(i) = -e^{-2\beta h_1} Q_i - e^{-2\beta h_1} R_i - e^{-2\beta h_2} U_i,$$
$$H_{(N+1)(N+2)}^l(i) = e^{-2\beta h_2} S_i^{lT}, \qquad H_{(N+1)(N+3)}^l(i) = H_{(N+2)(N+3)}^l(i) = H_{(N+1)(N+4)}^l(i) = 0$$
$$H_{(N+2)(N+4)}^l(i) = 0, \qquad H_{(N+2)(N+2)}^l(i) = -e^{-2\beta h_2} Q_i - e^{-2\beta h_2} R_i - e^{-2\beta h_2} U_i,$$
$$H_{(N+3)(N+3)}^l(i) = (h_2 - h_1) h_2 \Lambda_i + h_1^2 R_i + h_2^2 R_i + (h_2 - h_1)^2 U_i - 2P_i + \sum_{j=1, j \ne i}^{N} A_{ij}^l A_{ij}^{lT} + \varepsilon_i^l I.$$
$$H_{(N+3)(N+4)}^l(i) = 0, \qquad H_{(N+4),(N+4)}^l(i) = -2\frac{e^{-4\beta h_2}}{h_2^2 - h_1^2}\Lambda_i, \qquad H_{(N+5)(N+5)}^l(i) = -\frac{I}{2a_i^l}, \qquad H_{(N+5)1}^l = P_i,$$
$$H_{(N+4+k)(N+4+k)}^l(i) = -\frac{I}{2 + 2a_{ki}}, \qquad H_{(N+4+k)k}^l(i) = P_i, \quad k = \overline{2, N},\ i = 1,$$
$$H_{(N+4+k)(N+4+k)}^l(i) = -\frac{I}{2 + 2a_{(k-1)i}}, \qquad H_{(N+4+k)k}^l(i) = P_i, \quad k = \overline{2, N},\ i \ne 1, k \le i,$$
$$H_{(N+4+k)(N+4+k)}^l(i) = -\frac{I}{2 + 2a_{ki}}, \qquad H_{(N+4+k)k}^l(i) = P_i, \quad k = \overline{2, N}, i \ne 1, k \ge i + 1,$$

$$\varepsilon_i^l = a_i^l + \sum_{j\neq i, j=1}^{N} a_{ij}, \qquad \alpha_{i1} = \lambda_{\min}(P_{i1}), \qquad \alpha_1 = \min_{i=\overline{1,N}} \alpha_{i1}, \qquad \alpha_2 = \max_{i=\overline{1,N}} \alpha_{i2}.$$

$$\alpha_{i2} = \lambda_{\max}(P_{i1}) + \beta^{-1}\lambda_{\max}(Q_{i1}) + h_1^3\lambda_{\max}(R_{i1}) + h_2^3\lambda_{\max}(R_{i1}) + (h_2-h_1)^3\lambda_{\max}(U_{i1}) + (h_2-h_1)h_2^2\lambda_{\max}(\Lambda_{i1}),$$

$$L_i^l = P_iA_i^{lT} + A_i^lP_i + 2\beta P_i + 2Q_i - \frac{e^{-4\beta h_2}(h_2-h_1)}{h_2+h_1}\Lambda_i + \sum_{j=1, j\neq i}^{N} A_{ij}^l A_{ij}^{lT} + \varepsilon_i^l I,$$
$$i = 1,2,\ldots,N,\; l = 1,2,\ldots,s,$$

$$\Omega_i^l = \{x \in R^{n_i} : x^T P_{i1} L_i^l P_{i1} x < 0\}, \quad l = 1,\ldots,s,\; i = 1,2,\ldots,N,$$

$$\bar{\Omega}_i^1 = \Omega_i^1 \cup \{0\}, \qquad \bar{\Omega}_i^j = \Omega_i^j \setminus \bigcup_{k=1}^{j-1} \bar{\Omega}_i^k, \quad j = 2,3,\ldots,s,\; i = 1,2,\ldots,N.$$

The following is the main result of the paper, which gives sufficient conditions for designing switching rule of exponential stability of system (2.1).

Theorem 3.1. *Given $\beta > 0$. System (2.1) is exponentially stable if there exist symmetric positive definite matrices $P_i, Q_i, R_i, U_i, \Lambda_i, i = 1,2,\ldots,N$ and matrices $S_i^l, i = 1,2,\ldots,N,\ l = 1,2,\ldots,s$ such that the following conditions hold:*

(i) *The system $\{L_i^l\}, l = 1,2,\ldots,s$ is strictly complete for every $i = 1,2,\ldots,N$, i.e., there exist numbers $\xi_i^l \geq 0 : l = 1,2,\ldots,s, i = 1,2,\ldots,N, \sum_{l=1}^s \xi_i^l > 0, i = 1,2,\ldots,N$, such that*

$$\sum_{l=1}^{s} \xi_i^l L_i^l < 0, \quad i = 1,2,\ldots,N. \tag{3.1}$$

(ii) *For $i = 1,2,\ldots,N, l = 1,2,\ldots,s$,*

$$\begin{bmatrix} H_{11}^l(i) & H_{12}^l(i) & . & . & . & H_{1(2N+4)}^l(i) & 0 & 0 \\ * & H_{22}^l(i) & . & . & . & H_{2(2N+4)}^l(i) & 0 & 0 \\ . & . & . & . & . & . & . & . \\ * & * & . & . & . & H_{(2N+4)(2N+4)}^l(i) & 0 & 0 \\ * & * & . & . & . & * & -U_i & -S_i^l \\ * & * & . & . & . & * & * & -U_i \end{bmatrix} < 0. \tag{3.2}$$

The switching rule is chosen as $\sigma(x(t)) = (l_1, l_2, \ldots, l_N)$ whenever $x(t) \in \bar{\Omega}_1^{l_1} \times \bar{\Omega}_2^{l_2} \times \cdots \times \bar{\Omega}_N^{l_N}$. Moreover, the solution of this system satisfies

$$\|x(t)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-\beta t}\|\varphi\|, \quad \forall t \geq 0.$$

