

Impact of pre-existent Qanats on ground settlements due to mechanized tunneling



Mohammad-Reza Baghban Golpasand^a, Ngoc Anh Do^b, Daniel Dias^{c,d,*}

^a Department of Civil Engineering, Seraj Institute of Higher Education, Tabriz, Iran

^b Hanoi University of Mining and Geology, Faculty of Civil Engineering, Department of Underground and Mining Construction, Hanoi, Viet Nam

^c School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei, China

^d Antea Group, Antony, France

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ABSTRACT

Qanats have been used for a long time to extract groundwater in most historic cities of Iran. With the growth of cities and the development of new methods of irrigation and drainage, most of the qanats have been unused and abandoned. These abandoned and obsolete Qanats are sometimes used for drainage and disposal of urban sewage. Over time, due to environmental and geological factors, parts of these obsolete Qanats collapse and they become larger sized cavities which often cause problems for other underground structures and infrastructure and urban facilities. In the present study, the ground settlement due to the excavation of East-West lot of Line 7 of the Tehran Metro (EWL7) tunnel and its interaction with the pre-existing Qanats is studied. The monitoring of settlements, as well as the sinkhole in Molavi Street (South of Tehran), indicate that abnormal movements were caused by pre-existent qanat chains, minor branches, and conduits in this area. To investigate more precisely this problem, numerical models using the FLAC^{3D} finite difference software were used. The results show that in the case where the qanat is not turned into a cavity due to environmental factors, it would not cause a significant effect on the ground settlements induced by tunnel excavation. However, in the case where the qanat has turned to a bigger cavity due to environmental factors, its presence in the tunnel route can cause a settlement increase. This process can even lead to a ground surface collapse during the tunnel excavation.

Introduction

Qanats as an ancient approach to extract groundwater were mainly developed in major cities of Iran especially in arid and semi-arid regions. A qanat (Kanat) consists of a gently sloping tunnel, cut through alluvial material, which leads water by gravity flow from beneath the water table at its upper end to a ground surface outlet and irrigation canal at its lower end (Fig. 1). Qanats have been constructed manually by skilled workers. The initial stage of construction consists of the sinking of a shaft (well) to prove the presence and depth of the groundwater table [2]. Qanats are connected to the ground surface by a series of wells. The longer one is called mother well, as seen on Fig. 1. This system causes water to appear on the ground surface by gravity. Many studies have been carried out about the qanats and problems which can occur in ancient and historic cities. One of the initial studies on the qanats was done by Neol [31] which include the description of the subject and of its efficiency. English [14] reviewed many documents

about ancient qanats around the world, especially from Iran, focusing on the construction procedure and technical characteristics of them.

Fookes and Knill [16] with performing of the engineering geological studies on the Tehran-Varamin plain and its materials, pointed out that there are more than 200 qanats in this plain and many of them were in a state of disrepair. A comprehensive study on the Iranian qanats was done by Beaumont [2] who, dealing with the geographical distribution of qanats, analyzed their geometrical and hydrological properties. English [15] described the general characteristics of the qanats in the region of Middle-East, especially Iran, and presented the construction procedure and the geometrical dimension of the qanats. Yu [46] studied on the cavity expansion theory and dissolved the problem in terms of elastic, elastic-perfectly plastic, critical state and time-dependent states using finite element methods. Then, the application of this theory in geotechnical problems was evaluated. Atapour and Aftabi [1] investigated on the ground subsidence due to karstification in Kerman city and the effects of pre-existing qanats on this phenomenon.

* Corresponding author.

E-mail address: daniel.dias@anteagroup.com (D. Dias).

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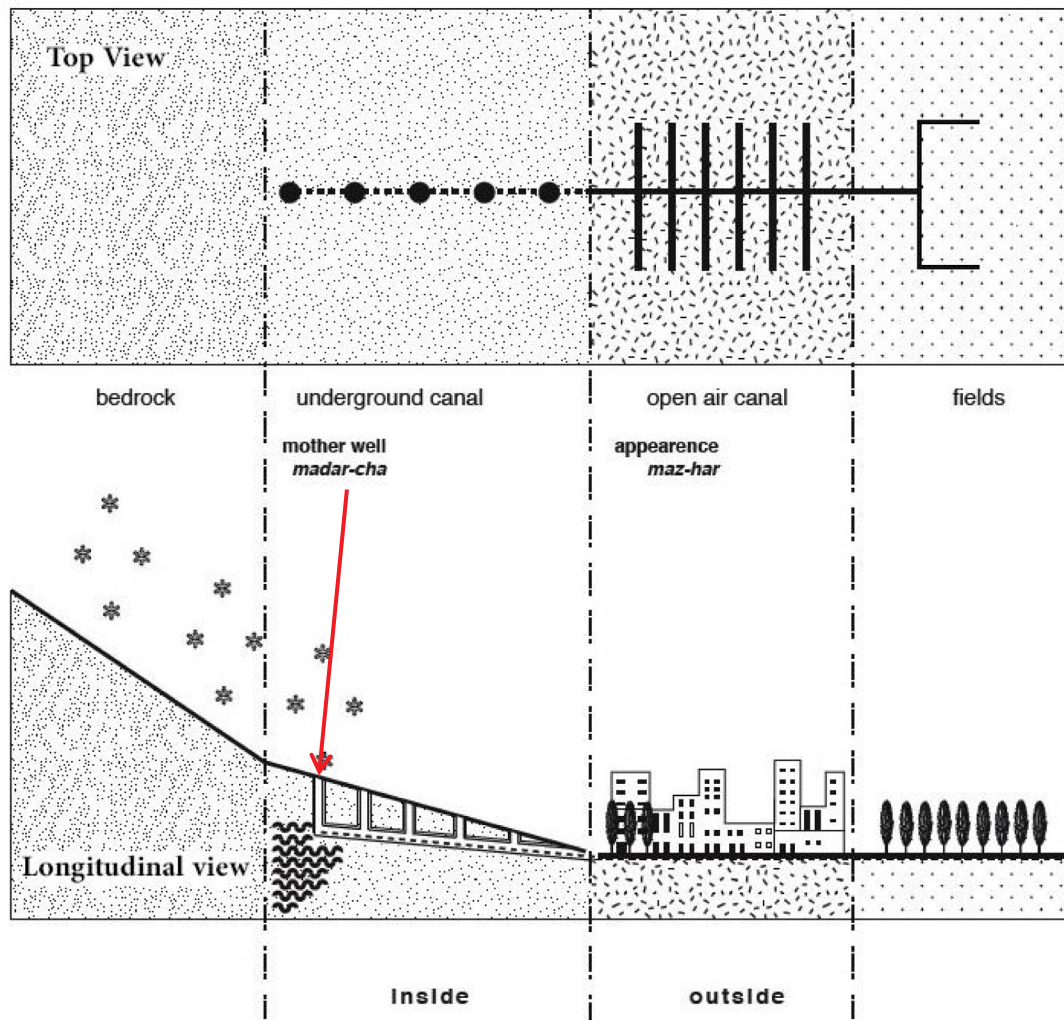


Fig. 1. Top and longitudinal views of a Qanat system.

Lightfoot [29] established a relationship between ancient ground subsidence and the location of qanats in Syria region. Pellet et al. [33] studied on the damages of the qanats occurred during the Bam earthquake in Iran, December 26, 2003. Stiros [40] presented the significant influence of the qanats on the human civilization in arid climates. Rayhani and El Naggar [35] studied collapses induced by qanats using numerical methods. They concluded that low strength soils and qanat walls erosion due to water flow are the most important factors that could increase the qanat wall displacements and cause its collapse. Hajian et al. [25] studied the detection of subsurface qanats using the artificial neural network and pointed out that the collapse of abandoned qanat in urban areas is a considerable hazard for structures and pipelines. Rezaei and Dadsetan [36] emphasized on the effect of the ancient qanats on settlements in the city of Eshtehard. Geranmayeh et al. [19] discussed on the underground obstacles of EWL7 tunnel such as cavities, wells, qanats, loose foundations, and historical buildings and the challenges induced from these obstacles for mechanized tunneling and proposed some technics for ground improvement. Bianchi Fasani et al. [5] outlined an experimental procedure that combines geophysical information with borehole data to assess the underground cavities which have been created by two different resources: (A) Naturally formed cavities mainly develop in geological formations that are prone to dissolution by groundwater and natural aggressive chemicals, (B) Man-made cavities usually result from mining or underground engineering

projects. Golpasand et al. [20] reviewed the effect of engineering geological factors on settlements due to a tunnel excavation (EWL7 tunnel) and emphasized the role of qanats on abnormal settlements along the tunnel route. Hamidian et al. [24] described the qanats and their internal and external structure, geometrical dimensions as well as the geographical distribution of them and pointed out the role of qanats in the life of Iranians. Castellanza et al. [7] proposed a methodological approach for the quantitative assessment of failure susceptibility of underground caves which comprises six steps including In-situ survey, choice of the conceptual model, experimental analysis, theoretical analysis, numerical analysis, and hazard assessment. Wang et al. [42] used a 3D numerical modeling to study the collapse of the roof of JK-A salt cavern, the first operating cavern of the Jintan underground gas storage and proposed a new index system consisting of volume shrinkage, dilatancy safety factor, displacement, vertical stress, and equivalent strain.

