Journal of Process Control 22 (2012) 1325–1339

Contents lists available at SciVerse ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Decentralized H_∞ control for large-scale interconnected nonlinear time-delay systems via LMI approach

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ARTICLE INFO

Article history:
Received 8 April 2012
Received in revised form 7 June 2012
Accepted 8 June 2012
Available online 10 July 2012

Keywords:
H_∞ control
Large-scale system
Decentralized control
Nonlinear systems
Interval time-varying delay
Lyapunov function
Linear matrix inequalities

ABSTRACT

In this paper, the problem of H_∞ control of nonlinear large-scale systems with interval time-varying delays in interconnection is considered. The time delays are assumed to be any continuous functions belonging to a given interval involved in both the state and observation output. By constructing a set of new Lyapunov–Krasovskii functionals, which are mainly based on the information of the lower and upper delay bounds, a new delay-dependent sufficient condition for the existence of decentralized H_∞ control is established in terms of linear matrix inequalities (LMIs). The approach is applied to decentralized H_∞ control of uncertain linear systems with interval time-varying delay. Numerical examples are given to show the effectiveness of the obtained results.

1. Introduction

The H_∞ control of time-delay systems are of practical and theoretical interest since time delay is often encountered in many industrial and engineering processes [1–4]. The main objective of the H_∞ control is to obtain a controller that makes the closed-loop system asymptotically stable for a maximum H_∞ performance bound [5]. Many practical systems are of large scale models and consist of interconnected subsystems in the real world and the control of large scale systems can become very complicated owing to the high dimensionality of the system equation, uncertainties and time delays [6–13].

During the last two decades, decentralized H_∞ control for large-scale systems has been one of the focused study topic in the past year and a lot of interesting results have been made, see [14–21]. There are two different approaches to study H_∞ control of time delay systems. They are the Lypunov–Krasovskii approach and the Lyapunov–Razumikhin approach. The obtained results using Lyapunov–Krasovskii approach are usually less conservative than those using the Lyapunov–Razumikhin approach. The Lyapunov function method was developed in [22–26] to decentralized H_∞ control of linear systems with interval time-varying delays, where the assumption on the derivative of the delay function is either strictly bounded, but the time-delay function is still assumed to be differentiable. Stability analysis of the above cited papers reveals some restrictions: (i) the time delays should be either time-invariant interconnected or the lower delay bound is restricted to being zero; and (ii) the time delay function should be differential and its derivative is bounded. In this paper, the above restricted conditions are removed on the large-scale systems. In fact, this problem is difficult to solve; particularly, when the time-varying delays are interval and non-differentiable in state and observation output. In this paper, the time delay is assumed to be any continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is bounded but not necessary to be differentiable. This allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero. It is clear that the application of any memoryless feedback controller to such time-delay systems would lead to closed-loop systems with interval time-varying delays. Difficulties then arise when one attempts to derive exponential stabilizability conditions and to extract the controllers parameters for these systems. Indeed, existing Lyapunov–Krasovskii functionals and their associated results in [10,17–22,24,25] cannot be applied to solve the problem posed in this paper as they would either fail to cope

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http://dx.doi.org/10.1016/j.jprocont.2012.06.005

with the non-differentiability aspects of the delays, or lead to very complex matrix inequality conditions. Moreover, any technique such as matrix computation of variables fails to extract the parameters of the memoryless feedback controllers. This has motivated our research.

In this paper, we consider a class of large-scale nonlinear systems with interval time-varying delays. Compared to the existing results, our result has its own advantages. First, the time delays are assumed to be any continuous functions belonging to a given interval involved in both the state and observation output. Second, both problems of exponential stabilization and H_∞ control will be treated simultaneously. For the former, the controllers are required to guarantee the global exponential stability for the closed-loop system. By constructing a set of new augmented Lyapunov–Krasovskii functionals, a new delay-dependent condition for the decentralized H_∞ control is established in terms of LMIs, that can be solved numerically in an efficient manner by using standard computational algorithms. It is worth mentioning that although the similar Lyapunov–Krasovskii functional was used in [21,23,25,29] to investigate the stability of systems with time-varying delays, the slack matrix variables in main theorem have not been introduced in [21,23,25,29] since in the derivation of stability only single integrals depending on the delay function were used, while we use additional triple integrals. This Lyapunov–Krasovskii functional is mainly based on the information of the lower and upper delay bounds, which allows us to avoid using additional free-weighting matrices and any assumption on the differentiability of the delay function. Therefore, our results are more comprehensive and effective. The approach allows us to apply to H_∞ control of uncertain linear systems with interval non-differentiable time-varying delay.

2. Preliminaries

The following notations will be used throughout this paper, R^+ denotes the set of all real-negative numbers; R^n denotes the n-dimensional space with the scalar product $(.,.)$ and the vector norm $\|\cdot\|$; $R^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimension. A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the all eigenvalues of A; $\lambda_{max}(A) = max\{Re\,\lambda : \lambda\in\lambda(A)\}$; $\lambda_{min}(A) = min\{Re\,\lambda : \lambda\in\lambda(A)\}$; $\lambda_A = \lambda_{max}(A^TA)$; $C^1([a,b],R^n)$ denotes the set of all R^n-valued differentiable functions on $[a,b]$; $L_2([0,\infty],R^r)$ stands for the set of all square-integrable R^r-valued functions on $[0,\infty]$. The symmetric terms in a matrix are denoted by *. Matrix A is semi-positive definite ($A\geq 0$) if $(Ax,x)\geq 0$, for all $x\in R^n$; A is positive definite ($A>0$) if $(Ax,x)>0$ for all $x\neq 0$; $A\geq B$ means $A-B\geq 0$. The segment of the trajectory $x(t)$ is denotes by $x_t=\{x(t+s): s\in[-\tau,0]\}$ with its norm

$$\|x_t\| = \sup_{s\in[-\tau,0]}\|x(t+s)\|.$$

Consider a class of large-scale nonlinear systems Σ with time-varying delays composed of N interconnected subsystems Σ_i described by the following equations:

$$\Sigma_i:\begin{cases}\dot{x}_i(t)=A_ix_i(t)+B_iu_i(t)+D_i\omega_i(t)+\sum\limits_{j\neq i,j=1}^{N}A_{ij}x_j(t-h_{ij}(t))+f_i(t,x_i(t),u_i(t),\omega_i(t),\{x_j(t-h_{ij}(t))\}_{j=1,j\neq i}^{N}),\\ z_i(t)=C_ix_i(t)+F_iu_i(t)+\sum\limits_{j\neq i,j=1}^{N}G_{ij}x_j(t-h_{ij}(t))+g_i(t,x_i(t),u_i(t),\{x_j(t-h_{ij}(t))\}_{j=1,j\neq i}^{N}),\\ x_i(t)=\varphi_i(t),\quad\forall t\in[-h,0],\end{cases}\tag{2.1}$$

where $x^T(t)=[x_1(t),\ldots,x_N(t)^T]$, $x_i(t)\in R^{n_i}$, is the state vector, $z_i(t)\in R^{q_i}$ is the output vector, $u_i\in R^{m_i}$ are the control input, $\omega_i\in L_2([0,\infty],R^{r_i})$ is the uncertain input, the systems matrices A_i, B_i, C_i, D_i and A_{ij}, G_{ij} are of appropriate dimensions, the time delays $h_{ij}(.)$ satisfy the following condition:

$$0\leq h_1\leq h_{ij}(t)<h_2,\quad t\geq 0,\ \ \forall i,j=\overline{1,N},\ \ h=h_2,$$

and the initial function $\varphi^T(t)=[\varphi_1(t)^T,\ldots,\varphi_N(t)^T]$, $\varphi_i(t)\in C^1([-h,0],R^{n_i})$, with the norm

$$\|\varphi_i\| = \sup_{-\bar{h}\leq t\leq 0}\{\|\varphi_i(t)\|,\quad\|\dot{\varphi}_i(t)\|\},\quad \|\varphi\|=\sqrt{\sum_{i=1}^{N}\|\varphi_i\|^2}.$$