Proof. Consider the following Lyapunov–Krasovskii functional for the closed loop system:

$$V(t, x_t) = \sum_{i=1}^{N}\sum_{j=1}^{7} V_{ij}(t, x_t),$$

where

$$V_{i1} = x_i(t)^T P_{i1} x_i(t), \qquad V_{i2} = \int_{t-h_1}^{t} e^{2\beta(s-t)} x_i(s)^T Q_{i1} x_i(s) ds,$$

$$V_{i3} = \int_{t-h_2}^{t} e^{2\beta(s-t)} x_i(s)^T Q_{i1} x_i(s) ds, \qquad V_{i4} = h_1 \int_{-h_1}^{0}\int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i(\tau) R_{i1} \dot{x}_i(\tau) d\tau ds,$$

$$V_{i5} = h_2 \int_{-h_2}^{0}\int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i(\tau) R_{i1} \dot{x}_i(\tau) d\tau ds, \qquad V_{i6} = (h_2-h_1) \times \int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i(\tau) U_{i1} \dot{x}_i(\tau) d\tau ds.$$

$$V_{i7}(t, x_t) = \int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+s}^{t} e^{2\beta(\tau+s-t)} \dot{x}_i(\tau)^T \Lambda_{i1} \dot{x}_i(\tau) d\tau ds d\theta.$$

It is easy to verify that

$$\alpha_1 \sum_{i=1}^{N} \|x_i(t)\|^2 \leq V(t, x_t), \qquad V(0, x_0) \leq \alpha_2 \sum_{i=1}^{N} \|\varphi_i\|^2. \tag{3.3}$$

Taking the derivative of $V(.)$ in t along the solution of the system, we have

$$\dot V_{i1} = 2x_i(t)^T P_{i1}\left[A_i^{l_i} x_i(t) + \sum_{j\neq i,j=1}^{N} A_{ij}^{l_i} x_j(t-h_{ij}(t)) + f_i^{l_i}\right]$$
$$\dot V_{i2} = x_i(t)^T Q_{i1} x_i(t) - 2\beta V_{i2} - e^{-2\beta h_1} x_i(t-h_1)^T Q_{i1} x_i(t-h_1),$$
$$\dot V_{i3} = x_i(t)^T Q_{i1} x_i(t) - 2\beta V_{i3} - e^{-2\beta h_2} x_i(t-h_2)^T Q_{i1} x_i(t-h_2),$$
$$\dot V_{i4} \le h_1^2 \dot x_i(t)^T R_{i1} \dot x_i(t) - 2\beta V_{i4} - h_1 e^{-2\beta h_1}\int_{t-h_1}^{t} \dot x_i(s)^T R_{i1}\dot x_i(s)ds,$$
$$\dot V_{i5} \le h_2^2 \dot x_i(t)^T R_{i1} \dot x_i(t) - 2\beta V_{i5} - h_2 e^{-2\beta h_2}\int_{t-h_2}^{t} \dot x_i(s)^T R_{i1}\dot x_i(s)ds,$$
$$\dot V_{i6} \le (h_2-h_1)^2 \dot x_i(t)^T U_{i1} \dot x_i(t) - 2\beta V_{i6} - (h_2-h_1) e^{-2\beta h_2}\int_{t-h_2}^{t-h_1} \dot x_i(s)^T U_{i1}\dot x_i(s)ds.$$
$$\dot V_{i7}(t,x_t) \le (h_2-h_1)h_2 \dot x_i(t)^T \Lambda_{i1} \dot x_i(t) - e^{-4\beta h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot x_i(s)^T \Lambda_{i1}\dot x_i(s)dsd\theta - 2\beta V_{i7}(t,x_t).$$

Applying Proposition 2.3 and the Newton–Leibniz formula $\int_{t-h}^{t}\dot x_i(s)ds = x_i(t) - x_i(t-h)$, we have

$$-h\int_{t-h}^{t} \dot x_i(s)^T R_{i1}\dot x_i(s)ds \le -[x_i(t)-x_i(t-h)]R_{i1}[x_i(t)-x_i(t-h)].$$

Note that

$$\int_{t-h_2}^{t-h_1} \dot x_i^T(s) U_{i1}\dot x_i(s), ds = \int_{t-h_2}^{t-h_{ji}(t)} \dot x_i^T(s) U_{i1}\dot x_i(s)ds + \int_{t-h_{ji}(t)}^{t-h_1} \dot x_i^T(s) U_{i1}\dot x_i(s), ds.$$

Using Proposition 2.3 and $h_2 - h_{ji}(t) \le h_2 - h_1$, $h_{ji}(t) - h_1 \le h_2 - h_1$ gives

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_{ji}(t)} \dot x_i^T(s) U_{i1}\dot x_i(s)ds\\
&\quad\le -\frac{h_2-h_1}{h_2-h_{ji}(t)}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)],\\
&-(h_2-h_1)\int_{t-h_{ji}(t)}^{t-h_1} \dot x_i^T(s) U_{i1}\dot x_i(s)ds\\
&\quad\le -\frac{h_2-h_1}{h_{ji}(t)-h_1}[x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))],
\end{aligned}$$

and hence

$$\begin{aligned}
&-(h_2-h_1)\int_{t-h_2}^{t-h_1} \dot x_i^T(s) U_{i1}\dot x_i(s)\,ds\\
&\quad\le -\frac{h_2-h_1}{h_2-h_{ji}(t)}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]\\
&\qquad -\frac{h_2-h_1}{h_{ji}(t)-h_1}[x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))].
\end{aligned} \tag{3.4}$$