It is noticeable that most of the previous studies focused on the introducing of the qanats and their structure as well as their efficiency in hydrological and hydrogeological problems. In other words, there is a lack of study in terms of interaction between qanats and tunnels from the ground settlement point of view. Given that, many urban tunnels are currently being excavated in major cities of Iran such as Tehran, Tabriz, and Mashhad (due to the construction of subway lines), the need for extensive studies in this regard is felt more than ever. Then, the

necessity and the novelty of the present study can be deduced according to these subjects.

In the present study, the interaction between pre-existing obsoleted qanats and the East-West Line 7 excavation of the Tehran Metro (EWL7) tunnel is investigated. The effect of dimensions of qanats and cavities as well as depth is discussed using numerical modeling (FLAC^{3D}). The outcomes of the models are compared with the real settlement, measured during the tunnel excavation.

Problems associated with the Qanats

In the last decades, high demand for fresh water and a considerable advance in drilling technology of deep wells has resulted in lowering the ground water table level, especially in arid and semi-arid environments. This phenomenon has resulted in land subsidence which can damage in structures and buildings as well as transport and transmission networks [17]. Most of the qanats loosed their efficiency and became obsolete. Qanats can be then abandoned or filled with natural materials such as soil and rock. The unfilled obsoleted qanats can cause important settlements during the construction of heavy surface structures or during the excavation of shallow tunnels. These settlements can sometimes lead to collapses at the ground surface that can create disorders in urban areas. Fig. 2 shows some of the collapses occurred at the ground surface caused by qanats in the city of Tehran.

Ancient and obsolete qanat chains are one of the most important engineering geological aspects of Tehran. Many studies have been carried out to determine the length and location of the qanat chains. Some of them have been added to maps which show their approximate length and location. Fig. 3 shows the main detected qanat chains around the route of the EWL7 tunnel between approximate chainages of 0 + 000 to 4 + 500 (4.5 km).

The ancient and obsolete qanat chains have often minor and unknown branches and conduits which can only be detected through a secondary exploratory drilling campaign. An example in Fig. 4 of this kind of minor branches was observed via an exploratory test pit drilling in the studied area.

Site information

The EWL7 tunnel, of approximately 12 km long, was excavated between the stations Navab-Qazvin Bridge and Amir-AL-Momennin town in the South of Tehran. The mechanized tunnel is circular shaped, with an excavation diameter of 9.164 m. In this research, the part of the EWL7 tunnel from chainages 0 + 000 to 4 + 500 has been selected for the analysis. Location of the study area has been shown in Fig. 5.



a)



b)

Fig. 2. Examples of ground subsidence and collapses occurred due to qanats in Tehran.

Geological setting

The city of Tehran has developed on Neogene and Quaternary sediments that have been originated from adjacent hills and mountains. Geological findings confirm that the quaternary and Pliocene alluviums and moraine deposit were developed in the Tehran desert. This Quaternary overburden typically consists of alluvial deposits with the extent range of grain sizes from cohesive fine-grained (clayey and silty clay) soils to coarse-grained alluvial deposits.

Tehran is divided into three structural and stratigraphic zones including the northern mountains, eastern and southern mountains, and the Tehran plain. The Tehran plain mainly consists of quaternary formations, which are often the result of erosion and re-deposition of former sediments. The plain extends to the south and generally consists of unsorted fluvial and river deposits [41].

Rieben [37] divided the Tehran alluvia into four categories based on geological factors and identified them as A, B, C and D formations (Fig. 5). The A formation is the oldest and D formation is the youngest. A simple description of these alluvia is presented in Table 1.

According to Mahmoudpour et al [30] the Tehran plain, as pediment zone, tectonically confined between surrounding blocks and reacted to the surrounding deformations after the Pliocene by the folding and faulting of the surface alluvium and probably down-faulting of the basement rock. The continuity of the directions and sense of tectonic movement that raised the Alborz range above the Central Basin from the late Eocene to the present day makes it difficult to establish a strict chronology for the movement of these blocks.

Geotechnical studies

Adequate geotechnical exploration practice has been conducted in this project. As a result, the geotechnical properties of the soil layer and groundwater level were determined, and the engineering geological profile of the tunnel route was delineated (Fig. 6). Soils along the tunnel route are categorized into four engineering geological types (soil types). According to ITA [27] and DAUB [9], the fine grain content of soil is the main factor for its categorization in mechanized tunneling. A layer of fill materials was detected at the ground surface with various thickness and low geotechnical properties. The engineering geological characteristics and the geotechnical parameters of soils are presented in Tables 2 and 3, respectively.

Ground settlements due to tunneling

Excavation of tunnels and other underground spaces causes movements of the surrounding soil mass and a disturbance of the ground initial stress state. Depending on the tunnel depth and on the excavation process, a part of the displacements induced in the ground

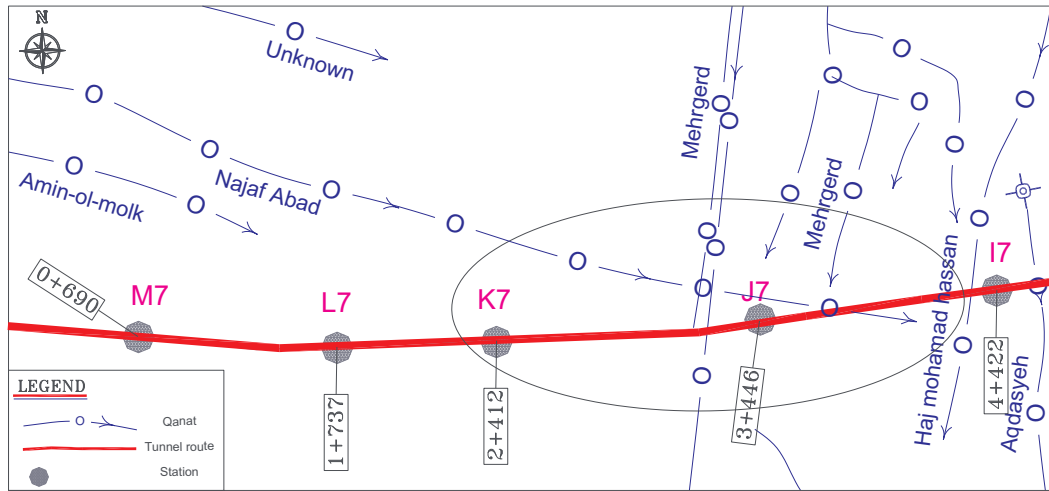


Fig. 3. Approximate location of qanat chains relative to the tunnel [20].

surrounding the tunnel can propagate to the surface which causes settlements. Peck [32] proposed an equation for the settlements evaluation due to tunnelling in the transverse direction:

$$S = S_{max} \exp(-x^2/2i^2) \tag{1}$$

where S is ground surface settlement at a selected x value (mm), S_{max} is the maximum ground surface settlement above the tunnel center line (Tunnel CL), x is the cross-sectional distance from the selected point to the center line of the tunnel (in meters) and i is the transverse distance between the center line of the tunnel and the settlement curve's inflection point (in meters).

Measuring of ground settlements due to the excavation of the EWL7 tunnel

Surface settlements, induced by the EWL7 tunnel excavation, were measured by Leveling and surveying methods. Some control points were selected at the ground surface above the tunnel center line (C.L.) and measuring equipment (pins) were then installed on the selected points. In other words, ground settlements were measured by leveling techniques.

Installation of the measuring equipment and leveling of the points were done using the main recommendations of Dunncliff [13]. Regarding the main purpose of this study, and the practical limitations in urban areas, only the S_{max} , maximum vertical displacement of the ground surface above the tunnel center line (C.L.), was measured.

Measuring the displacements of the benchmarks was started before the passing of the shield machine at that point and continued until the

level of points reached a constant value and so the S_{max} was recorded. Fig. 7 shows the real maximum settlements (S_{max}) due to excavation of the studied tunnel between chainages 0 + 610 and 4 + 275. It is seen that the amounts of measured settlements (S_{max}) vary between 5 mm and more than 155 mm.

Further attention on the Fig. 7 clears that the tunnel route can be divided into two parts based on the S_{max} values:

Part A: From the chainage 0 + 600 to 2 + 300. The values of S_{max} in this part are in normal ranges, mostly between 5 and 20 mm.