Let $x_j^{h_{ij}}(t):=x_j(t-h_{ij}(t)),\ \ i\neq j$, the nonlinear functions $f_i(.), g_i(.)$ satisfy the following growth conditions

$$\begin{aligned}&\exists\ a_i,b_i,d_i,a_{ij}>0:\|f_i(.)\|\leq a_i\|x_i(t)\|+b_i\|u_i(t)\|+d_i\|\omega_i(t)\|+\sum_{j\neq i,j=1}^{N}a_{ij}\|x_j^{h_{ij}}(t)\|\\ &\exists\ c_i,e_i,g_{ij}>0:\|g_i(.)\|^2\leq c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ij}\|x_j^{h_{ij}}(t)\|^2\end{aligned}\tag{2.2}$$

Definition 1. Given $\beta>0$. The zero solution of system (2.1), where $u_i(t)=0$, $\omega_i(t)=0$, is β- stable if there is a positive number $N_0>0$ such that every solution of the system satisfies:

$$\|x(t)\|\leq N_0\|\varphi\|e^{-\beta t},\quad\forall t\geq 0.$$

Definition 2. Given $\beta>0$, $\gamma>0$. The H_∞ control problem for system (2.1) has a solution if there exists memoryless state feedback controllers $u_i(t)=K_ix_i(t)$, satisfying the following two requirements:

(a) The zero solution of the nonlinear closed-loop system

$$\begin{cases} \dot{x}_i(t) = [A_i + B_iK_i]x_i(t) + \sum_{j\neq i, j=1}^{N} A_{ij}x_j(t-h_{ij}(t)) + f_i(t, x_i(t), K_ix_i(t), 0, \{x_j(t-h_{ij}(t))\}_{j=1, j\neq i}^{N}(t)), \\ x_i(t) = \varphi_i(t), \quad \forall t \in [-h, 0], \end{cases}$$

is β- stable.

(b) There is a number $c_0 > 0$ such that

$$\sup \frac{\int_0^\infty ||z(t)||^2 dt}{c_0||\varphi||^2 + \int_0^\infty |\omega(t)||^2 dt} \leq \gamma,$$

where the super is taken over all $\varphi_i \in C^1([-\tau, 0], R^{n_i})$ and the non-zero uncertainty $\omega_i(t) \in L_2([0, \infty], R^{r_i})$. In this case we say that the feedback controls $u_i(t) = K_i x_i(t)$ exponentially stabilizes the system.

Proposition 2.1. *For any $x, y \in R^n$ and positive definite matrix $M \in R^{n\times n}$, we have*

$$2x^Ty \leq y^TMy + x^TM^{-1}x.$$

Proposition 2.2. [27] *Given matrices X, Y, Z, where $Y = Y^T > 0$. Then $X + Z^TY^{-1}Z < 0$ if and only if*

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0.$$

Proposition 2.3 ([28]). *For any constant matrix $Z = Z^T > 0$ and scalar $h, \bar{h}, \ 0 < h < \bar{h}$ such that the following integrations are well defined, then*

$$-\int_{t-h}^{t} x^T(s)Zx(s)ds \leq -\frac{1}{h}\left(\int_{t-h}^{t} x(s)ds\right)^T Z \left(\int_{t-h}^{t} x(s)ds\right).$$

$$-\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x^T(\tau)Zx(\tau)d\tau\, ds \leq -\frac{2}{\bar{h}^2 - h^2}\left(\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x(\tau)d\tau\, ds\right)^T Z \left(\int_{-\bar{h}}^{-h}\int_{t+s}^{t} x(\tau)d\tau\, ds\right).$$

Proposition 2.4 (Lower bounds lemma [29]). *Let $f_1, f_2, \ldots, f_N : R^m \to R$ have positive values in an open subset D of R^m. Then*

$$\min_{r_i | r_i > 0, \sum_i r_i = 1} \sum_i \frac{1}{r_i} f_i(t) \geq \sum_i f_i(t) + \sum_{i\neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \ R^m \to R, g_{i,j}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j} \\ g_{i,j} & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

3. Main results

In this section, we investigate the decentralized H_∞ control of nonlinear system (2.1) with interval time-varying delays. It will be seen from the following theorem that neither free-weighting matrices nor any transformation are employed in our derivation. Before introducing main result, the following notations of several matrix variables are defined for simplicity.

$$P_{i1} = P_i^{-1}, \quad Q_{i1} = P_i^{-1}Q_iP_i^{-1}, \quad R_{i1} = P_i^{-1}R_iP_i^{-1}, \quad U_{i1} = P_i^{-1}U_iP_i^{-1},$$
$$\Lambda_{i1} = P_i^{-1}\Lambda_iP_i^{-1}, \quad S_{i1} = P_i^{-1}S_iP_i^{-1}$$
$$H_{11}^i = P_iA_i^T + A_iP_i + B_iY_i + Y_i^TB_i^T + 2\beta P_i + 2Q_i - e^{-2\beta h_1}R_i - e^{-2\beta h_2}R_i - 2\frac{e^{-4\beta h_2}(h_2-h_1)}{h_2+h_1}\Lambda_i + \sum_{j=1, j\neq i}^{N} A_{ij}A_{ij}^T + \frac{4}{\gamma_i}D_iD_i^T + \varepsilon_i I,$$
$$H_{1k}^i = 0, \ \forall k = \overline{2,N}, \quad H_{1(N+1)}^i = e^{-2\beta h_1}R_i, \quad H_{1(N+2)}^i = e^{-2\beta h_2}R_i,$$
$$H_{1(N+3)}^i = P_iA_i^T + Y_i^TB_i^T, \quad H_{1(N+4)}^i = 2\frac{e^{-2\beta h_2}}{h_2+h_1}\Lambda_i, \quad H_{kj}^i = 0, \ \forall k \neq j, \ k, j = \overline{2,N},$$
$$H_{kk}^i = -2\frac{e^{-2\beta h_2}}{N-1}U_i + \frac{e^{-2\beta h_2}}{N-1}(S_i + S_i^T),$$
$$H_{k(N+1)}^i = \frac{e^{-2\beta h_2}}{N-1}U_i - \frac{e^{-2\beta h_2}}{N-1}S_i, \quad H_{k(N+2)}^i = \frac{e^{-2\beta h_2}}{N-1}U_i - \frac{e^{-2\beta h_2}}{N-1}S_i^T,$$
$$H_{k(N+3)}^i = H_{k(N+4)}^i = 0, \ \forall k = \overline{2,N},$$
$$H_{(N+1)(N+1)}^i = -e^{-2\beta h_1}Q_i - e^{-2\beta h_1}R_i - e^{-2\beta h_2}U_i,$$
$$H_{(N+1)(N+2)}^i = e^{-2\beta h_2}S_i^T,$$
$$H_{(N+1)(N+3)}^i = H_{(N+2)(N+3)}^i = H_{(N+1)(N+4)}^i = H_{(N+2)(N+4)}^i = 0,$$
$$H_{(N+2)(N+2)}^i = -e^{-2\beta h_2}Q_i - e^{-2\beta h_2}R_i - e^{-2\beta h_2}U_i,$$
$$H_{(N+3)(N+3)}^i = (h_2-h_1)h_2\Lambda_i + h_1^2R_i + h_2^2R_i - 2P_i + (h_2-h_1)^2U_i + \frac{4}{\gamma_i}D_iD_i^T + \sum_{j=1, j\neq i}^{N} A_{ij}A_{ij}^T + \varepsilon_i I,$$
$$H_{(N+3)(N+4)}^i = 0, \quad H_{(N+4)(N+4)}^i = -2\frac{e^{-4\beta h_2}}{h_2^2-h_1^2}\Lambda_i,$$
$$H_{(N+5)(N+5)}^i = -\frac{I}{N+2}, \quad H_{1(N+5)}^i = P_iC_i^T, H_{k(N+5)}^i = 0, \ \forall k = \overline{2,(N+4)},$$
$$H_{(N+4+k)(N+4+k)}^i = -\frac{I}{N+2}, \quad H_{j(N+4+k)}^i = 0, \ \forall k = \overline{2,N}, \ j = \overline{1,(N+3+k)}, \ j \neq k,$$
$$H_{k(N+4+k)}^i = P_iG_{ki}^T, \quad i = 1, \ k = \overline{2,N},$$
$$H_{k(N+4+k)}^i = P_iG_{(k-1)i}^T, \quad i \neq 1, \ k \leq i, \ k = \overline{2,N},$$
$$H_{k(N+4+k)}^i = P_iG_{ki}^T, \quad i \neq 1, \ i < k = \overline{2,N},$$
$$H_{(2N+3+k)(2N+3+k)}^i = -\frac{I}{2+2a_{ki}+[N+2]g_{ki}}, \quad H_{k(2N+3+k)}^i = P_i, \ i = 1, \ k = \overline{2,N},$$
$$H_{(2N+3+k)(2N+3+k)}^i = -\frac{I}{2+2a_{(k-1)i}+[N+2]g_{(k-1)i}},$$
$$H_{k(2N+3+k)}^i = P_i, \quad i \neq 1, \ k \leq i, \ k = \overline{2,N},$$
$$H_{(2N+3+k)(2N+3+k)}^i = -\frac{I}{2+2a_{ki}+[N+2]g_{ki}}, \quad H_{k(2N+3+k)}^i = P_i, \ i \neq 1, \ i < k = \overline{2,N},$$
$$H_{j(2N+3+k)}^i = 0, \quad \forall k = \overline{2,N}, \ j = \overline{1,(2N+3+k)}, \ j \neq k, j \neq (2N+3+k),$$
$$H_{(3N+4),(3N+4)}^i = -\frac{I}{2a_i + c_i[N+2]}, \quad H_{1(3N+4)}^i = P_i, \quad H_{j(3N+4)}^i = 0, \ j \neq 1, \ j \neq (3N+4),$$
$$H_{(3N+5),(3N+5)}^i = -\frac{I}{2b_i + (N+2)(e_i+1)}, \quad H_{1,(3N+5)}^i = Y_i^T.$$
$$\varepsilon_i = a_i + b_i + \frac{4d_i^2}{\gamma_i} + \sum_{j\neq i, j=1}^{N} a_{ij}, \quad \alpha_{i1} = \lambda_{min}(P_{i1}),$$
$$\alpha_{i2} = \lambda_{max}(P_{i1}) + \beta^{-1}\lambda_{max}(Q_{i1}) + h_1^3\lambda_{max}(R_{i1}) + h_2^3\lambda_{max}(R_{i1}) + (h_2-h_1)^3\lambda_{max}(U_{i1}) + (h_2-h_1)h_2^2\lambda_{max}(\Lambda_{i1}),$$
$$\alpha_1 = \min_{i=\overline{1,N}}\alpha_{i1}, \quad \alpha_2 = \max_{i=\overline{1,N}}\alpha_{i2}, \quad \gamma = \max_{i=\overline{1,N}}\gamma_i.$$