To estimate inequality (3.4) by using Proposition 2.4 we set $r_1 = \frac{h_2-h_{ji}(t)}{h_2-h_1}$, $r_2 = \frac{h_{ji}(t)-h_1}{h_2-h_1}$, and

$$f_1(t) = [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)],$$
$$f_2(t) = [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))],$$
$$g_{1,2}(t) = [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T S_{i1}^{l_i}[x_i(t-h_1) - x_i(t-h_{ji}(t))]$$
$$g_{2,1}(t) = [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T S_{i1}^{l_i T}[x_i(t-h_{ji}(t)) - x_i(t-h_2)],$$

we have from condition (3.2):

$$\begin{bmatrix} f_1(t) & g_{1,2}(t) \\ g_{1,2}(t) & f_2(t) \end{bmatrix} = \begin{bmatrix} [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T & 0 \\ 0 & [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T \end{bmatrix}$$
$$\times \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i1} \end{bmatrix} \begin{bmatrix} U_i & S_i^{l_i} \\ S_i^{l_iT} & U_i \end{bmatrix} \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i1} \end{bmatrix}$$
$$\times \begin{bmatrix} [x_i(t-h_{ji}(t)) - x_i(t-h_2)] & 0 \\ 0 & [x_i(t-h_1) - x_i(t-h_{ji}(t))] \end{bmatrix} \geq 0,$$

where $g_{1,2}(t) = g_{2,1}(t)$ and $r_1 + r_2 = 1$. Applying Proposition 2.4 gives

$$\begin{aligned} -(h_2 - h_1)\int_{t-h_2}^{t-h_1} \dot{x}_i^T(s)U_{i1}\dot{x}_i(s)\,ds &\leq -\frac{1}{r_1}f_1(t) - \frac{1}{r_2}f_2(t) \\ &\leq -f_1(t) - f_2(t) - g_{1,2}(t) - g_{2,1}(t) \\ &= -[x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)] \\ &\quad - [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))] \\ &\quad - [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T S_{i1}^{l_i}[x_i(t-h_1) - x_i(t-h_{ji}(t))] \\ &\quad - [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T S_{i1}^{l_iT}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]. \end{aligned}$$

Therefore, we have

$$\begin{aligned} \dot{V}_{i4} &\leq h_1^2\dot{x}_i(t)^T R_{i1}\dot{x}_i(t) - 2\beta V_{i4} - e^{-2\beta h_1}[x_i(t) - x_i(t-h_1)]^T R_{i1}[x_i(t) - x_i(t-h_1)], \\ \dot{V}_{i5} &\leq h_2^2\dot{x}_i(t)^T R_{i1}\dot{x}_i(t) - 2\beta V_{i5} - e^{-2\beta h_2}[x_i(t) - x_i(t-h_2)]^T R_{i1}[x_i(t) - x_i(t-h_2)], \\ \dot{V}_{i6} &\leq (h_2 - h_1)^2\dot{x}_i(t)^T U_{i1}\dot{x}_i(t) - 2\beta V_{i6} - \frac{e^{-2\beta h_2}}{N-1} \\ &\quad \times \sum_{j=1,j\neq i}^{N} [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)] \\ &\quad - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N} [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))] \\ &\quad - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N} [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T S_{i1}^{l_i}[x_i(t-h_1) - x_i(t-h_{ji}(t))] \\ &\quad - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N} [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T S_{i1}^{l_iT}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]. \end{aligned} \tag{3.5}$$

Note that if $r_1 = 0$ or $r_2 = 0$ or $f_1(t) = 0$ or $f_2(t) = 0$ and $[x_i(t-h_1) - x_i(t-h_{ji}(t))]^T = 0$ or $[x_i(t-h_{ji}(t)) - x_i(t-h_2)] = 0$, then relation (3.5) still holds. Besides, using Proposition 2.3 again, we have

$$\begin{aligned} &-e^{-4\beta h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}_i(s)^T \Lambda_{i1}\dot{x}_i(s)dsd\theta \\ &\leq -\frac{2e^{-4\beta h_2}}{h_2^2 - h_1^2}\left((h_2 - h_1)x_i(t) - \int_{t-h_2}^{t-h_1} x_i(\theta)d\theta\right)^T \Lambda_{i1}\left((h_2 - h_1)x_i(t) - \int_{t-h_2}^{t-h_1} x_i(\theta)d\theta\right). \end{aligned}$$

Hence,

$$\begin{aligned} \dot{V}_7(.) &\leq (h_2 - h_1)h_2\dot{x}_i(t)^T \Lambda_{i1}\dot{x}_i(t) - 2\beta V_7(t, x_t) - \frac{2e^{-4\beta h_2}}{h_2^2 - h_1^2}\left((h_2 - h_1)x_i(t) - \int_{t-h_2}^{t-h_1} x_i(\theta)d\theta\right)^T \\ &\quad \times \Lambda_{i1}\left((h_2 - h_1)x_i(t) - \int_{t-h_2}^{t-h_1} x_i(\theta)d\theta\right). \end{aligned} \tag{3.6}$$

From the following identity relation

$$-2\dot{x}_i(t)^T P_{i1} \times \left[\dot{x}_i(t) - A_i^{l_i}x_i(t) - \sum_{j\neq i,j=1}^{n} A_{ij}^{l_i}x_j(t - h_{ij}(t)) - f_i^{l_i}(.)\right] = 0,$$

applying Proposition 2.1 and condition (2.2), we obtain

$$2x_i(t)^T P_{i1}\left[\sum_{j\neq i,j=1}^{N} A_{ij}^{l_i} x_j(t-h_{ij}(t))\right] \le \sum_{j\neq i,j=1}^{N} x_i(t)^T P_{i1} A_{ij}^{l_i} A_{ij}^{l_i T} P_{i1} x_i(t), + \sum_{j\neq i,j=1}^{N} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)),$$

$$2\dot{x}_i(t)^T P_{i1}\left[\sum_{j\neq i,j=1}^{N} A_{ij}^{l_i} x_j(t-h_{ij}(t))\right] \le \sum_{j\neq i,j=1}^{N} \dot{x}_i(t)^T P_{i1} A_{ij}^{l_i} A_{ij}^{l_i T} P_{i1} \dot{x}_i(t), + \sum_{j\neq i,j=1}^{N} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)),$$

$$2x_i(t)^T P_{i1} f_i^{l_i}(.) \le \varepsilon_i^{l_i}\|x_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 + \sum_{j\neq i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2,$$

$$2\dot{x}_i(t)^T P_{i1} f_i^{l_i}(.) \le \varepsilon_i^{l_i}\|\dot{x}_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 + \sum_{j\neq i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2,$$

$$\begin{aligned} 0 &= -2\dot{x}_i(t)^T P_{i1}\left[\dot{x}_i(t) - A_i^{l_i} x_i(t) - \sum_{j\neq i,j=1}^{N} A_{ij}^{l_i} x_j(t-h_{ij}(t)) - f_i^{l_i}(.)\right] \\ &\le -2\dot{x}_i(t)^T P_{i1}\left[\dot{x}_i(t) - A_i^{l_i} x_i(t)\right] + \sum_{j\neq i,j=1}^{N} \dot{x}_i(t)^T P_{i1} A_{ij}^{l_i} A_{ij}^{l_i T} P_{i1}\dot{x}_i(t) + \sum_{j\neq i,j=1}^{N} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)) \\ &\quad + \varepsilon_i^{l_i}\|\dot{x}_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 + \sum_{j\neq i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2, \end{aligned} \tag{3.7}$$