Part B: From the chainage 2 + 300 to the end of the studied area. High values of settlements even more than 155 mm are seen in this part of the tunnel.

The settlements measured and recorded in part B are out of the pre-designed range and can cause ground surface collapses. Based on the geological and geometrical similarities along the studies area of the tunnel, it can be simply deduced that the high and abnormal settlements observed in part B were not occurred merely due to the excavation of the EWL7 tunnel. In other words, another factor(s) contributed to the occurrence of abnormal settlements and sinkholes.

Abnormal settlements and collapses

Abnormal settlements occurred and some of them led to sinkholes. Fig. 8 shows the collapse in Molavi Street (South of Tehran) located along the tunnel route. Site observations indicated that the settlements of ground surface increased continually and exceeded allowable values and finally, led to collapse and occurrence of a sinkhole shown in Fig. 8.



Fig. 4. Samples of qanat minor branches in the studied area [6].

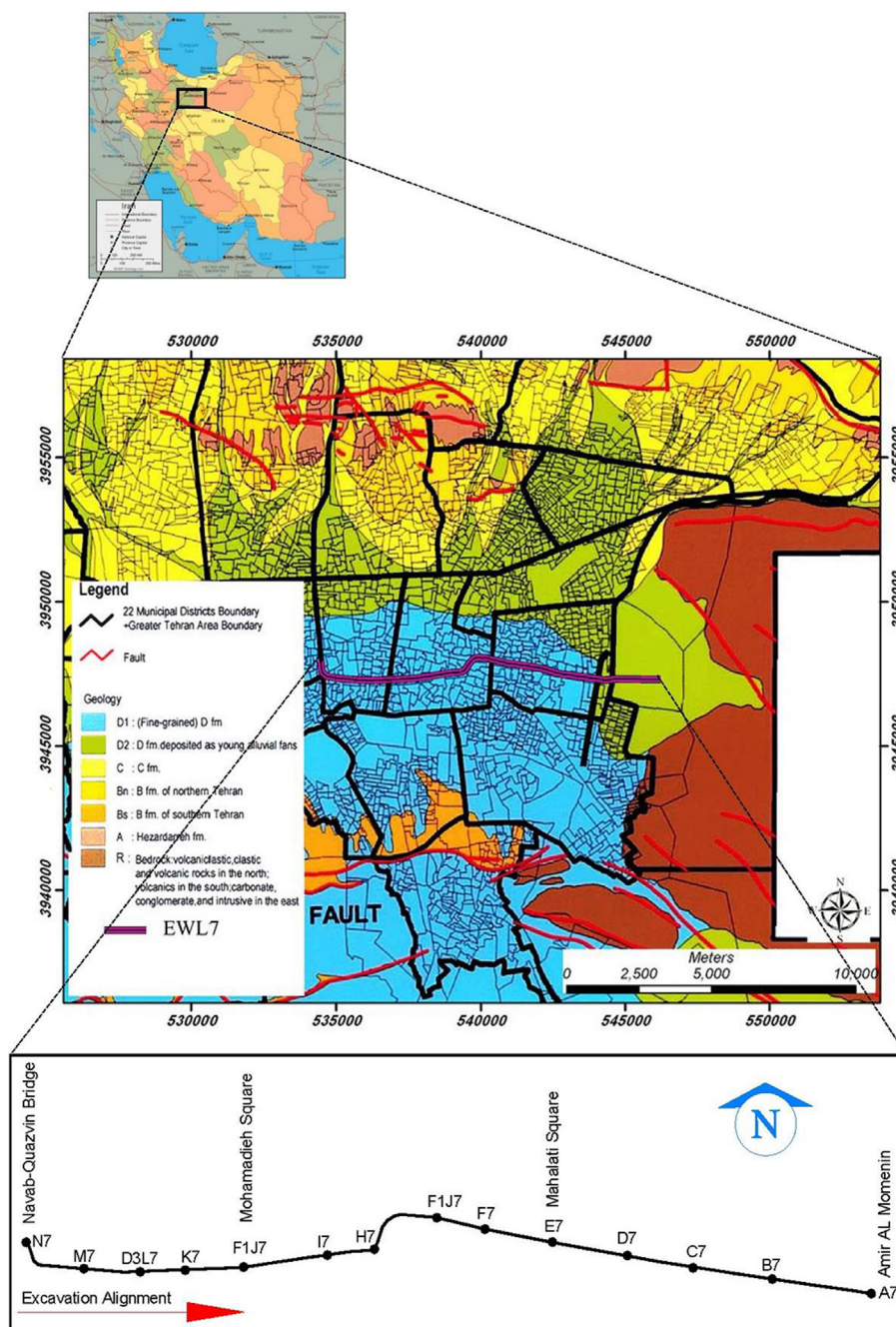


Fig. 5. Location of the study area with the geological situation.

As seen in this figure, because of the presence of a sinkhole which has nearly 7 m in diameter and 10 m in depth, most of the pipelines and infrastructures in this zone were damaged. Fig. 9 shows the studied area plan with the collapse which occurred during the tunnel excavation as well as the position of the pins installed on the ground to measure settlements. Variations of the settlements recorded by the pins are shown in Fig. 10.

In-situ measurements indicated that the vertical displacements are highly dependent on the pins location and distance to the sinkhole. As seen in Fig. 10, the pins of CS.LP.128 (C, L, and R) are clearly affected by the collapse. Pin CS.LP.128 C indicates normal settlements of around 15 mm before the collapse occurrence. Then, abnormal settlements up to 155 mm are seen. Pin CS.LP.128 L indicates abnormal settlements (nearly 200 mm) developing after normal settlements of being lower than 15 mm. by increasing the settlement, this Pin was fallen into the

sinkhole. Pin CS.LP.128 R is located at a distance of 5 m from the sinkhole, so it was not strongly affected by the collapse. The behaviors of the pins are summarized in Table 4. According to Figs. 9 and 10 as well as Table 4, the behavior of the pins is highly affected by the sinkhole. Field observations showed that the qanats are the main reason for this phenomenon. More investigations and literature review confirm that the several chains of ancient and obsolete qanats are existent in the studied region (Fig. 3).

The relationship between qanats and abnormal settlements

The qanat chains map along the tunnel route, developed via historical documents and geophysical studies from the tunnel, were used. Part of this map was shown in Fig. 3. It can be seen that the qanat chains are gathered dominantly after the chainage 3 + 000. It coincides

Table 1
Engineering Geological characteristics of Tehran alluvia after [20].

Characteristic	Alluvium			
	A	B	C	D
Age	5 Ma	700 ka	50 ka	10 ka
Lithology	Homogeneous conglomerate	Heterogeneous conglomerate	Alluvial fan	Recent alluvial
Cementation	Strongly cemented	Variable, usually weakly cemented	Moderately cemented	Non-cemented
Grain size	Clay to 100–250 mm	Very variable up to several meters	Clay to 100–200 mm	Clay up to several meters
Dip of layer (degrees)	0–90	0–15	0	0
N _(SPT)	Most of them exceed 50	Variable, usually exceeds 50	slightly lower than those of the A and B fm., averagely exceeds 50	Variable, generally lower than those of others.
Thickness	Maximum 1200 m	Maximum 60 m (thickness decreases toward south)	Maximum 60 m	< 10 m
Sedimentary environment	Fluvial	Fluvioglacial and periglacial	Fluvial	Fluvial
Another name (local name)	Hezardareh alluvial formation	Kahrizak formation	Tehran alluvial formation	Recent alluvial

with the tunnel route part where abnormal settlements were measured. Consequently, it can be said that the abnormal settlements, as well as the collapse of the ground surface, were created due to pre-existing qanats.

A precise comparison between Figs. 7 and 3 indicates that the qanat chains location doesn't exactly coincide with the location of abnormal settlements. According to Fig. 7, abnormal settlements were seen after chainage 2 + 500, whereas no qanat chain is apparently presented at this location in Fig. 3. An important fact about this issue is that the ancient and obsolete qanat chains have minor and unknown branches and conduits which have probably converted to cavities along the time via geological and environmental factors. These secondary induced cavities can highly influence the qanats technical characteristics and can be the main reason for the collapse. In addition, the walls and roof of the qanats can be subjected to secondary changes because of the groundwater level fluctuation and other natural or artificial factors and can lead to an increase of the qanats dimension.

Evaluation of ground settlements due to the EWL7 tunnel excavation

Several methods have been suggested to evaluate ground settlements due to tunneling such as semi-empirical, analytical and numerical methods. Golpasand [21] studied on the ground settlements due to the EWL7 tunnel excavation using semi-empirical, analytical and numerical (FLAC^{3D}) methods and compared the results with real in-situ settlements. The comparisons indicated that the numerical method, which is used in the following sections, is more accurate and releases reliable results. In this section, ground settlements due to the tunnel

excavation are evaluated using numerical methods and the effect of pre-existing qanats in terms of settlements occurred due to tunnel excavation is studied.