Then, for simplicity of expression [5], we assume that

$$F_i^T F_i = I, \quad F_i^T [C_i, G_{ij}] = 0, \ \forall j, i = \overline{1, N}, \ j \neq i.$$

The following is the main result of the paper, which gives sufficient conditions for the decentralized H_∞ control of system (2.1). Essentially, the proof is based on the construction of Lyapunov–Krasovskii functions satisfying Lyapunov stability theorem for time-delay system [30].

Theorem 3.1. *The H_∞ control of the system (2.1) has a solution if there exist symmetric positive definite matrices $P_i, Q_i, R_i, U_i, \Lambda_i$ and matrices S_i, Y_i such that the following LMIs hold:*

$$\begin{bmatrix} H_{11}^i & H_{12}^i & \dots & H_{1(3N+5)}^i & 0 & 0 \\ * & H_{22}^i & \dots & H_{2(3N+5)}^i & 0 & 0 \\ . & . & \dots & . & . & . \\ * & * & \dots & H_{(3N+5)(3N+5)}^i & 0 & 0 \\ * & * & \dots & * & -U_i & -S_i \\ * & * & \dots & * & * & -U_i \end{bmatrix} < 0, \quad i = 1, 2, \dots, N, \tag{3.1}$$

Moreover, stabilizing feedback controls are defined as

$$u_i(t) = Y_i P_{i1} x_i(t), \ t \geq 0, \quad i = 1, 2, \dots, N,$$

and the zero solution of the closed-loop system is β-stable, i.e. the solution satisfies

$$||x(t)|| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-\beta t} ||\varphi||, \quad \forall t \geq 0.$$

Proof. Consider the following Lyapunov–Krasovskii functional for the closed loop system:

$$V(t, x_t) = \sum_{i=1}^{N} \sum_{j=1}^{7} V_{ij}(t, x_t),$$

where

$$\begin{aligned} V_{i1} &= x_i^T(t) P_{i1} x_i(t), \quad V_{i2} = \int_{t-h_1}^{t} e^{2\beta(s-t)} x_i^T(s) Q_{i1} x_i(s) ds, \\ V_{i3} &= \int_{t-h_2}^{t} e^{2\beta(s-t)} x_i^T(s) Q_{i1} x_i(s) ds, \\ V_{i4} &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i^T(\tau) R_{i1} \dot{x}_i(\tau) d\tau \, ds, \\ V_{i5} &= h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i^T(\tau) R_{i1} \dot{x}_i(\tau) d\tau \, ds, \\ V_{i6} &= (h_2 - h_1) \times \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{2\beta(\tau-t)} \dot{x}_i^T(\tau) U_{i1} \dot{x}_i(\tau) d\tau \, ds. \\ V_{i7}(t, x_t) &= \int_{-h_2}^{-h_1} \int_{\theta}^{0} \int_{t+s}^{t} e^{2\beta(\tau+s-t)} \dot{x}_i^T(\tau) \Lambda_{i1} \dot{x}_i(\tau) d\tau \, ds \, d\theta. \end{aligned}$$

It is easy to verify that

$$\sum_{i=1}^{N} \alpha_{i1} ||x_i(t)||^2 \leq V(t, x_t), \quad V(0, x_0) \leq \sum_{i=1}^{N} \alpha_{i2} ||\varphi_i||^2. \tag{3.2}$$

Taking the derivative of V in t along the solution of the system, we have

$$\dot{V}_{i1} = 2x_i^T(t)P_{i1}[A_ix_i(t) + \sum_{j\neq i,j=1}^{N} A_{ij}x_j(t-h_{ij}(t)) + B_iu_i(t) + D_i\omega_i(t) + f_i(.)] = 2x_i^T(t)P_{i1}\left[A_ix_i(t) + \sum_{j\neq i,j=1}^{N} A_{ij}x_j(t-h_{ij}(t)) + B_iY_iP_{i1}x_i(t) + D_i\omega_i(t) + f_i(.)\right],$$

$$\dot{V}_{i2} = x_i^T(t)Q_{i1}x_i(t) - 2\beta V_{i2} - e^{-2\beta h_1}x_i^T(t-h_1)Q_{i1}x_i(t-h_1),$$

$$\dot{V}_{i3} = x_i^T(t)Q_{i1}x_i(t) - 2\beta V_{i3} - e^{-2\beta h_2}x_i^T(t-h_2)Q_{i1}x_i(t-h_2),$$

$$\dot{V}_{i4} \le h_1^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t) - 2\beta V_{i4} - h_1e^{-2\beta h_1}\int_{t-h_1}^{t}\dot{x}_i^T(s)R_{i1}\dot{x}_i(s)ds,$$

$$\dot{V}_{i5} \le h_2^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t) - 2\beta V_{i5} - h_2e^{-2\beta h_2}\int_{t-h_2}^{t}\dot{x}_i^T(s)R_{i1}\dot{x}_i(s)ds,$$

$$\dot{V}_{i6} \le (h_2-h_1)^2\dot{x}_i^T(t)U_{i1}\dot{x}_i(t) - 2\beta V_{i6} - (h_2-h_1)e^{-2\beta h_2}\int_{t-h_2}^{t-h_1}\dot{x}_i^T(s)U_{i1}\dot{x}_i(s)ds.$$

$$\dot{V}_{i7}(t,x_t) \le (h_2-h_1)h_2\dot{x}_i^T(t)\Lambda_{i1}\dot{x}_i(t) - e^{-4\beta h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}_i^T(s)\Lambda_{i1}\dot{x}_i(s)ds\,d\theta - 2\beta V_{i7}(t,x_t)$$

Applying Proposition 2.3 and the Newton–Leibniz formula

$$\int_{t-h}^{t}\dot{x}_i(s)ds = x_i(t) - x_i(t-h),$$

we have

$$-h\int_{t-h}^{t}\dot{x}_i^T(s)R_{i1}\dot{x}_i(s)ds \le -\left[\int_{t-h}^{t}\dot{x}_i(s)ds\right]^T R_{i1}\left[\int_{t-h}^{t}\dot{x}_i(s)ds\right] = -[x_i(t)-x_i(t-h)]^TR_{i1}[x_i(t)-x_i(t-h)].$$