$$\begin{aligned} \dot{V}_{i1}(.) &\le 2x_i(t)^T P_{i1} A_i^{l_i} x_i(t) + \sum_{j\neq i,j=1}^{N} x_i(t)^T P_{i1} A_{ij}^{l_i} A_{ij}^{l_i T} P_{i1} x_i(t) + \varepsilon_i^{l_i}\|x_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 \\ &\quad + \sum_{j\neq i,j=1}^{N} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)) + \sum_{j\neq i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2. \end{aligned} \tag{3.8}$$

Therefore, applying inequalities (3.5)–(3.8) and note that

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)) = \sum_{i=1}^{N}\left[\sum_{j=1,i\neq j}^{N} x_i(t-h_{ji}(t))^T x_i(t-h_{ji}(t))\right],$$

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} a_{ij} x_j(t-h_{ij}(t))^T x_j(t-h_{ij}(t)) = \sum_{i=1}^{N}\left[\sum_{j=1,i\neq j}^{N} a_{ji} x_i(t-h_{ji}(t))^T x_i(t-h_{ji}(t))\right],$$

we have

$$\begin{aligned} \dot{V}(.) + 2\beta V(.) \le \sum_{i=1}^{N} \Bigg[& 2x_i(t)^T P_{i1} A_i^{l_i} x_i(t) + 2\beta x_i(t)^T P_{i1} x_i(t) \\ & + x_i(t)^T Q_{i1} x_i(t) - e^{-2\beta h_1} x_i(t-h_1)^T Q_{i1} x_i(t-h_1) + x_i(t)^T Q_{i1} x_i(t) \\ & - e^{-2\beta h_2} x_i(t-h_2)^T Q_{i1} x_i(t-h_2) + h_1^2 \dot{x}_i(t)^T R_{i1} \dot{x}_i(t) \\ & - e^{-2\beta h_1}[x_i(t) - x_i(t-h_1)]^T R_{i1}[x_i(t) - x_i(t-h_1)] + \varepsilon_i^{l_i}\|x_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 \\ & - e^{-2\beta h_2}[x_i(t) - x_i(t-h_2)]^T R_{i1}[x_i(t) - x_i(t-h_2)] + \varepsilon_i^{l_i}\|\dot{x}_i(t)^T P_{i1}\|^2 + a_i^{l_i}\|x_i(t)\|^2 \\ & + (h_2-h_1)^2 \dot{x}_i(t)^T U_{i1} \dot{x}_i(t) + h_2^2 \dot{x}_i(t)^T R_{i1} \dot{x}_i(t) \\ & - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N} [x_i(t-h_{ji}(t)) - x_i(t-h_2)]^T U_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)] \\ & - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N} [x_i(t-h_1) - x_i(t-h_{ji}(t))]^T U_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))] \end{aligned}$$

$$
\begin{aligned}
& -\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^T S_{i1}^{l_i}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \\
& -\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\neq i}^{N}[x_i(t-h_1)-x_i(t-h_{ji}(t))]^T S_{i1}^{l_iT}[x_i(t-h_{ji}(t))-x_i(t-h_2)] \\
& +(h_2-h_1)h_2\dot{x}_i(t)^T\Lambda_{i1}\dot{x}_i(t)+\sum_{j\neq i,j=1}^{N}x_i(t-h_{ji}(t))^T x_i(t-h_{ji}(t)) \\
& -\frac{2e^{-4\beta h_2}}{h_2^2-h_1^2}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right)^T\Lambda_{i1}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right) \\
& -[2\dot{x}_i(t)^T P_{i1}]\times[\dot{x}_i(t)-A_i^{l_i}x_i(t)]+\sum_{j\neq i,j=1}^{N}x_i(t)^T P_{i1}A_{ij}^{l_i}A_{ij}^{l_iT}P_{i1}x_i(t) \\
& +\sum_{j\neq i,j=1}^{N}x_i(t-h_{ji}(t))^T x_i(t-h_{ji}(t))+\sum_{j\neq i,j=1}^{N}\dot{x}_i(t)^T P_{i1}A_{ij}^{l_i}A_{ij}^{l_iT}P_{i1}\dot{x}_i(t) \\
& \left.+\sum_{j\neq i,j=1}^{N}a_{ji}\|x_i^{h_{ji}}(t)\|^2+\sum_{j\neq i,j=1}^{N}a_{ji}\|x_i^{h_{ji}}(t)\|^2\right].
\end{aligned}
$$

Setting $y_i(t)=P_{i1}x_i(t)$, we obtain

$$
\dot{V}(t,x_t)+2\beta V(t,x_t)\le\sum_{i=1}^{N}y_i(t)^T L_i^{l_i}y_i(t)+\sum_{i=1}^{N}\xi_i(t)^T M^{l_i}(i)\xi_i(t), \tag{3.9}
$$

where

$$
\xi_i(t)^T=\left[y_i(t)^T\{y_i(t-h_{ji}(t))^T\}_{j=1,j\neq i}^{N}\,y_i(t-h_1)^T\,y_i(t-h_2)^T\,\dot{y}_i(t)^T\,\int_{t-h_2}^{t-h_1}y_i(\theta)^T d\theta\right],
$$

$$
M^{l_i}(i)=\begin{bmatrix} M_{11}^{l_i}(i) & M_{12}^{l_i}(i) & . & . & . & M_{1(N+4)}^{l_i}(i) \\ * & M_{22}^{l_i}(i) & . & . & . & M_{2(N+4)}^{l_i}(i) \\ . & . & . & . & . & . \\ * & * & . & . & . & M_{(N+4)(N+4)}^{l_i}(i) \end{bmatrix},\quad i=\overline{1,N},\ l_i=1,2\ldots,s,
$$