Numerical method

Numerical modeling is one of the useful and current methods used to evaluate the ground settlements induced by shallow tunneling. In this study, the finite difference code FLAC^{3D} [28] was used to build the 3D numerical model and study the effect of pre-existent qanats on ground settlements. To aim this goal, several cases were considered and the modeling was done according to these cases as follows:

Case 1: without qanat

In this case, it is supposed that no qanats are located around the tunnel. The numerical model was constructed according to the procedure defined in Golpasand et al. [22], Golpasand et al. [23], Dias and Kastner [10], and Do et al. [11,12]. Fig. 11 shows the geometrical characteristics of the model.

The constructed model includes 117,600 zones and 124,011 grid-points. Because of symmetry conditions, only a tunnel half is simulated. The model vertical boundaries were fixed in their normal directions. The bottom of the model was fixed in all three directions to prevent any movements. The geometrical characteristics of the model were adopted from a mean situation of the tunnel route from chainage 3 + 000 to 4 + 500.

Excavation and advancing of the Tunnel Boring Machine (TBM) were simulated based on the processes were taken place in the site. The mechanized tunneling is a sequential process; so all of its stages were

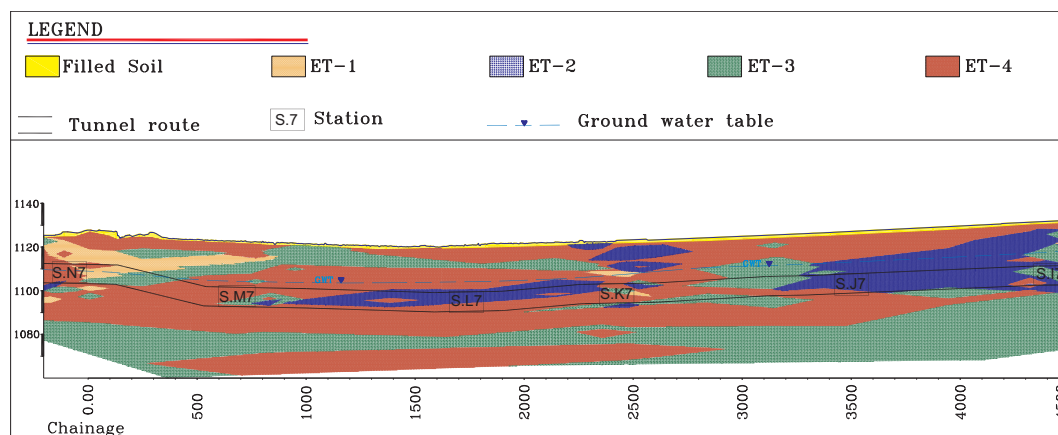


Fig. 6. Geological profile and tunnel location [41].

Table 2
Engineering geological characteristics of categorized soil types [21].

Engineering geological types	Fill	ET-1	ET-2	ET-3	ET-4
Description	Very soft sandy clay	Poorly graded sandy gravels	Gravelly sands with silt/clay	Very silty/clayey sands with gravel,	Clayey silts with sand
Passing 200 Soil type	Various –	Less than 12% GW, GW-GM, GP-GC, SW, SP	12 to 30% SC, SC-SM, GC	30 to 60% SC, SM, CL	More than 60% CL, ML, CL-ML, rarely CH

considered during numerical modeling. The considered stages are respectively:

- Excavation of tunnel equal to the length of each segment (1.5 m).
- Generation of the Earth Pressure Balance (EPB) machine elements for the new tunnel excavation length.
- Application of the face pressure on the new excavated tunnel face.
- Allowing the model for relaxation and convergence of the soil, equal to the gap between the outer diameter of the shield and the inner diameter of the excavated tunnel.
- Application of the grout pressure on the converging section of the tunnel.
- Generation of the segmental lining elements installed after the shield.
- Equilibrium of the model.
- Repeating the above steps.

The geology of the studied area as well as the geotechnical properties of soil layers used for the model construction are presented in Fig. 6 and Table 3, respectively. Technical properties of the shield and segment elements are presented in Table 5. The grouting pressure has been modeled as a linearly varied normal stress applied to the excavation wall for one segmental ring length behind the shield. The face pressure has been modeled as a linearly varied normal stress applied to the tunnel face. Values of grouting and face pressures have been selected according to the site information and shown in Table 6. To simulate the initial stress conditions, vertical stresses considering gravity effect were applied. The coefficient of lateral earth pressure (K_0) was used to define the ratio of the horizontal to the vertical in-situ stress.

The soil behavior was modeled using a linear elastic perfectly plastic constitutive model using the Mohr-Coulomb shear failure criterion. This criterion is widely used in tunnel modeling because of its simplicity and the availability of the necessary geotechnical parameters. To model the weight of over-ground neighboring structures, a uniform pressure of 20 kPa is applied on the upper surface of the model. The numerical results are presented in Figs. 12 and 13 in terms of contours of vertical stresses and contours of vertical displacements, respectively. As shown in Fig. 13, the maximum vertical displacement at the ground surface, above the tunnel center line (S_{max}), is in the range between 15 mm and 20 mm. The trough of ground settlements in a transverse section is shown in Fig. 14. A good agreement between the numerical and monitoring settlements results permits to validate the numerical modeling.

Case 2: with pre-existent qanats

In this case, qanats are considered between the tunnel crown and

Table 3
Geotechnical parameters of soils [21].

Parameters	Effective cohesion	Friction angle	Young Modulus	Poisson's ratio	Dry Density	Saturated Density	Over Consolidation ratio	Coefficient of Earth Pressure at rest	Dilatancy angle
Soil types	C' (kPa)	ϕ' (degrees)	E' (MPa)	ν	γ_d (kN/m ³)	γ_{sat} (kN/m ³)	OCR	K_0	Ψ (degrees)
Fill	6	20	15	0.35	16	19	1	0.65	0
ET-1	14	33	53	0.31	18.5	20.5	2	0.66	3
ET-2	20	31	35	0.33	19	21.5	2	0.70	1
ET-3	25	28	25	0.35	18.5	21	2	0.77	0

ground surface as shown in Figs. 15 and 16. As shown, the qanat is considered perpendicularly to the tunnel route, because this situation of the qanats is the more encountered one during the tunnel excavation. The qanat depth position is an important factor.

In this study, three models were considered with different depths of qanats ($h1$), that $h1 = 17, 10$ and 3 m. Each of the constructed models includes 125,280 zones and 132,009 grid-points.

Execution steps of the model are the following ones:

- Initial stress of state.
- Qanat excavation.
- Installation of the support on the qanata's interior wall.
- Equilibrium of the model.
- Execution of the steps presented in Section 5.1.1.

It is seen that generally two separate phases can be specified for the above stages: (1) Only qanat excavation, (2) Tunnel excavation after the qanat excavation. However, these steps are carried out for all intended cases and the outcomes of one of the models, for example the model of $h1 = 10$ m, are shown in Figs. 17 and 18. The values of the displacements of ground surface and qanat's crown are presented in Table 7. Comparison of the values in this Table shows that the existence of qanats in this study did not have much effect on the settlements. This is probably due to the relatively small diameter of qanats in relation to the tunnel diameter.

Deformability characteristic of the qanat's lining

Practically, the lining should be installed immediately after the qanat excavation in weak soil zones. In areas of weakly consolidated material, baked clay rings known as Kavulls are used to avoid roof and wall collapse.

The vertical shafts are usually strengthened in their upper portion by mud linings or mud brickwork [2]. Fig. 19 shows two cases of the situation of the interior wall and roof of qanats: (a) using Kavulls to protect against collapse, (b) without lining (in soils of better characteristics). The important point in this issue is the fact that the strength and deformability parameters of the qanat's lining are not constant and can change with time due to environmental and geological factors. This can lead to a significant decrease in the value of these parameters. Therefore, it is often observed that due to various environmental factors (such as changes in the groundwater level), the roof of the qanat can fail and the qanat became a bigger cavity. This phenomenon was illustrated in Fig. 4. In this case, due to the former collapse, the qanat has no lining. Salehi et al. [38] modeled the qanats of Mashhad by the finite element software Plaxis 2D using three assumptions:

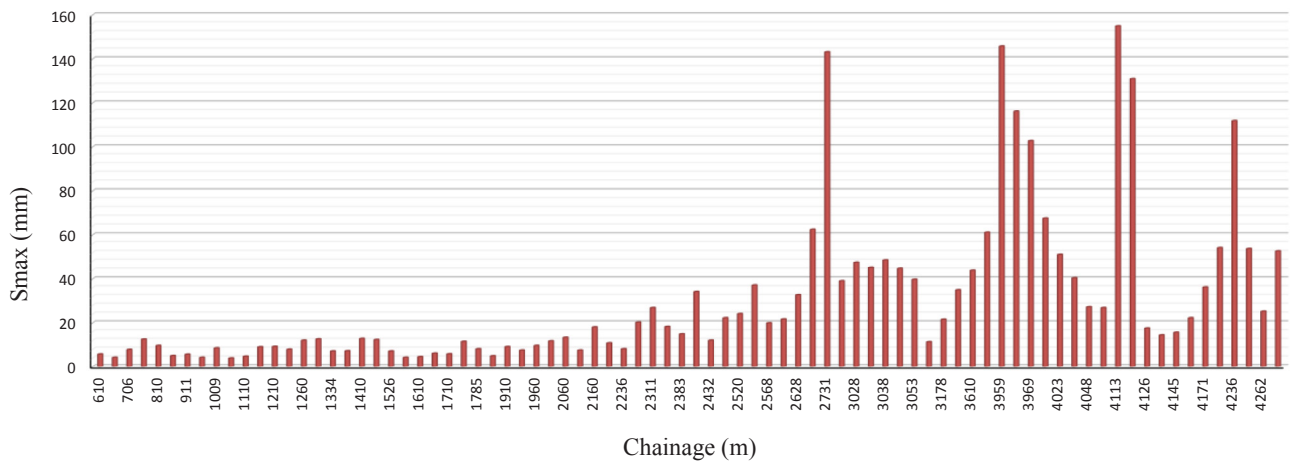


Fig. 7. Bar chart of the measured settlements [21].



Fig. 8. Collapses in Molavi Street, nearly chainage 4 + 100.

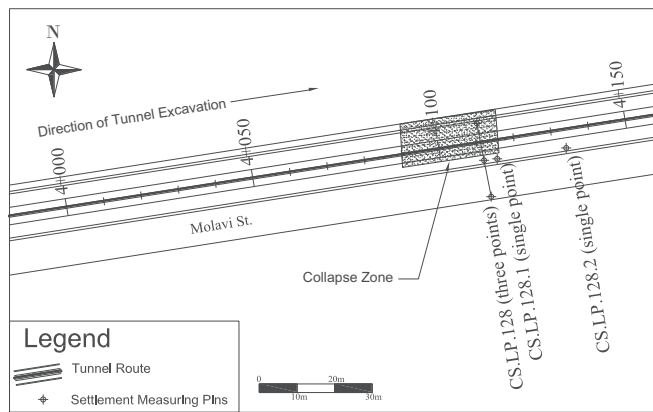


Fig. 9. Studied area with the location of the collapse zone.

- a) Without lining,
- b) Qanats with brick and mortar lining
- c) Qanats with concrete lining.

They used the following parameters in their numerical modeling (Table 8):

Using the above deformability characteristics will result in very small displacements of the qanat’s roof, which is not consistent with field observations. Thus, it should be selected a more rational value for this parameter. Castellanza et al. [7] carried out some experimental tests such as uniaxial compressive tests (UCT) and Brazilian tests (BT) on the rocks selected from the cavity's interior wall in dry, partially saturated and saturated (wet) states. A reduction of both strength and stiffness up to 50% of the dry corresponding values is recorded after a

few minutes of soaking of a dry specimen. According to these results, it can be concluded that the strength parameters and deformability characteristics of the materials decrease considerably over the time from the age of qanat construction to nowadays. Based on field observations (such as those found on the collapse of Molavi Street), and former field studies, the following values were considered and the models were constructed based on these values (Table 9):

Case 3: with pre-existent qanats which failed and converted into cavities

The qanats are not often in a steady state during the time. Generally, secondary changes take place due to geological and environmental factors and cause the qanats to be converted into a cavity with larger dimensions. Some numerical models were built to investigate on this case. Figs. 20 and 21 show the spatial position of a tunnel, qanat, and cavity and the numerical simulation model of this case, respectively. As seen, a square-shaped cavity has been assumed with a diameter of 4 m at a depth of 17 m. The cavities are often shapeless. However, they are modeled in this study with a square shape for the sake of simplification, similar to Helm et al. [26] as well as Fuenkajorn and Archeeploha [18]. The dimension of the cavity (dc = 4 m) has been certainly selected as an average value of the diameters of qanat and tunnel. In fact, when the qanat is collapsed and converted into the cavity, the dimension of the cavity can be different from case to case.

Two different approaches are considered to model the progress of converting qanats to cavities:

- A) The qanat is converted into a cavity before the tunnel excavation. In order to model this approach, the cavity is excavated and equilibrated. Then, the displacements are set equal to zero before the tunnel excavation. In the next stage, the tunnel is excavated and equilibrated after the installation of the segmental lining. It is

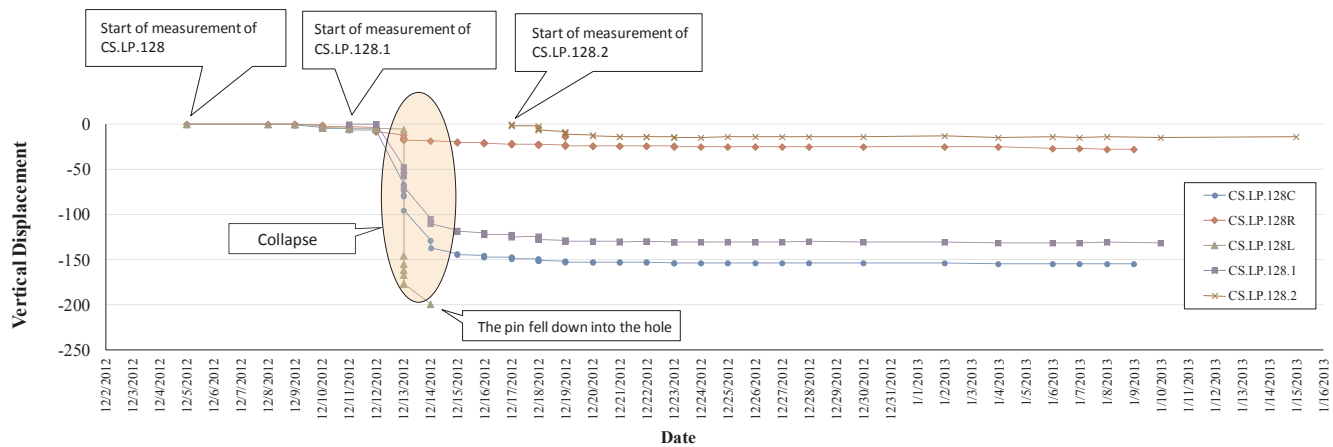


Fig. 10. Settlement values near the collapse of Molavi Street.

Table 4 Behavior of the pins near the collapse of Molavi Street.

Pin	Relative position to the hole	Maximum recorded settlement (mm)	Final situation
CS.LP.128 C	Vicinity of hole	155	Fixed in a constant level
CS.LP.128 L	Center of hole	200	Fallen into the hole
CS.LP.128 R	5 m distance	28.6	Fixed in a constant level
CS.LP.128.1	Vicinity of hole	131	Fixed in a constant level
CS.LP.128.2	15 m distance	14.5	Fixed in a constant level

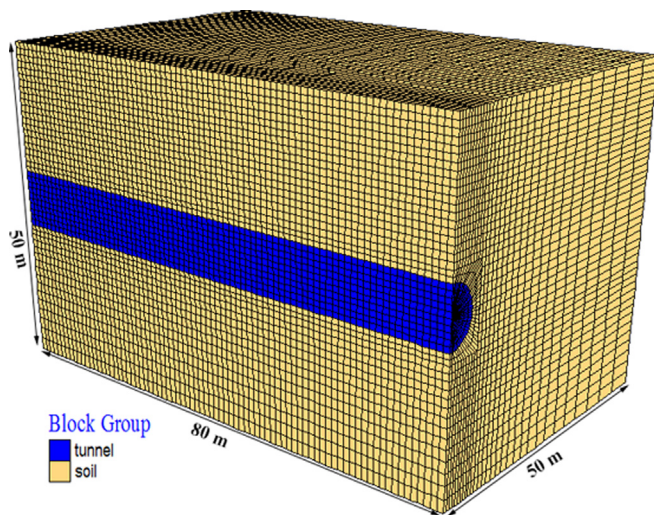


Fig. 11. Dimensions of the numerical model.

Table 5 Properties of the shield and segments.

Element	Thickness (m)	Elasticity modulus (GPa)	Poisson's ratio	Density (kg/m ³)
Segment	0.35	30	0.25	2500
Shield	0.35	200	0.20	7800

Table 6 Values of grouting and face pressures applied to the numerical model.

Face pressure (kPa)	Grouting pressure (kPa)
150	170

observed that the final displacement values at the cavity roof and the ground surface are 27 mm and 21 mm, respectively.