Note that

$$\int_{t-h_2}^{t-h_1}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds = \int_{t-h_2}^{t-h_{ji}(t)}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds + \int_{t-h_{ji}(t)}^{t-h_1}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds.$$

Using Proposition 2.3 again gives

$$[h_2-h_{ji}(t)]\int_{t-h_2}^{t-h_{ji}(t)}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds \ge \left[\int_{t-h_2}^{t-h_{ji}(t)}\dot{x}_i(s)ds\right]^T U_{i1}\left[\int_{t-h_2}^{t-h_{ji}(t)}\dot{x}_i(s)ds\right] \ge [x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]$$

Because of $h_2-h_{ji}(t)\le h_2-h_1$, we have

$$-(h_2-h_1)\int_{t-h_2}^{t-h_{ji}(t)}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds \le -\frac{h_2-h_1}{h_2-h_{ji}(t)}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)],$$

$$-(h_2-h_1)\int_{t-h_{ji}(t)}^{t-h_1}\dot{x}_i{}^T(s)U_{i1}\dot{x}_i(s)ds \le -\frac{h_2-h_1}{h_{ji}(t)-h_1}[x_i(t-h_1)-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))].$$

On the other hand, condition (3.1) figures out $\begin{bmatrix} U_i & S_i \\ S_i^T & U_i \end{bmatrix} \ge 0$, so using Proposition 2.4 with $r_1 = \frac{h_2-h_{ji}(t)}{h_2-h_1}$; $r_2 = \frac{h_{ji}(t)-h_1}{h_2-h_1}$ gives the following inequalities:

$$\begin{bmatrix} \sqrt{\frac{r_2}{r_1}}[x_i(t-h_{ji}(t)-x_i(t-h_2))] \\ -\sqrt{\frac{r_1}{r_2}}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \end{bmatrix}^T \begin{bmatrix} U_{i1} & S_{i1} \\ S_{i1}^T & U_{i1} \end{bmatrix} \begin{bmatrix} \sqrt{\frac{r_2}{r_1}}[x_i(t-h_{ji}(t)-x_i(t-h_2))] \\ -\sqrt{\frac{r_1}{r_2}}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{r_2}{r_1}}[x_i(t-h_{ji}(t)-x_i(t-h_2))] \\ -\sqrt{\frac{r_1}{r_2}}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \end{bmatrix}^T \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i1} \end{bmatrix} \begin{bmatrix} U_i & S_i \\ S_i^T & U_i \end{bmatrix}$$
$$\times \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i1} \end{bmatrix} \begin{bmatrix} \sqrt{\frac{r_2}{r_1}}[x_i(t-h_{ji}(t)-x_i(t-h_2))] \\ -\sqrt{\frac{r_1}{r_2}}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \end{bmatrix} \ge 0,$$

equivalently,

$$-\frac{r_2}{r_1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)] - \frac{r_1}{r_2}[x_i(t-h_1)-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \le$$
$$-[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TS_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))] - [x_i(t-h_1)-x_i(t-h_{ji}(t))]^TS_{i1}^T[x_i(t-h_{ji}(t))-x_i(t-h_2)],$$

and

$$-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}_i^{\,T}(s)U_{i1}\dot{x}_i(s)ds \le -\frac{h_2-h_1}{h_2-h_{ji}(t)}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]-\frac{h_2-h_1}{h_{ji}(t)-h_1}[x_i(t-h_1)$$

$$-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))] = -\frac{1}{r_1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]-\frac{1}{r_2}[x_i(t-h_1)$$

$$-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))] \le -[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]$$

$$-[x_i(t-h_1)-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))]-[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TS_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))]-[x_i(t-h_1)$$

$$-x_i(t-h_{ji}(t))]^TS_{i1}^T[x_i(t-h_{ji}(t))-x_i(t-h_2)].$$

Therefore,

$$\dot{V}_{i4} \le h_1^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t)-2\beta V_{i4}-e^{-2\beta h_1}[x_i(t)-x_i(t-h_1)]^TR_{i1}[x_i(t)-x_i(t-h_1)],$$

$$\dot{V}_{i5} \le h_2^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t)-2\beta V_{i5}-e^{-2\beta h_2}[x_i(t)-x_i(t-h_2)]^TR_{i1}[x_i(t)-x_i(t-h_2)],$$

$$\dot{V}_{i6} \le (h_2-h_1)^2\dot{x}_i^T(t)U_{i1}\dot{x}_i(t)-2\beta V_{i6}-\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t))-x_i(t-h_2)]$$

$$-\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_1)-x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)-x_i(t-h_{ji}(t))]-\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_{ji}(t))-x_i(t-h_2)]^TS_{i1}[x_i(t-h_1) \tag{3.3}$$

$$-x_i(t-h_{ji}(t))]-\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_1)-x_i(t-h_{ji}(t))]^TS_{i1}^T[x_i(t-h_{ji}(t))-x_i(t-h_2)].$$

Note that when $h_{ji}(t)=h_1$ or $h_{ji}(t)=h_2$, we have

$$[x_i(t-h_1)-x_i(t-h_{ji}(t))]^T=0 \quad \text{or} \quad [x_i(t-h_{ji}(t))-x_i(t-h_2)]=0,$$

respectively. So the relation (3.3) still holds. Besides, using Proposition 2.3 again, we have

$$-e^{-4\beta h_2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}_i^T(s)\Lambda_{i1}\dot{x}_i(s)ds\,d\theta \le -e^{-4\beta h_2}\frac{2}{h_2^2-h_1^2}\left(\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}_i(s)ds\,d\theta\right)^T\Lambda_{i1}\left(\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}_i(s)ds\,d\theta\right) \le$$

$$-\frac{2e^{-4\beta h_2}}{h_2^2-h_1^2}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right)^T\Lambda_{i1}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right).$$

Hence,

$$\dot{V}_7(.) \le (h_2-h_1)h_2\dot{x}_i^T(t)\Lambda_{i1}\dot{x}_i(t)-2\beta V_7(t,x_t)-\frac{2e^{-4\beta h_2}}{h_2^2-h_1^2}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right)^T\Lambda_{i1}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right) \tag{3.4}$$

From the following identity relation

$$-2\dot{x}_i^T(t)P_{i1}\times\left[\dot{x}_i(t)-A_ix_i(t)-\sum_{j\ne i,j=1}^{N}A_{ij}x_j(t-h_{ij}(t))-B_iu_i(t)-D_i\omega_i(t)-f_i(.)\right]=0,$$

and applying Proposition 2.1 and condition (2.2), we obtain

$$2x_i^T(t)P_{i1}\left[\sum_{j\ne i,j=1}^{N}A_{ij}x_j(t-h_{ij}(t))\right] \le \sum_{j\ne i,j=1}^{N}x_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}x_i(t)+\sum_{j\ne i,j=1}^{N}x_j^T(t-h_{ij}(t))x_j(t-h_{ij}(t)),$$

$$2\dot{x}_i^T(t)P_{i1}\left[\sum_{j\ne i,j=1}^{N}A_{ij}x_j(t-h_{ij}(t))\right] \le \sum_{j\ne i,j=1}^{N}\dot{x}_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}\dot{x}_i(t)+\sum_{j\ne i,j=1}^{N}x_j^T(t-h_{ij}(t))x_j(t-h_{ij}(t)),$$

$$2x_i^T(t)P_{i1}D_i\omega_i(t) \le \frac{4}{\gamma_i}x_i^T(t)P_{i1}D_iD_i^TP_{i1}x_i(t) + 0.25\gamma_i\omega_i^T(t)\omega_i(t),$$

$$2\dot{x}_i^T(t)P_{i1}D_i\omega_i(t) \le \frac{4}{\gamma_i}\dot{x}_i^T(t)P_{i1}D_iD_i^TP_{i1}\dot{x}_i(t) + 0.25\gamma_i\omega_i^T(t)\omega_i(t),$$

$$2x_i^T(t)P_{i1}f_i(.) \le 2||x_i^T(t)P_{i1}||.||f_i(t, x_i(t), u_i(t), \omega_i(t), x_1^{h_{i1}}(t), \ldots, x_j^{h_{ij}}(t), \ldots, x_N^{h_{iN}}(t))|| \le 2||x_i^T(t)P_{i1}||$$
$$\left[a_i\|x_i(t)\| + b_i\|u_i(t)\| + d_i\|\omega_i(t)\| + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|\right] \le (a_i + b_i + \frac{4d_i^2}{\gamma_i} + \sum_{j\ne i,j=1}^{N} a_{ij})||x_i^T(t)P_{i1}||^2 + a_i||x_i(t)||^2 + b_i||u_i(t)||^2$$
$$+0.25\gamma_i||\omega_i(t)||^2 + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2 \le \varepsilon_i||x_i^T(t)P_{i1}||^2 + a_i||x_i(t)||^2 + b_i||u_i(t)||^2 + 0.25\gamma_i||\omega_i(t)||^2 + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2.$$

$$2\dot{x}_i(t)^TP_{i1}f_i(.) \le \varepsilon_i||\dot{x}_i^T(t)P_{i1}||^2 + a_i||x_i(t)||^2 + b_i||u_i(t)||^2 + 0.25\gamma_i||\omega_i(t)||^2 + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2.$$