$$
\begin{aligned}
& M_{11}^{l_i}(i)=-\frac{e^{-4\beta h_2}(h_2-h_1)}{h_2+h_1}\Lambda_i-e^{-2\beta h_1}R_i-e^{-2\beta h_2}R_i+2a_i^{l_i}P_i^2 \\
& M_{1k}^{l_i}(i)=0,\quad \forall k=\overline{2,N},\qquad M_{1(N+1)}^{l_i}(i)=e^{-2\beta h_1}R_i,\qquad M_{1(N+2)}^{l_i}(i)=e^{-2\beta h_2}R_i, \\
& M_{1(N+3)}^{l_i}(i)=P_iA_i^{l_iT},\qquad M_{1,(N+4)}^{l_i}(i)=2\frac{e^{-2\beta h_2}}{h_2+h_1}\Lambda_i,\qquad M_{kj}^{l_i}(i)=0,\quad \forall k\neq j,\ k,j=\overline{2,N}, \\
& M_{kk}^{l_i}(i)=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i^{l_i}+S_i^{l_iT})+(2+2a_{ki})P_i^2,\quad \forall k=\overline{2,N},\ i=1, \\
& M_{kk}^{l_i}(i)=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i^{l_i}+S_i^{l_iT})+(2+2a_{(k-1)i})P_i^2,\quad k=\overline{2,N},\ i\neq 1,\ k\le i, \\
& M_{kk}^{l_i}(i)=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i^{l_i}+S_i^{l_iT})+(2+2a_{ki})P_i^2,\quad k=\overline{2,N},\ i\neq 1,\ k\ge i+1, \\
& M_{k(N+1)}^{l_i}(i)=\frac{e^{-2\beta h_2}}{N-1}U_i-\frac{e^{-2\beta h_2}}{N-1}S_i^{l_i};\qquad M_{k(N+2)}^{l_i}(i)=\frac{e^{-2\beta h_2}}{N-1}U_i-\frac{e^{-2\beta h_2}}{N-1}S_i^{l_iT}, \\
& M_{(N+1)(N+1)}^{l_i}(i)=-e^{-2\beta h_1}Q_i-e^{-2\beta h_1}R_i-e^{-2\beta h_2}U_i,\qquad M_{(N+1)(N+2)}^{l_i}(i)=e^{-2\beta h_2}S_i^{l_iT}, \\
& M_{(N+1)(N+3)}^{l_i}(i)=M_{(N+2)(N+3)}^{l_i}(i)=M_{(N+1)(N+4)}^{l_i}(i)=M_{(N+2)(N+4)}^{l_i}(i)=0, \\
& M_{(N+2)(N+2)}^{l_i}(i)=-e^{-2\beta h_2}Q_i-e^{-2\beta h_2}R_i-e^{-2\beta h_2}U_i, \\
& M_{(N+3)(N+3)}^{l_i}(i)=(h_2-h_1)h_2\Lambda_i+h_1^2R_i+h_2^2R_i+(h_2-h_1)^2U_i-2P_i+\sum_{j=1,j\neq i}^{N}A_{ij}^{l_i}A_{ij}^{l_iT}+\varepsilon_i^{l_i}I. \\
& M_{(N+3)(N+4)}^{l_i}(i)=0,\qquad M_{(N+4),(N+4)}^{l_i}(i)=-2\frac{e^{-4\beta h_2}}{h_2^2-h_1^2}\Lambda_i,\qquad M_{k(N+3)}^{l_i}(i)=M_{k(N+4)}^{l_i}(i)=0,\quad k=\overline{2,N}.
\end{aligned}
$$

Since matrix $P_i^{-1} > 0$, the system of matrices $\{P_i^{-1}L_i^lP_i^{-1} : l = 1, 2, \dots, s\}$ is strictly complete. Then, we have

$$\bigcup_{l=1}^{s} \Omega_i^l = R^{n_i} \setminus \{0\}$$

and it follows by constructing the sets $\bar{\Omega}_i^l$ that

$$\bigcup_{l=1}^{s} \bar{\Omega}_i^l = R^{n_i}, \qquad \bar{\Omega}_i^{l_1} \cap \bar{\Omega}_i^{l_2} = \emptyset, \quad \text{for } l_1 \neq l_2.$$

Therefore, for $x(t) = (x_1(t)^T, x_2(t)^T, \dots, x_N(t)^T)^T \in R^{n_1} \times R^{n_2} \times \dots \times R^{n_N}$, there exists $l_i \in \{1, 2, \dots, s\}$ such that $x_i(t) \in \bar{\Omega}_i^{l_i}$ and

$$x_i(t)^T P_i^{-1} L_i^{l_i} P_i^{-1} x_i(t) \le 0 \quad \text{and hence } y_i(t)^T L_i^{l_i} y_i(t) \le 0.$$

Choosing the switching rule as $\sigma(x(t)) = (l_1, l_2, \dots, l_N)$ whenever

$$x(t) \in \bar{\Omega}_1^{l_1} \times \bar{\Omega}_2^{l_2} \times \dots \times \bar{\Omega}_N^{l_N}.$$

Thus, from (3.9), (3.1), (3.2) we obtain

$$\dot{V}(t, x_t) + 2\beta V(t, x_t) \le 0. \tag{3.10}$$

Therefore, we have

$$V(t, x_t) \le V(0, x_0)e^{-2\beta t}, \quad \forall t \ge 0.$$

Taking inequality (3.3) into account, we finally obtain that

$$\alpha_1 \sum_{i=1}^{N} \|x_i(t)\|^2 \le V(t, x_t) \le \alpha_2 \sum_{i=1}^{N} \|\varphi_i\|^2 e^{-2\beta t}, \quad \text{for all } t \ge 0.$$

This completes the proof of the theorem. □

Remark 3.1. Theorem 3.1 provides sufficient conditions for designing switching law of the nonlinear large-scale system (2.1) in terms of the solutions of LMIs, which guarantees the closed-loop system to be exponentially stable with a prescribed decay rate β. The developed method using new inequalities for lower bounding cross terms eliminate the need for over bounding and provide larger values of the admissible delay bound. Note that the time-varying delays are non-differentiable, therefore, the methods proposed in [9,10,15,18,19,22,23] are not applicable to system (2.1). The LMI condition (3.2) depends on parameters of the system under consideration as well as the delay bounds. The feasibility of the LMIs can be tested by the reliable and efficient Matlab LMI Control Toolbox [32].