B) The qanat is simultaneously converted into a cavity with the tunnel excavation. In this approach, the displacements are not set equal to zero before the tunnel excavation. The final displacements due to the tunnel excavation, at the roof of the qanat and at the ground surface are respectively equal to 850 mm and 30 mm. Table 10 summarizes the results obtained from the numerical models. The outcomes of the model are shown in Figs. 22, 23 and 24 similar to previous cases. It should be noted that the displacement of 850 mm at the roof of the qanat is not a real value. It means that the qanat roof is subjected to large displacements without collapse occurrence. Both of the above approaches can happen in the qanat tunnel interaction cases, but it seems that the approach B is closer to the case of Molavi Street. This case will be modeled by the next section.

The effect of the cavity depth

To investigate the effect of the cavity depth, three models were considered with different depths of cavities ($h1$) of 17, 8.5 and 3 m. Each of the numerical models includes 104,642 zones and 110,896 grid-points. The results of the investigation are summarized in Table 11. According to the obtained results, the vertical displacement of the ground surface, as well as, vertical displacement of cavity roof decreases when decreasing the cavity depth.

In the case of $h1 = 3$ m, the equilibrium state was not achieved in the model that means the cavity was collapsed. It seems that the excavation of tunnel after cavity creation is the main factor in the continuance of vertical displacement of the ground surface and cavity roof. Then, the value of vertical displacement of the ground surface and cavity roof in the case of $h1 = 17$ m is higher because of the lower distance of the cavity from the tunnel.

Numerical modeling of the Molavi Street case

The collapse of Molavi Street occurred during the excavation of the EWL7 tunnel. Based on the site observations, a cavity with a diameter of about 6 to 7 m and a depth of about 10 m was created on the ground due to collapse (Fig. 8). For simulation purpose, a square-shaped cavity

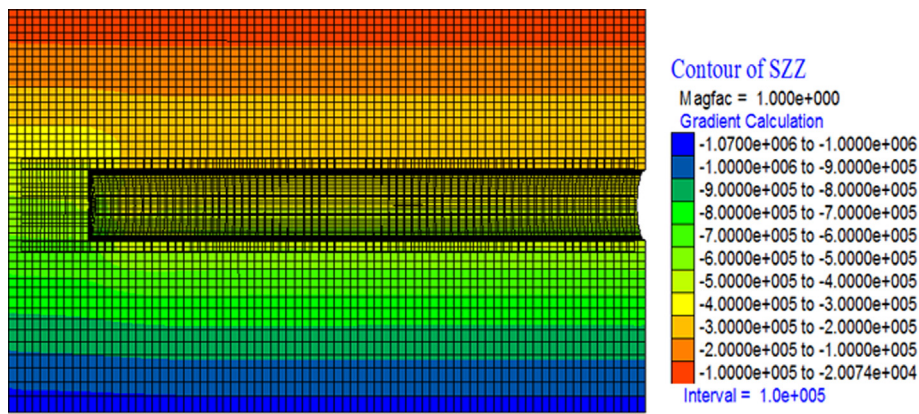


Fig. 12. Contours of vertical stresses at the end of tunnel excavation (case 1), unity Pascals.

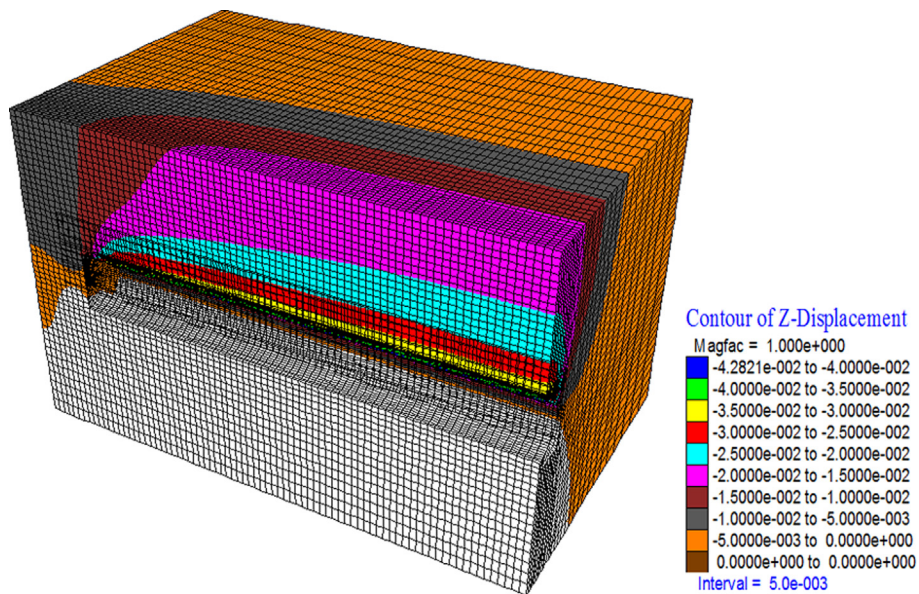


Fig. 13. Contours of vertical displacements at the end of the tunnel excavation (case 1), unity: meters.

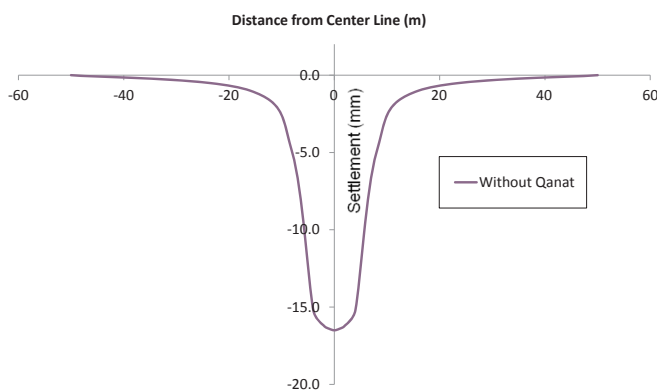


Fig. 14. Trough of ground settlements in a transversal section.

with a length of 6 m at the end of the qanat was modeled. The geometry of the model is shown in Fig. 25. Similar to the previous ones, the cavity was excavated in the first stage and the model was executed. It is found that the model would not reach to equilibrium state with this condition, which confirms the collapse occurrence of the mentioned cavity. It is noticeable that no plastic zones were appeared around the Molavi Street cavity.

Modeling of failure

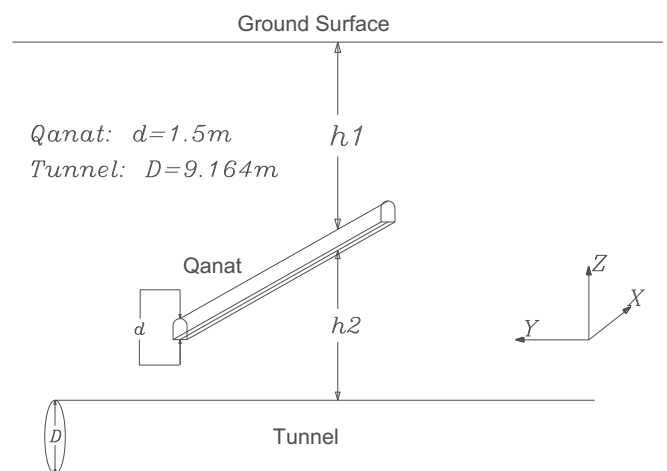


Fig. 15. Spatial position of the qanat.

The modeling of failure and collapse of cavities is an important aspect in numerical studies for the interaction of tunnels, qanats and cavities. If the host materials consist of rock masses, the presence of the discontinuities and the geometrical and geological characteristics of them play a main role in the failure process. Generally in this case, the

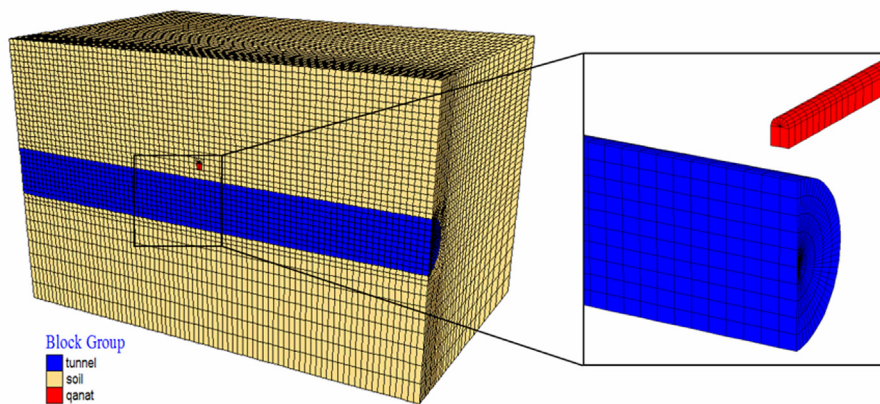


Fig. 16. Comparison between diameters of the qanat and tunnel.