Moreover, the following estimations hold

$$0 = -2\dot{x}_i^T(t)P_{i1}\left[\dot{x}_i(t) - A_ix_i(t) - \sum_{j\ne i,j=1}^{N} A_{ij}x_j(t-h_{ij}(t)) - B_iu_i(t) - D_i\omega_i(t) + f_i(.)\right] \le -[2\dot{x}_i^T(t)P_{i1}][\dot{x}_i(t) - A_ix_i(t) - B_iY_iP_{i1}x_i(t)]$$
$$+\sum_{j\ne i,j=1}^{N}\dot{x}_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}\dot{x}_i(t) + \sum_{j\ne i,j=1}^{N}x_j^T(t-h_{ij}(t))x_j(t-h_{ij}(t)) + \frac{4}{\gamma_i}\dot{x}_i^T(t)P_{i1}D_iD_i^TP_{i1}\dot{x}_i(t) + 0.25\gamma_i\omega_i^T(t)\omega_i(t) + \varepsilon_i||\dot{x}_i(t)^TP_{i1}||^2$$
$$+a_i||x_i(t)||^2 + b_i||u_i(t)||^2 + 0.25\gamma_i||\omega_i(t)||^2 + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2, \tag{3.5}$$

and

$$\dot{V}_{i1}(.) = 2x_i^T(t)P_{i1}\left[A_ix_i(t) + \sum_{j\ne i,j=1}^{N} A_{ij}x_j(t-h_{ij}(t)) + B_iu_i(t) + D_i\omega_i(t) + f_i(.)\right] \le 2x_i^T(t)P_{i1}[A_ix_i(t) + B_iY_iP_{i1}x_i(t)]$$
$$+\sum_{j\ne i,j=1}^{N}x_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}x_i(t) + \sum_{j\ne i,j=1}^{N}x_j^T(t-h_{ij}(t))x_j(t-h_{ij}(t)) + \frac{4}{\gamma_i}x_i^T(t)P_{i1}D_iD_i^TP_{i1}x_i(t) + 0.25\gamma_i\omega_i^T(t)\omega_i(t) + \varepsilon_i||x_i(t)^TP_{i1}||^2$$
$$+a_i||x_i(t)||^2 + b_i||u_i(t)||^2 + 0.25\gamma_i||\omega_i(t)||^2 + \sum_{j\ne i,j=1}^{N} a_{ij}\|x_j^{h_{ij}}(t)\|^2. \tag{3.6}$$

Therefore, applying the inequalities from (3.3) to (3.6) and note that

$$\sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}x_j^T(t-h_{ij}(t))x_j(t-h_{ij}(t)) = \sum_{j=1}^{N}\sum_{i=1,i\ne j}^{N}x_i^T(t-h_{ji}(t))x_i(t-h_{ji}(t)) = \sum_{i=1}^{N}\left[\sum_{j=1,i\ne j}^{N}x_i^T(t-h_{ji}(t))x_i(t-h_{ji}(t))\right],$$

we have

$$\dot{V}(.) + 2\beta V(.) \le \sum_{i=1}^{N}\Big[2x_i^T(t)P_{i1}[A_ix_i(t) + B_iY_iP_{i1}x_i(t)] + 2\beta x_i^T(t)P_{i1}x_i(t) + x_i^T(t)Q_{i1}x_i(t) - e^{-2\beta h_1}x_i^T(t-h_1)Q_{i1}x_i(t-h_1) + x_i^T(t)Q_{i1}x_i(t)$$
$$-e^{-2\beta h_2}x_i^T(t-h_2)Q_{i1}x_i(t-h_2) + h_1^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t) - e^{-2\beta h_1}[x_i(t) - x_i(t-h_1)]^TR_{i1}[x_i(t) - x_i(t-h_1)]$$
$$-e^{-2\beta h_2}[x_i(t) - x_i(t-h_2)]^TR_{i1}[x_i(t) - x_i(t-h_2)] + (h_2-h_1)^2\dot{x}_i^T(t)U_{i1}\dot{x}_i(t) + h_2^2\dot{x}_i^T(t)R_{i1}\dot{x}_i(t)$$
$$-\frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]^TU_{i1}[x_i(t-h_{ji}(t)) - x_i(t-h_2)] - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_1) - x_i(t-h_{ji}(t))]^TU_{i1}[x_i(t-h_1)$$
$$-x_i(t-h_{ji}(t))] - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_{ji}(t)) - x_i(t-h_2)]^TS_{i1}[x_i(t-h_1) - x_i(t-h_{ji}(t))] - \frac{e^{-2\beta h_2}}{N-1}\sum_{j=1,j\ne i}^{N}[x_i(t-h_1)$$
$$-x_i(t-h_{ji}(t))]^TS_{i1}^T[x_i(t-h_{ji}(t)) - x_i(t-h_2)] + (h_2-h_1)h_2\dot{x}_i^T(t)\Lambda_{i1}\dot{x}_i(t)$$

$$-\frac{2e^{-4\beta h_2}}{h_2^2-h_1^2}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right)^T\Lambda_{i1}\left((h_2-h_1)x_i(t)-\int_{t-h_2}^{t-h_1}x_i(\theta)d\theta\right)-[2\dot{x}_i^T(t)P_{i1}]\times[\dot{x}_i(t)-A_ix_i(t)-B_iY_iP_{i1}x_i(t)]$$

$$+\sum_{j\neq i,j=1}^{N}x_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}x_i(t)+\sum_{j\neq i,j=1}^{N}x_i^T(t-h_{ji}(t))x_i(t-h_{ji}(t))+\sum_{j\neq i,j=1}^{N}\dot{x}_i^T(t)P_{i1}A_{ij}A_{ij}^TP_{i1}\dot{x}_i(t)+\sum_{j\neq i,j=1}^{N}x_i^T(t-h_{ji}(t))x_i(t-h_{ji}(t))$$

$$+\frac{4}{\gamma_i}x_i^T(t)P_{i1}D_iD_i^TP_{i1}x_i(t)+\gamma_i\omega_i(t)^T\omega_i(t)+\frac{4}{\gamma_i}\dot{x}_i^T(t)P_{i1}D_iD_i^TP_{i1}\dot{x}_i(t)+2a_i||x_i(t)||^2+2b_i||u_i(t)||^2$$

$$\left.+\sum_{j\neq i,j=1}^{N}2a_{ji}\|x_i^{h_{ji}}(t)\|^2+\varepsilon_i||x_i^T(t)P_{i1}||^2+\varepsilon_i||\dot{x}_i^T(t)P_{i1}||^2\right].$$