4. Numerical examples

In this section, we give two numerical examples to show the effectiveness of the proposed result.

Example 4.1. This example is a large-scale model composed of two machine subsystems [14] as follows:

$$\begin{cases} \dot{x}_1(t) = A_1^{\sigma_1} x_1(t) + A_{12}^{\sigma_1} x_2(t - h_{12}(t)) + f_1^{\sigma_1}(t, x_1(t), x_2(t - h_{12}(t))), \\ x_1(t) = \varphi_1(t), \quad \forall t \in [-h_2, 0], \end{cases}$$

$$\begin{cases} \dot{x}_2(t) = A_2^{\sigma_2} x_2(t) + A_{21}^{\sigma_2} x_1(t - h_{21}(t)) + f_2^{\sigma_2}(t, x_2(t), x_1(t - h_{21}(t))), \\ x_2(t) = \varphi_2(t), \quad \forall t \in [-h_2, 0], \end{cases}$$

where the absolute rotor angle and angular velocity of the machine in each subsystem are denoted by $x_1 = (x_{11}, x_{12})$, and $x_2 = (x_{21}, x_{22})$, respectively; the ith system coefficient A_i; the control and nonlinear perturbations $f_i(.)$ and the modulus of the transfer admittance A_{ij}; the initial input φ_i; the time-varying delays $h_{ij}(t)$ between the two machines in the subsystem:

$$h_{12} = \begin{cases} 0.1 + 0.4\sin^2(t), & t \in H, \\ 0.1, & t \notin H, \end{cases} \qquad h_{21} = \begin{cases} 0.2 + 0.3\sin^2(t), & t \in H, \\ 0.2, & t \notin H, \end{cases}$$

$$H = \cup_{k \in N}(2k\pi, (2k+1)\pi),$$

$$A_1^1 = \begin{bmatrix} -1.2 & 0.1 \\ 0.2 & -1.3 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1.3 \end{bmatrix}, \quad A_2^1 = \begin{bmatrix} -1.1 & 0.2 \\ 0.1 & -1 \end{bmatrix}, \quad A_2^2 = \begin{bmatrix} -0.9 & 0.1 \\ 0 & -1.2 \end{bmatrix},$$

$$A_{12}^1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A_{12}^2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A_{21}^1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A_{21}^2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$a_1^1 = a_1^2 = a_2^1 = a_2^2 = a_{12}^1 = a_{21}^1 = a_{12}^2 = a_{21}^2 = 0.01.$$

It is worth nothing that, the delay functions $h_{12}(t),\ h_{21}(t)$ are non differentiable, therefore, the methods in [9,10,15,18,19,22,23] are not applicable to this system. By using LMI Toolbox in Matlab, LMIs (3.1), (3.2) are feasible with $h_1 = 0.1,\ h_2 = 0.5,\ \beta = 0.01$, and

$$P_1 = \begin{bmatrix} 2.4277 & 0.9127 \\ 0.9127 & 2.5280 \end{bmatrix}, \qquad Q_1 = \begin{bmatrix} 2.5516 & 0.8833 \\ 0.8833 & 3.1343 \end{bmatrix}, \qquad R_1 = \begin{bmatrix} 4.8699 & 1.7459 \\ 1.7459 & 4.9864 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 8.1686 & 4.5412 \\ 4.5412 & 6.9322 \end{bmatrix}, \qquad \Lambda_1 = \begin{bmatrix} 0.2137 & 0.1384 \\ 0.1384 & 0.1571 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 4.2938 & 0.4716 \\ 0.4716 & 3.4603 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 4.1022 & 0.1090 \\ 0.1090 & 3.6822 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 7.7860 & 0.7776 \\ 0.7776 & 6.5886 \end{bmatrix},$$

$$U_2 = \begin{bmatrix} 17.5932 & 3.7369 \\ 3.7369 & 12.1542 \end{bmatrix}, \qquad \Lambda_2 = \begin{bmatrix} 0.4503 & 0.1251 \\ 0.1251 & 0.2933 \end{bmatrix}, \qquad S_1^1 = \begin{bmatrix} 5.3224 & 3.1503 \\ 3.1529 & 4.9629 \end{bmatrix},$$

$$S_1^2 = \begin{bmatrix} 5.2757 & 3.2732 \\ 3.2713 & 5.0573 \end{bmatrix}, \qquad S_2^1 = \begin{bmatrix} 12.8735 & 2.5152 \\ 2.5163 & 8.4604 \end{bmatrix}, \qquad S_2^2 = \begin{bmatrix} 12.5743 & 2.6729 \\ 2.6715 & 8.7318 \end{bmatrix}.$$

In this case, it can be shown that

$$L_1^1 = \begin{bmatrix} -0.6018 & 0.1510 \\ 0.1510 & 0.0388 \end{bmatrix}, \qquad L_1^2 = \begin{bmatrix} 0.2168 & -0.4048 \\ -0.4048 & -0.3263 \end{bmatrix},$$

$$L_2^1 = \begin{bmatrix} -1.2317 & 0.2767 \\ 0.2767 & 0.4757 \end{bmatrix}, \qquad L_2^2 = \begin{bmatrix} 0.4215 & -0.4987 \\ -0.4987 & -1.0027 \end{bmatrix}.$$

Moreover,

$$L_1^1 + L_1^2 = \begin{bmatrix} -0.3850 & -0.2539 \\ -0.2539 & -0.2874 \end{bmatrix} < 0, \qquad L_2^1 + L_2^2 = \begin{bmatrix} -0.8102 & -0.2220 \\ -0.2220 & -0.5270 \end{bmatrix} < 0.$$

The sets Ω_i^l are given as

$$\Omega_1^1 = \{(x_1, x_2) : (x_1 - 0.7744x_2)(x_1 - 0.2698x_2) > 0\}$$
$$\Omega_1^2 = \{(x_1, x_2) : (x_1 - 1.7336x_2)(x_1 - 0.0098x_2) < 0\}$$
$$\Omega_2^1 = \{(x_1, x_2) : (x_1 - 1.1021x_2)(x_1 + 0.4299x_2) > 0\}$$
$$\Omega_2^2 = \{(x_1, x_2) : (x_1 - 2.8152x_2)(x_1 + 0.9016x_2) < 0\}.$$