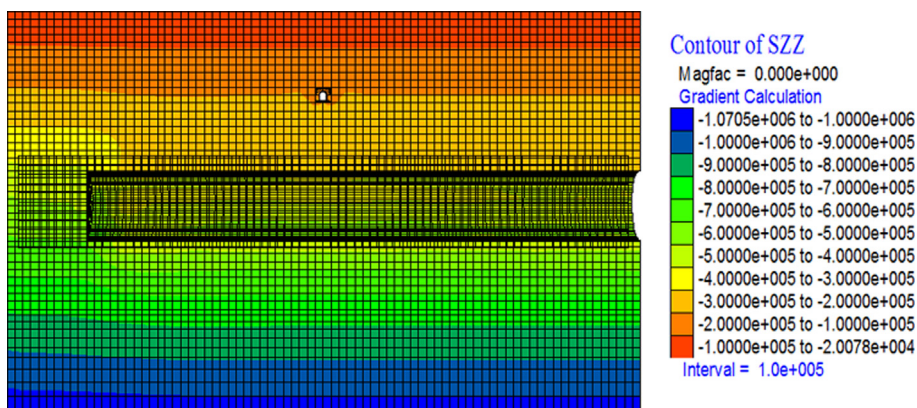


Fig. 17. Contours of vertical stresses at the end of the tunnel excavation (case 2).

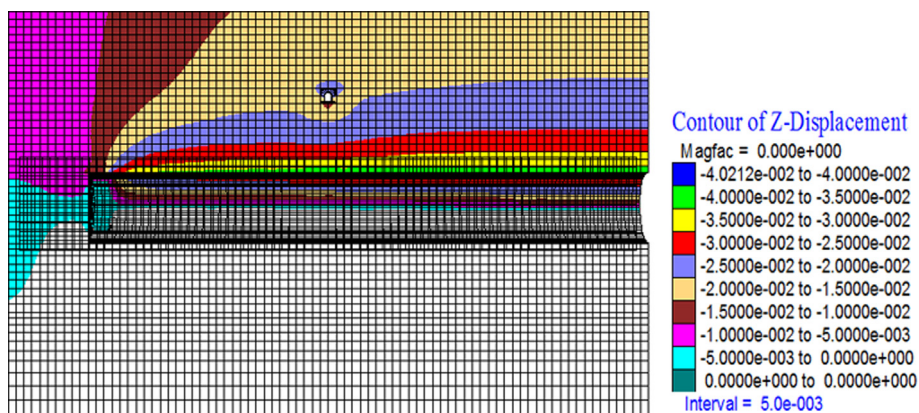


Fig. 18. Contours of vertical displacements at the end of the tunnel excavation (case 2), unity: meters.

Table 7

Vertical displacements for several values of $h1$.

$h1$ (m)	Only qanat excavation		Tunnel excavation after the qanat excavation	
	Vertical displacement of qanat's crown (mm)	S_{max} of ground surface (mm)	Vertical displacement of qanat's crown (mm)	S_{max} of ground surface (mm)
17	15.7	1.8	36	17
10	4	0.9	29	17
3	1.2	0.5	17	16

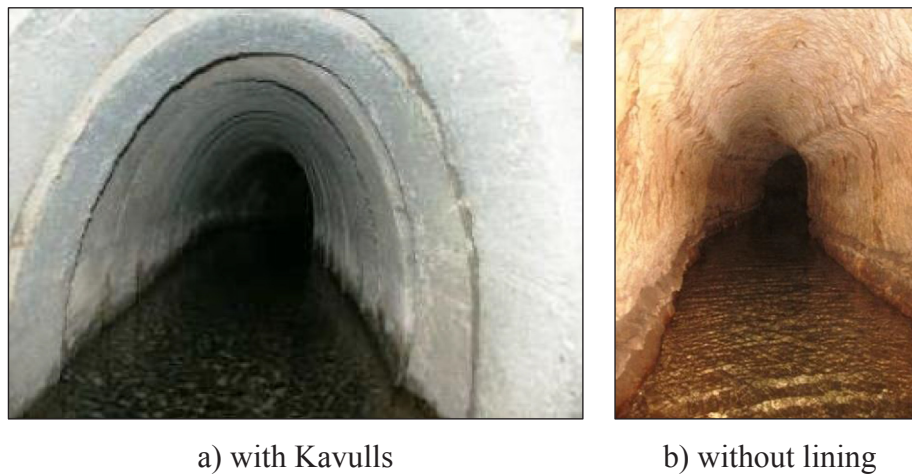


Fig. 19. The interior cover of qanat.

Table 8
Deformability characteristics of qanat's lining [38].

Cover Type	E (GPa)	Thickness (m)	Poisson's ratio
Brick and Mortar	4.5	0.05	0.22
Concrete	21	0.05	0.15

Table 9
Deformability characteristics of quanta's interior cover used in the present study.

Cover Type	E (MPa)	Poisson's ratio	Thickness (m)	Density (kg/m ³)
Weakened Kavullis	150	0.25	0.1	2500

only permits to evaluate the extension of the failure by showing the plastic zones. If the equilibrium cannot be achieved, it means that a failure occurs. This phenomenon is in a good agreement with the field observations in the study area (Fig. 8). It can therefore be said that the numerical results, obtained in this study, have permitted to simulate the failure zone.

Analysis of the findings

Comparison between the results

The in-situ settlements caused by the excavation of the EWL7 tunnel were experimentally measured on site by leveling points installed at the ground surface. These results are used for comparison with the numerical ones from the FLAC^{3D} models. As mentioned above, in-situ normal settlements change in ranges from 5 mm to 20 mm while the abnormal settlements reached up to 155 mm (Fig. 7).

Using the geometrical and geotechnical data of the tunnel in studied section, numerical models were employed to evaluate the ground settlement caused by the tunnel excavations and the impact of pre-existent qanats. According to the numerical results, in the case without qanats, the settlement obtained was equal to 16 mm. Similarly, the values of 17 mm and 30 mm were obtained in the cases of pre-existent qanats and pre-existent qanats and cavities, respectively, for normal settlements. For the cases 1 (without qanat) and 2 (with pre-existent qanats), abnormal settlements were not seen in numerical modeling. However abnormal settlements appeared for case 3. A collapse of the pre-existent qanats is observed. The values of the maximum settlements (S_{max}) obtained by two methods are compared in Table 12. A good agreement is seen between the real (in-situ) settlements and the numerical ones especially for normal settlements.

It is noticeable that settlements take place in the vicinity of the cavity. Their values are dependent on the distance of the measuring point from the center of the cavity. Generally, by moving away from the center of the cavity, the values of settlements are reduced.

Probable Mechanism of the collapse of Molavi Street

The collapse of Molavi Street occurred due to pre-existing qanat in the route of the EWL7 tunnel. To investigate this collapse, in addition to field observations, numerical models are used to simulate all of the possible cases which could be happened in the studied region. Several models were constructed to study this problem and generally, three situations were considered in the numerical models: (1) the model without qanat, (2) the model with qanat, and (3) the model with qanat which converted into a cavity.

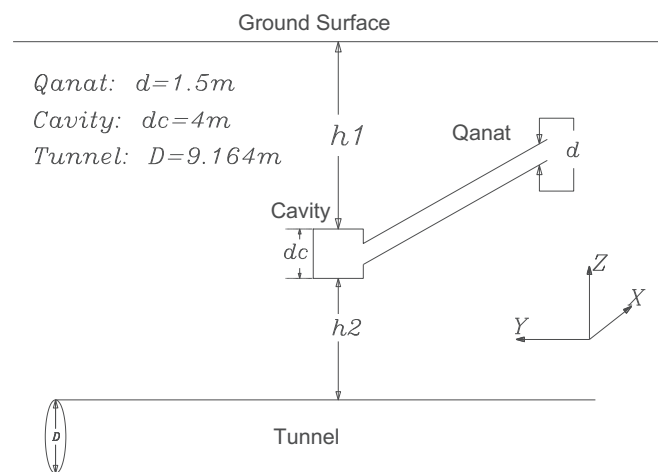


Fig. 20. The spatial position of the qanat.

collapse occurs by rock wedges creation at the intersection of discontinuities. The dimensions of the wedges are controlled by the spacing of the discontinuities. Many studies have been carried out on this subject using the Discrete Element method [34,8], General Particle Dynamics [47,3,4], Peridynamics [44,43,45,39].