Setting $y_i(t)=P_{i1}x_i(t)$, it leads to

$$\dot{V}(t,x_t)+2\beta V(t,x_t)\leq\sum_{i=1}^{N}\gamma_i||\omega_i(t)||^2+\sum_{i=1}^{N}\xi_i^T(t)M^i\xi_i(t)-(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||u_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2\right]$$

$$-(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2\right] \quad (3.7)$$

where

$$\xi_i^T(t)=\left[y_i^T(t)\{y_i^T(t-h_{ji}(t))\}_{j=1,j\neq i}^N\ y_i^T(t-h_1)\ y_i^T(t-h_2)\ \dot{y}_i^T(t)\ \int_{t-h_2}^{t-h_1}y_i^T(\theta)d\theta\right],$$

$$M^i=\begin{bmatrix}M_{11}^i & M_{12}^i & \dots & M_{1(N+4)}^i\\ * & M_{22}^i & \dots & M_{2(N+4)}^i\\ . & . & \dots & .\\ * & * & \dots & M_{(N+4)(N+4)}^i\end{bmatrix},\quad i=\overline{1,N},$$

$$M_{11}^i=P_iA_i^T+A_iP_i+B_iY_i+Y_i^TB_i^T+[2b_i+(N+2)(e_i+1)]Y_i^TY_i+2\beta P_i+2Q_i-e^{-2\beta h_1}R_i-e^{-2\beta h_2}R_i-2\frac{e^{-4\beta h_2}(h_2-h_1)}{h_2+h_1}\Lambda_i$$

$$+\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T+\frac{4}{\gamma_i}D_iD_i^T+(N+2)P_iC_i^TC_iP_i+(2a_i+c_i[N+2])P_i^2+\varepsilon_iI,$$

$$M_{1k}^i=0,\ \ \forall k=\overline{2,N},\quad M_{1(N+1)}^i=e^{-2\beta h_1}R_i,\ M_{1(N+2)}^i=e^{-2\beta h_2}R_i,$$

$$M_{1(N+3)}^i=P_iA_i^T+Y_i^TB_i^T,\quad M_{1,(N+4)}^i=2\frac{e^{-2\beta h_2}}{h_2+h_1}\Lambda_i,\quad M_{kj}^i=0,\ \ \forall k\neq j,\ \ k,j=\overline{2,N},$$

$$M_{kk}^i=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i+S_i^T)+(2+2a_{ki}+[N+2]g_{ki})P_i^2+(N+2)P_iG_{ki}^TG_{ki}P_i,\ \ \forall k=\overline{2,N},\ \ i=1,$$

$$M_{kk}^i=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i+S_i^T)+(2+2a_{(k-1)i}+[N+2]g_{(k-1)i})P_i^2+(N+2)P_iG_{(k-1)i}^TG_{(k-1)i}P_i,\quad k=\overline{2,N},\ \ i\neq 1,\ \ k\leq i,$$

$$M_{kk}^i=-2\frac{e^{-2\beta h_2}}{N-1}U_i+\frac{e^{-2\beta h_2}}{N-1}(S_i+S_i^T)+(2+2a_{ki}+[N+2]g_{ki})P_i^2+(N+2)P_iG_{ki}^TG_{ki}P_i,\quad k=\overline{2,N},\ \ i\neq 1,\ \ k\geq i+1,$$

$$M_{k(N+1)}^i=\frac{e^{-2\beta h_2}}{N-1}U_i-\frac{e^{-2\beta h_2}}{N-1}S_i,\quad M_{k(N+2)}^i=\frac{e^{-2\beta h_2}}{N-1}U_i-\frac{e^{-2\beta h_2}}{N-1}S_i^T,$$

$$M_{k(N+3)}^i=M_{k(N+4)}^i=0,\quad k=\overline{2,N},$$

$$M_{(N+1)(N+1)}^i=-e^{-2\beta h_1}Q_i-e^{-2\beta h_1}R_i-e^{-2\beta h_2}U_i,\quad M_{(N+1)(N+2)}^i=e^{-2\beta h_2}S_i^T,$$

$$M_{(N+1)(N+3)}^i=M_{(N+2)(N+3)}^i=M_{(N+1)(N+4)}^i=M_{(N+2)(N+4)}^i=0,$$

$$M_{(N+2)(N+2)}^i=-e^{-2\beta h_2}Q_i-e^{-2\beta h_2}R_i-e^{-2\beta h_2}U_i,$$

$$M_{(N+3)(N+3)}^i=(h_2-h_1)h_2\Lambda_i+h_1^2R_i+h_2^2R_i+(h_2-h_1)^2U_i-2P_i+\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T+\frac{4}{\gamma_i}D_iD_i^T+\varepsilon_iI.$$

$$M_{(N+3)(N+4)}^i=0,\quad M_{(N+4),(N+4)}^i=-2\frac{e^{-4\beta h_2}}{h_2^2-h_1^2}\Lambda_i$$

Using the Schur complement lemma, condition (3.1) leads to $M^i<0, \forall i\in\overline{1,N}$ and from the inequality (3.7), it follows that

$$\dot{V}(t,x_t)+2\beta V(t,x_t)\leq -(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||u_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2\right]$$

$$-(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2\right]+\sum_{i=1}^{N}\gamma_i||\omega_i(t)||^2. \tag{3.8}$$

Letting $\omega_i(t) = 0$, and since

$$-(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||u_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2\right]\leq 0,$$

$$-(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2\right]\leq 0,$$

we obtain from the inequality (3.8) that

$$\dot{V}(t,x_t)+2\beta V(t,x_t)\leq 0. \tag{3.9}$$

Differentiating the inequality (3.9) from 0 to t, we have

$$V(t,x_t)\leq V(0,x_0)e^{-2\beta t},\quad \forall t\geq 0.$$

Taking inequality (3.2) in account, we finally obtain that

$$\alpha_1\sum_{i=1}^{N}||x_i(t)||^2\leq V(t,x_t)\leq \alpha_2\sum_{i=1}^{N}||\varphi_i||^2e^{-2\beta t},\ \ \forall\ t\geq 0,$$

which implies that the zero solution of the close loop system is the β-stable. To complete the proof of the theorem, it remains to show the γ_i-optimal level condition. For this, we consider the following relation:

$$\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2\right]dt=\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2+\dot{V}(t,x_t)\right]dt-\int_0^s\dot{V}(t,x_t)dt,\quad \forall s\geq 0.$$

Since $V(t, x_t) \geq 0, \quad \forall\, t \geq 0$, we have

$$-\int_0^s\dot{V}(t,x_t)dt=V(0,x_0)-V(s,x_s)\leq V(0,x_0).$$

Therefore, for all $s \geq 0$, we have

$$\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2\right]dt\leq\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2+\dot{V}(t,x_t)\right]dt+V(0,x_0) \tag{3.10}$$

Combining (3.8) and the inequality

$$V(t,x_t)\geq\sum_{i=1}^{N}x_i(t)^TP_{i1}x_i(t)=\sum_{i=1}^{N}y_i(t)^TP_iy_i(t),$$

we obtain

$$\dot{V}(t,x_t)\leq\sum_{i=1}^{N}\gamma_i||\omega_i(t)||^2-2\beta\sum_{i=1}^{N}y_i(t)^TP_iy_i(t)-(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||u_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2\right]$$

$$-(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2\right] \tag{3.11}$$

Observe that the value of $||z_i(t)||^2$ is defined due to (2.1) as

$$||z_i(t)||^2=||C_ix_i(t)+\sum_{j\neq i,j=1}^{N}G_{ij}x_j(t-h_{ij}(t))+F_iu_i(t)+g_i()||^2\leq(N+2)||C_ix_i(t)||^2+(N+2)||F_iu_i(t)||^2+(N+2)||g_i()||^2$$

$$+\sum_{j\neq i,j=1}^{N}(N+2)||G_{ij}x_j(t-h_{ij}(t))||^2 \le (N+2)||C_ix_i(t)||^2+(N+2)||F_iu_i(t)||^2+\sum_{j\neq i,j=1}^{N}(N+2)||G_{ij}x_j(t-h_{ij}(t))||^2$$
$$+(N+2)\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ij}\|x_j^{h_{ij}}(t)\|^2\right]$$

Then, from the expressions

$$\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N}||G_{ij}x_j(t-h_{ij}(t))||^2=\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2$$
$$\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N}g_{ij}\|x_j^{h_{ij}}(t)\|^2=\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2$$

and the assumption

$$F_i^TF_i=I,\quad F_i^T[C_i,G_{ij}]=0,\quad \forall j,i=\overline{1,N},\ j\neq i,$$

we have

$$\sum_{i=1}^{N}||z_i(t)||^2\le(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||F_iu_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ij}x_j(t-h_{ij}(t))||^2\right]$$
$$+(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ij}\|x_j^{h_{ij}}(t)\|^2\right]$$
$$\le(N+2)\sum_{i=1}^{N}\left[||C_ix_i(t)||^2+||u_i(t)||^2+\sum_{j\neq i,j=1}^{N}||G_{ji}x_i(t-h_{ji}(t))||^2\right]$$
$$+(N+2)\sum_{i=1}^{N}\left[c_i\|x_i(t)\|^2+e_i\|u_i(t)\|^2+\sum_{j\neq i,j=1}^{N}g_{ji}\|x_i^{h_{ji}}(t)\|^2\right] \tag{3.12}$$