The union of Ω_1^1 and Ω_1^2 is equal to $R^2 \setminus \{0\}$. The union of Ω_2^1 and Ω_2^2 is equal to $R^2 \setminus \{0\}$.
The sets $\bar{\Omega}_i^l$ are given as

$$\bar{\Omega}_1^1 = \{(x_1, x_2) : (x_1 - 0.7744x_2)(x_1 - 0.2698x_2) \geq 0\}$$
$$\bar{\Omega}_1^2 = \{(x_1, x_2) : (x_1 - 0.7744x_2)(x_1 - 0.2698x_2) < 0\}$$
$$\bar{\Omega}_2^1 = \{(x_1, x_2) : (x_1 - 1.1021x_2)(x_1 + 0.4299x_2) \geq 0\}$$
$$\bar{\Omega}_2^2 = \{(x_1, x_2) : (x_1 - 1.1021x_2)(x_1 + 0.4299x_2) < 0\}.$$

According to Theorem 3.1, the system with the switching rule $\sigma(x(t)) = (l_1, l_2)$ whenever $x(t) \in \bar{\Omega}^{l_1} \times \bar{\Omega}^{l_2}$ is exponentially stable. Finally, the solution $x(t, \varphi)$ of the system satisfies (see Figs. 1–4)

$$\|x(t, \varphi)\| \leq 15.7140e^{-0.01t}\|\varphi\|.$$

Example 4.2. Consider system (2.1) composed of two interconnected subsystems with switching rule which takes its values in the finite set of modes $\{1, 2\}$, where

$$A_1^1 = \begin{bmatrix} -0.06 & 0.01 \\ 0.01 & -0.06 \end{bmatrix}, \qquad A_1^2 = \begin{bmatrix} -0.06 & 0.01 \\ 0.02 & -0.06 \end{bmatrix}, \qquad A_2^1 = \begin{bmatrix} -0.06 & 0.02 \\ 0.01 & -0.06 \end{bmatrix},$$

$$A_2^2 = \begin{bmatrix} -0.06 & 0.02 \\ 0.02 & -0.06 \end{bmatrix}, \qquad A_{12}^1 = A_{12}^2 = A_{21}^1 = A_{21}^2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$a_1^1 = a_1^2 = a_2^1 = a_2^2 = a_{12}^1 = a_{21}^1 = a_{12}^2 = a_{21}^2 = 0.001; \qquad \beta = 0.01.$$

Table 1
Upper bound of the delay.

Methods	Theorem 3.1	Hi–Ph [9]	Ma–Fo [4]	Hua–Wa–Gu [19]	Pa–Ko–Je [36]
$\max\{h_2 - h_1\}$	7.1	0.2	3.229	1	1.86
h_2	10	0.6	4.113	1	4.09

We have $h_1 = 5,\ h_2 = 10$, and LMI (3.1) is feasible with

$$P_1 = \begin{bmatrix} 0.0397 & 0.0111 \\ 0.0111 & 0.0431 \end{bmatrix}, \qquad Q_1 = 10^{-3}\begin{bmatrix} 0.7610 & 0.0113 \\ 0.0113 & 0.8537 \end{bmatrix}, \qquad R_1 = 10^{-3}\begin{bmatrix} 0.1880 & 0.0249 \\ 0.0249 & 0.1796 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.0012 & 0.0007 \\ 0.0007 & 0.0013 \end{bmatrix}, \qquad \Lambda_1 = 10^{-4}\begin{bmatrix} 0.2054 & 0.1446 \\ 0.1446 & 0.2232 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.0406 & 0.0155 \\ 0.0155 & 0.0406 \end{bmatrix}, \qquad Q_2 = 10^{-3}\begin{bmatrix} 0.6272 & 0.0222 \\ 0.0222 & 0.6948 \end{bmatrix}, \qquad R_2 = 10^{-3}\begin{bmatrix} 0.1748 & 0.0365 \\ 0.0365 & 0.1756 \end{bmatrix},$$

$$U_2 = \begin{bmatrix} 0.0013 & 0.0009 \\ 0.0009 & 0.0013 \end{bmatrix}, \qquad \Lambda_2 = 10^{-4}\begin{bmatrix} 0.2050 & 0.1903 \\ 0.1903 & 0.2098 \end{bmatrix}, \qquad S_1^1 = \begin{bmatrix} 0.0011 & 0.0006 \\ 0.0006 & 0.0012 \end{bmatrix},$$

$$S_1^2 = \begin{bmatrix} 0.0011 & 0.0006 \\ 0.0006 & 0.0012 \end{bmatrix}, \qquad S_2^1 = \begin{bmatrix} 0.0012 & 0.0008 \\ 0.0008 & 0.0012 \end{bmatrix}, \qquad S_2^2 = \begin{bmatrix} 0.0012 & 0.0008 \\ 0.0008 & 0.0012 \end{bmatrix}.$$

In this case, it can be shown that

$$L_1^1 = 10^{-3}\begin{bmatrix} -0.1287 & -0.2669 \\ -0.2669 & -0.2851 \end{bmatrix}, \qquad L_1^2 = 10^{-3}\begin{bmatrix} -0.1287 & 0.1300 \\ 0.1300 & -0.0622 \end{bmatrix},$$

$$L_2^1 = 10^{-3}\begin{bmatrix} -0.0960 & -0.2865 \\ -0.2865 & -0.2675 \end{bmatrix}, \qquad L_2^2 = 10^{-3}\begin{bmatrix} -0.0960 & 0.1199 \\ 0.1199 & 0.0416 \end{bmatrix}.$$

Moreover,

$$L_1^1 + L_1^2 = 10^{-3}\begin{bmatrix} -0.2574 & -0.1370 \\ -0.1370 & -0.3473 \end{bmatrix} < 0, \qquad L_2^1 + L_2^2 = 10^{-3}\begin{bmatrix} -0.1920 & -0.1666 \\ -0.1666 & -0.2260 \end{bmatrix} < 0.$$

The sets Ω_i^l are given as

$$\Omega_1^1 = \{(x_1, x_2) : (x_1 + 32.9221x_2)(x_1 + 0.3835x_2) > 0\}$$
$$\Omega_1^2 = \{(x_1, x_2) : (x_1 - 1.2849x_2)(x_1 - 0.4793x_2) > 0\}$$
$$\Omega_2^1 = \{(x_1, x_2) : (x_1 - 4.719x_2)(x_1 + 0.1617x_2) < 0\}$$
$$\Omega_2^2 = \{(x_1, x_2) : (x_1 - 1.5121x_2)(x_1 - 0.2319x_2) > 0\}.$$

The union of Ω_1^1 and Ω_1^2 is equal to $R^2 \setminus \{0\}$. The union of Ω_2^1 and Ω_2^2 is equal to $R^2 \setminus \{0\}$.
The sets $\bar{\Omega}_i^l$ are given as

$$\bar{\Omega}_1^1 = \{(x_1, x_2) : (x_1 + 32.9221x_2)(x_1 + 0.3835x_2) \ge 0\}$$
$$\bar{\Omega}_1^2 = \{(x_1, x_2) : (x_1 + 32.9221x_2)(x_1 + 0.3835x_2) < 0\}$$
$$\bar{\Omega}_2^1 = \{(x_1, x_2) : (x_1 - 4.719x_2)(x_1 + 0.1617x_2) \le 0\}$$
$$\bar{\Omega}_2^2 = \{(x_1, x_2) : (x_1 - 4.719x_2)(x_1 + 0.1617x_2) > 0\}.$$

Table 1 illustrates the numerical results for different values of upper bound delay. This shows that our result provides a larger allowable upper bound than the one obtained by the technique of [4,9,19,36].

5. Conclusion

In this paper, the problem of the decentralized stability for switched large-scale time-varying delay systems with nonlinear perturbations has been studied. The time delay is assumed to be a function belonging to a given interval, but not necessary to be differentiable. By effectively combining appropriate Lyapunov functionals with the Newton–Leibniz formula and free-weighting parameter matrices, this paper has derived new delay-dependent conditions for the exponential stability in terms of linear matrix inequalities, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution. The developed method using new inequalities for lower bounding cross terms eliminate the need for overbounding and provide larger values of the delay bound. Numerical examples are given to show the effectiveness of the obtained result.

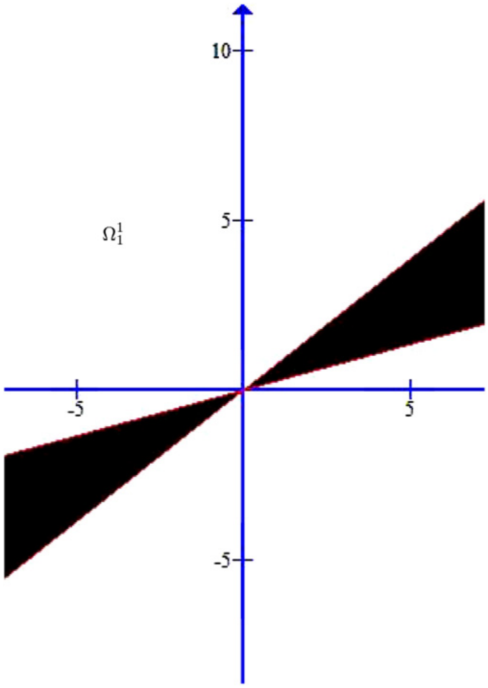

Fig. 1. The region Ω_1^1 of Example 4.1.

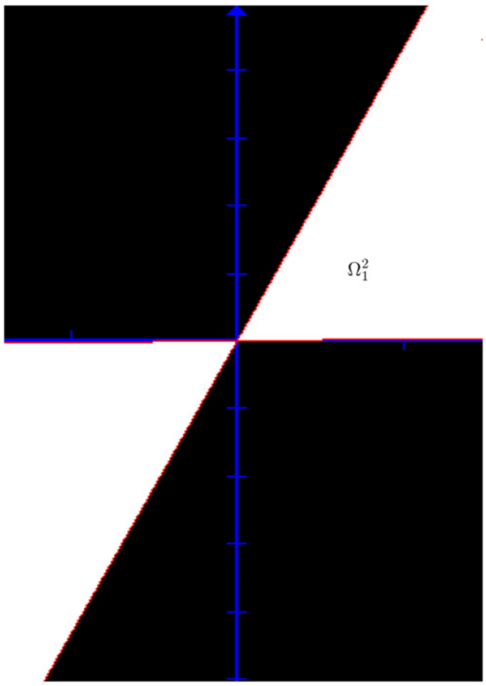

Fig. 2. The region Ω_1^2 of Example 4.1.

It is worth noting that the proposed method can be applied for large-scale switched systems with multiple (mixed) interval time-varying delays; however, it may have difficulty in obtaining the solution within a reasonable amount of the time delays, when the number of the delays is extremely large. Future research will focus on extending the method to solve the robust stabilization problem for uncertain large-scale switched systems with interval time-varying delays in state and control.

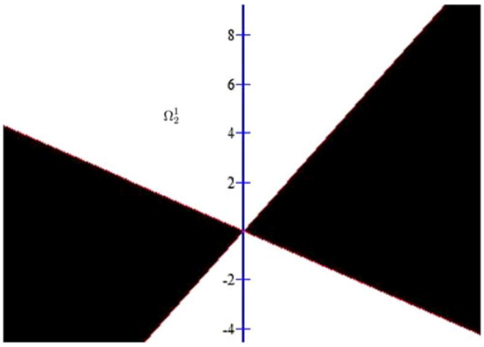

Fig. 3. The region Ω_2^1 of Example 4.1.

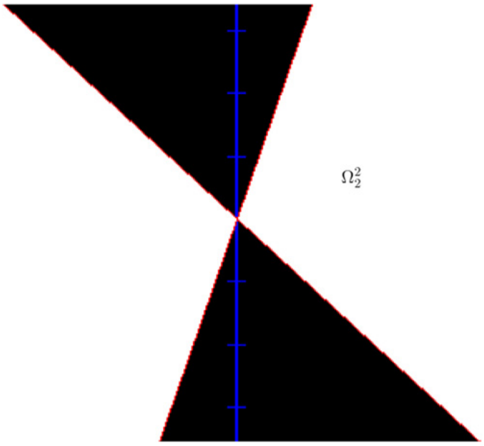

Fig. 4. The region Ω_2^2 of Example 4.1.

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