Submitting the estimation of $\dot{V}(.)$ and $||z_i(t)||^2$ defined by (3.12) and (3.11), respectively into (3.10), we obtain

$$\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2\right]dt\le\int_0^s\sum_{i=1}^{N}\left[-2\beta\sum_{i=1}^{N}y_i^T(t)P_iy_i(t)\right]dt+V(0,x_0),\quad\forall s\ge0. \tag{3.13}$$

Hence from (3.13) it follows that

$$\int_0^s\sum_{i=1}^{N}\left[||z_i(t)||^2-\gamma_i||\omega_i(t)||^2\right]dt\le V(0,x_0)\le\sum_{i=1}^{N}\alpha_{i2}||\varphi_i||^2,\quad\forall s\ge0,$$

equivalently, $\forall s\ge0$,

$$\int_0^s\sum_{i=1}^{N}||z_i(t)||^2dt\le\int_0^s\sum_{i=1}^{N}\gamma_i||\omega_i(t)||^2dt+\sum_{i=1}^{N}\alpha_{i2}||\varphi_i||^2\le\gamma\int_0^s\sum_{i=1}^{N}||\omega_i(t)||^2dt+\alpha_2\sum_{i=1}^{N}||\varphi_i||^2.$$

Letting $s\to+\infty$, and setting $c_0=\frac{\alpha_2}{\gamma}>0$, we obtain

$$\int_0^\infty\sum_{i=1}^{N}||z_i(t)||^2dt\le\gamma\int_0^\infty\sum_{i=1}^{N}||\omega_i(t)||^2dt+\alpha_2\sum_{i=1}^{N}||\varphi_i||^2,$$

implies

$$\frac{\int_0^\infty||z(t)||^2dt}{c_0||\varphi||^2+\int_0^\infty||\omega(t)||^2dt}\le\gamma.$$

This completes the proof of the theorem. □

In the sequel, we give an application to H_∞ control of uncertain linear systems with interval time-varying delay. Consider the following uncertain linear systems with time-varying delay:

$$\begin{cases} \dot{x}_i(t) = [A_i + \Delta A_i]x_i(t) + [B_i + \Delta B_i]u_i(t) + [D_i + \Delta D_i]\omega_i(t) + \sum_{j \neq i, j=1}^{N} [A_{ij} + \Delta A_{ij}]x_j(t - h_{ij}(t)), \\ z_i(t) = [C_i + \Delta C_i]x_i(t) + [F_i + \Delta F_i]u_i(t) + \sum_{j \neq i, j=1}^{N} [G_{ij} + \Delta G_{ij}]x_j(t - h_{ij}(t)), \\ x_i(t) = \varphi_i(t), \quad \forall t \in [-h, 0], \end{cases} \tag{3.14}$$

where the time-varying uncertainties ΔA_i, ΔB_i, ΔD_i, ΔA_{ij}, ΔC_i, ΔF_i, ΔG_{ij}, satisfy

$$\Delta M = K_M^i H_M^i(t) L_M^i,$$

and K_M^i, L_M^i, $M \in \{A_i, B_i, D_i, A_{ij}, C_i, F_i, G_{ij}\}$ are known real constant matrices of appropriate dimensions, and $H_M^i(t)$ is an unknown matrix uncertainty satisfying

$$H_M^i(t)^T H_M^i(t) \le I, \quad \forall t \ge 0.$$

To apply Theorem 3.1, let us denote

$$f_i(.) = \Delta A_i x_i(t) + \Delta B_i u_i(t) + \Delta D_i \omega_i(t) + \sum_{j \neq i, j=1}^{N} \Delta A_{ij} x_j^{h_{ij}}, \, g_i(.) = \Delta C_i x_i(t) + \Delta F_i u_i(t) + \sum_{j \neq i, j=1}^{N} \Delta G_{ij} x_j^{h_{ij}}, \quad \lambda(M) = \lambda_{max}(M^T M).$$

Observe that

$$||\Delta M x||^2 = x^T L_M^{iT} H_M^{iT}(t) K_M^{iT} K_M^i H_M^i(t) L_M^{iT} x \le \lambda(K_M^i) x^T L_M^{iT} H_M^{iT}(t) H_M^i(t) L_M^{iT} x \le \lambda(K_M^i)\lambda(L_M^i) x^T x$$

and using the boundedness condition of the functions f_i, g_i, we have

$$|g_i(.)||^2 \le (N+1)||\Delta C_i x_i(t)||^2 + (N+1)||\Delta F_i u_i(t)||^2 + \sum_{j \neq i, j=1}^{N} (N+1)||\Delta G_{ij} x_j^{h_{ij}}||^2 \le (N+1)\lambda(K_{C_i}^i)\lambda(L_{C_i}^i)||x_i(t)||^2$$

$$+(N+1)\lambda(K_{F_i}^i)\lambda(L_{F_i}^i)||u_i(t)||^2 + \sum_{j \neq i, j=1}^{N} (N+1)\lambda(K_{G_{ij}}^i)\lambda(L_{G_{ij}}^i)||x_j^{h_{ij}}(t)||^2$$

$$||f_i(.)|| \le ||\Delta A_i x_i(t)|| + ||\Delta B_i u_i(t)|| + ||\Delta D_i \omega_i(t)|| + \sum_{j \neq i, j=1}^{N} ||\Delta A_{ij} x_j^{h_{ij}}|| \le \sqrt{\lambda(K_{A_i}^i)\lambda(L_{A_i}^i)}||x_i(t)|| + \sqrt{\lambda(K_{B_i}^i)\lambda(L_{B_i}^i)}||u_i(t)||$$

$$+\sqrt{\lambda(K_{D_i}^i)\lambda(L_{D_i}^i)}||\omega_i(t)|| + \sum_{j \neq i, j=1}^{N} \sqrt{\lambda(K_{A_{ij}}^i)\lambda(L_{A_{ij}}^i)}||x_j^{h_{ij}}(t)||$$

Theorem 3.1 is applied for the values of $\{a_i, b_i, c_i, d_i, e_i, a_{ij}, g_{ij}\}$ defined as

$$\begin{aligned} & a_i \ge \sqrt{\lambda(K_{A_i}^i)\lambda(L_{A_i}^i)}, \quad b_i \ge \sqrt{\lambda(K_{B_i}^i)\lambda(L_{B_i}^i)}, \quad d_i \ge \sqrt{\lambda(K_{D_i}^i)\lambda(L_{D_i}^i)}, \\ & a_{ij} \ge \sqrt{\lambda(K_{A_{ij}}^i)\lambda(L_{A_{ij}}^i)}, \quad c_i \ge (N+1)\lambda(K_{C_i}^i)\lambda(L_{C_i}^i), \quad e_i \ge (N+1)\lambda(K_{F_i}^i)\lambda(L_{F_i}^i), \\ & g_{ij} \ge (N+1)\lambda(K_{G_{ij}}^i)\lambda(L_{G_{ij}}^i). \end{aligned}$$

With the same notation stated in Theorem 3.1, we have

Corollary 3.1. *The H_∞ control of system (3.14) has a solution if there exist symmetric positive definite matrices P_i, Q_i, R_i, U_i, and matrices S_i, Y_i such that the following LMIs hold*

$$\begin{bmatrix} H_{11}^i & H_{12}^i & \dots & H_{1(3N+5)}^i & 0 & 0 \\ * & H_{22}^i & \dots & H_{2(3N+5)}^i & 0 & 0 \\ . & . & \dots & . & . & . \\ * & * & \dots & H_{(3N+5)(3N+5)}^i & 0 & 0 \\ * & * & \dots & * & -U_i & -S_i \\ * & * & \dots & * & * & -U_i \end{bmatrix} < 0, \quad i = 1, 2, \dots, N, \tag{3.15}$$

Moreover, stabilizing feedback controls are defined as

$$u_i(t) = Y_i P_{i1} x_i(t), \quad t \ge 0,$$

and the zero solution of the closed-loop system is β-stable, i.e. the solution satisfies

$$||x(t)|| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-\beta t} ||\varphi||, \quad \forall t \geq 0.$$

Remark 3.1. Theorem 3.1 provides sufficient conditions for the closed-loop system to be exponential stable with a prescribed decay rate β, while the existing method can provides asymptotic stability of the closed-loop system. Moreover, in these papers the time delays are assumed to be differentiable and its derivative is bounded. In Theorem 3.1 this assumption is removed and LMI conditions (3.1) is less conservative, since they do not contain less free weighting matrix unknowns and then reduce computational complexity.

4. Illustrative example

In this section, we give a numerical example to show the validity of the H_∞ controller designed in previous section. This example is a large-scale model composed of two machine subsystems [31] as follows:

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t) + D_1 \omega_2(t) + A_{12} x_1(t - h_{12}(t)) + f_1(.), \\ z_1(t) = C_1 x_1(t) + F_1 u_1(t) + G_{12} x_1(t - h_{12}(t)) + g_1(.), \\ x_1(t) = \varphi_1(t), \quad \forall t \in [-2.1, 0], \\ \dot{x}_2(t) = A_2 x_1(t) + B_2 u_2(t) + D_2 \omega_2(t) + A_{21} x_2(t - h_{21}(t)) + f_2(.) \\ z_2(t) = C_2 x_2(t) + F_2 u_2(t) + G_{21} x_2(t - h_{21}(t)) + g_2(.) \\ x_2(t) = \varphi_2(t), \quad \forall t \in [-2.1, 0], \end{cases} \tag{4.1}$$

where the absolute rotor angle and angular velocity of the machine in each subsystem are denoted by $x_1 = (x_{11}, x_{12})$, and $x_2 = (x_{21}, x_{22})$, respectively; the ith system coefficient A_i; the control and uncertain coefficients B_i and D_i; the ith system perturbations $f_i(.)\,;g_i(.)$ and the modulus of the transfer admittance A_{ij}; output observation z_i; the initial input φ_i; the time-varying delays $h_{ij}(t)$ between the two machine in the subsystem:

$$h_{12} = \begin{cases} 1 + \sin^2(t), & t \in H, \\ 1, & t \notin H, \end{cases} \quad h_{21} = \begin{cases} 1.5 + 0.6\sin^2(t), & t \in H, \\ 1.5, & t \notin H, \end{cases}$$
$$H = \cup_{k \in N} (2k\pi, (2k+1)\pi),$$
$$A_1 = \begin{bmatrix} -1 & 0.5 \\ 1 & -1.5 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.01 & 0.02 \\ 0.025 & -0.04 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -0.02 & 0.01 \\ 0.02 & -0.03 \end{bmatrix},$$
$$C_1 = G_{21} = C_2 = G_{12} = \begin{bmatrix} 0.06 & -0.06 \\ -0.08 & 0.08 \end{bmatrix},$$
$$F_1 = F_2 = \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -0.03 & 0.03 \\ 0.01 & -0.05 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.03 & 0.01 \\ 0.02 & -0.01 \end{bmatrix},$$
$$f_1(.) = 0.01 \begin{bmatrix} \sqrt{x_{11}(t)^2 + x_{21}(t - h_{12}(t))^2} \\ \sqrt{x_{12}(t)^2 + x_{22}(t - h_{12}(t))^2} \end{bmatrix}, \quad g_1(.) = 0.1 \begin{bmatrix} \sqrt{x_{11}(t)^2 + x_{21}(t - h_{12}(t))^2} \\ \sqrt{x_{12}(t)^2 + x_{22}(t - h_{12}(t))^2} \end{bmatrix}$$
$$f_2(.) = 0.01 \begin{bmatrix} \sqrt{x_{21}(t)^2 + x_{11}(t - h_{21}(t))^2} \\ \sqrt{x_{22}(t)^2 + x_{12}(t - h_{21}(t))^2} \end{bmatrix}, \quad g_2(.) = 0.1 \begin{bmatrix} \sqrt{x_{21}(t)^2 + x_{11}(t - h_{21}(t))^2} \\ \sqrt{x_{22}(t)^2 + x_{12}(t - h_{21}(t))^2} \end{bmatrix}$$
$$a_1 = b_1 = c_1 = d_1 = e_1 = 0.01, \quad a_2 = b_2 = c_2 = d_2 = e_2 = 0.01,$$
$$a_{12} = a_{21} = g_{12} = g_{21} = 0.01.$$

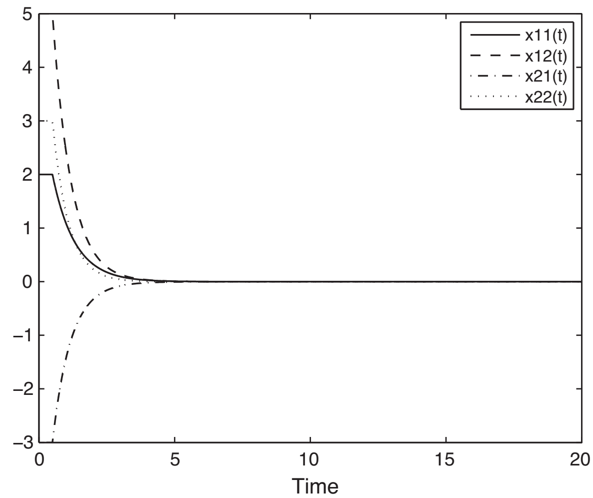

Fig. 1. Response solution of the system (4.1).

It is worth nothing that, the delay functions $h_{12}(t)$, $h_{21}(t)$ are non differentiable, therefore, the controller designed in [10,17–22,24,25] are not applicable to this system. By using LMI Toolbox in MATLAB [32], the LMI (3.1) is feasible with $h_1 = 1$, $h_2 = 2.1$, $\beta = 0.1$, $\gamma_1 = \gamma_2 = 4$, and

$$P_1 = \begin{bmatrix} 0.1072 & 0.0269 \\ 0.0269 & 0.1500 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.0258 & 0.0131 \\ 0.0131 & 0.0587 \end{bmatrix}, \quad R_1 = 10^{-3}\begin{bmatrix} 0.8407 & 0.7267 \\ 0.7267 & 0.6756 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.0341 & 0.0213 \\ 0.0213 & 0.0476 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.1031 & 0.0200 \\ 0.0200 & 0.1497 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.0243 & 0.0070 \\ 0.0070 & 0.0546 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.0007 & 0.0079 \\ 0.0009 & 0.0013 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.0297 & 0.0212 \\ 0.0212 & 0.0551 \end{bmatrix}.$$

$$\Lambda_1 = \begin{bmatrix} 0.0018 & 0.0016 \\ 0.0016 & 0.0015 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.0015 & 0.0019 \\ 0.0019 & 0.0029 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.0240 & -0.0282 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 0.0110 & -0.0292 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} -0.0284 & -0.0166 \\ -0.0161 & -0.0430 \end{bmatrix}, \quad S_2 = \begin{bmatrix} -0.0247 & -0.0149 \\ -0.0145 & -0.0459 \end{bmatrix},$$

The feedback control can be obtained as

$$u_1(t) = Y_1 P_{11} x_1(t) = [-0.1853 \quad -0.1545] x_1(t),$$

$$u_2(t) = Y_2 P_{21} x_2(t) = [0.1488 \quad -0.2152] x_2(t).$$

Moreover, the solution $x(t, \varphi)$ of the system satisfies

$$||x(t, \varphi)|| \le 89.1266 e^{-0.1t} ||\varphi||.$$

Fig. 1 shows the trajectories of $x_1(t)$ and $x_2(t)$ of the closed loop system with the initial conditions $\varphi_1(t) = [2\ 5]^T$, $\varphi_2(t) = [-3\ 3]^T$

5. Conclusion

In this paper, the problem of decentralized H_∞ control for large-scale nonlinear systems with interval time-varying delays in state and observation has been studied. By introducing a set of improved Lyapunov–Krasovskii functionals and using new bounding estimation technique, delay-dependent conditions for the H_∞ control and exponential stability have been established in terms of linear matrix inequalities. An application to decentralized H_∞ control of uncertain linear systems with interval time-varying delay has been given. Numerical examples are given to show the effectiveness of the obtained results.

Acknowledgement

This work was supported by the National Foundation for Science and Technology Development, Vietnam under grant 101.01-2011.51.

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