In the present study, a continuous media is used, the situation is completely different from the above studies and the Mohr-Coulomb shear failure criterion is used to model the failure. Regarding the Molavi Street failure case, as mentioned previously, at the end of the model execution, the equilibrium was not achieved. Using a continuous media code do not permit to study what happens after the failure. It

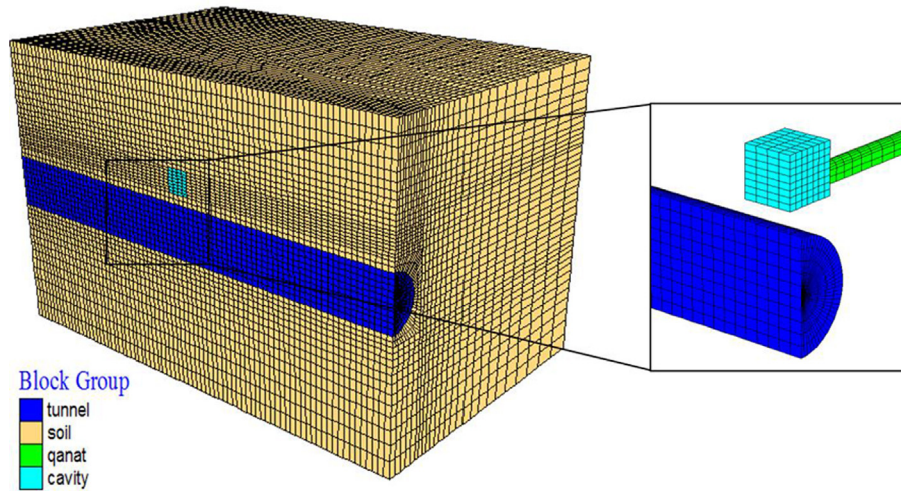


Fig. 21. 3D view of the cavity, qanat, and tunnel.

Table 10
Results of numerical modeling of the case of cavity.

Approach		A		B	
Vertical displacement (mm)	Location Value	Cavity roof 27 mm	Ground surface (S_{max}) 21 mm	Cavity roof 850 mm	Ground surface (S_{max}) 30 mm

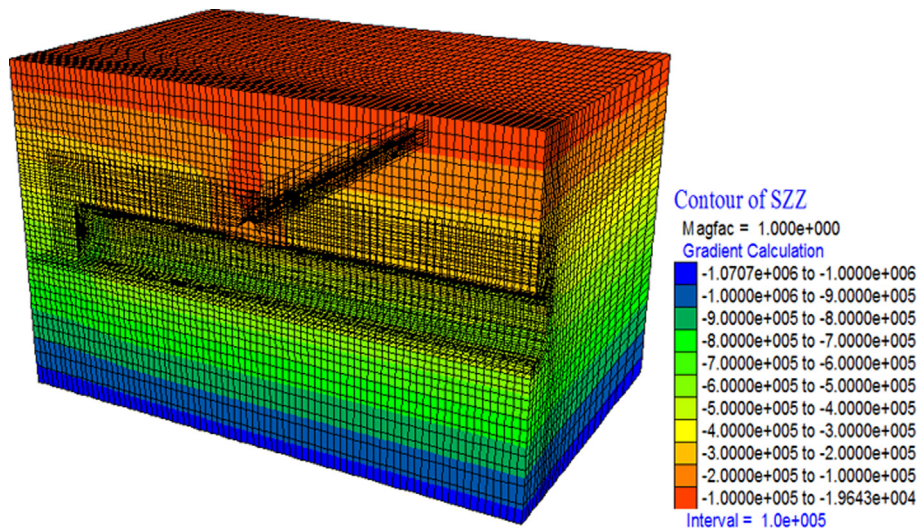


Fig. 22. Contours of vertical stresses at the end of the tunnel excavation (case 3), unity: Pascals.

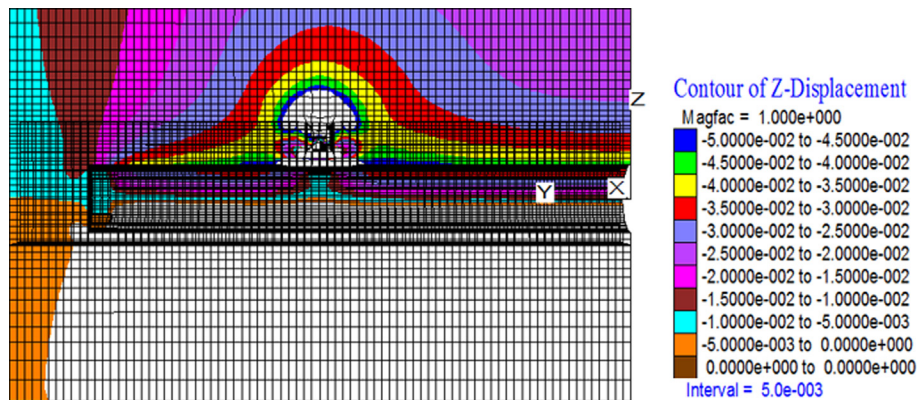


Fig. 23. Contours of vertical displacements (case 3), unity: meters.

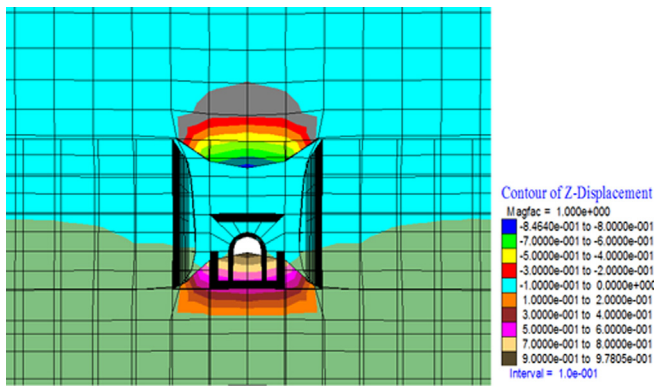


Fig. 24. Contours of vertical displacements of the cavity roof (case 3).

Table 11
Effect of the cavity depth.

h1 (m)	Approach			
	A		B	
	Cavity roof	Ground surface (S _{max})	Cavity roof	Ground surface (S _{max})
17	27	21	850	30
8.5	17	15	100	22
3	Collapse reported			

The dimensions of the qanat and cavity are selected based on the field evidence of the hole of Molavi Street as well as preceding literature about this subject. The numerical results show that unless in the case of the shallow cavity ($h1 = 3$ m), the other numerical models with the higher depth of cavity (i.e., $h1 = 17$ and 8.5 m) can reach the equilibrium state (Table 11) meaning that no collapse occurs in qanats or cavities. In other words, based on the numerical results, the collapse was occurred in the case that the shallower qanat has been converted into a cavity. The conversion process of a qanat into a cavity is very important. According to historical and technical evidence, this conversion can be occurred due to two groups of factors:

- A) Natural factors which mainly include geological and environmental effects, such as ground-water fluctuation, water drainage and passage (especially in the qanats used for drainage and disposal of urban sewage), erosion of qanat wall or Kavulls, earthquake and etc.
- B) Artificial (man-made) factors which mainly include urbanization activities such as building construction, tunnel excavation and etc.

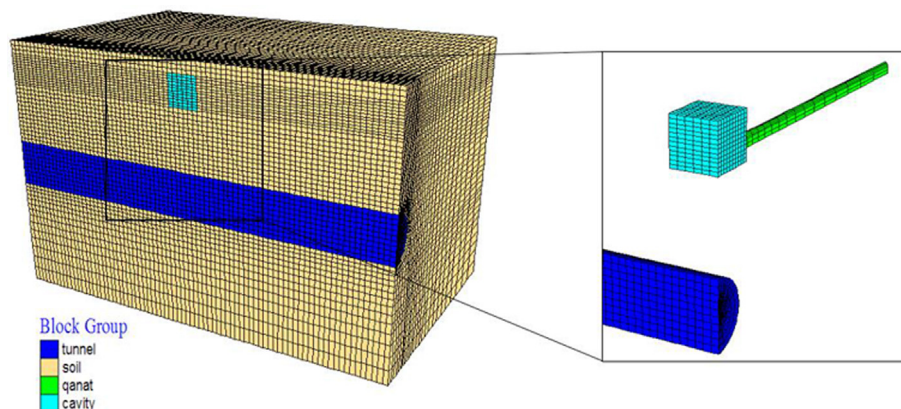


Fig. 25. The Model Constructed for the Cavity of the Molavi Street.

Table 12
Comparison between real and evaluated settlements.

Settlements	S _{max} (mm)			
	Measured (Real)	Numerical (FLAC ^{3D})		
		Case 1*	Case 2**	Case 3***
Normal	5–20	16	17	30
Abnormal	Up to 155	–	–	Collapse

* Case without qanats.

** Case with pre-existent qanats.

*** Case with pre-existent qanats which failed and were converted into cavities.

Field observations and technical records show that the hole of Molavi Street was created simultaneously with the excavation of the EWL7 tunnel. Also, it is found that the dimensions of the hole are larger than the identified qanats existing in the studied area. Therefore, according to the numerical modeling results, it can reasonably be said that in the case of Molavi Street, the pre-existing qanat in the vicinity of EWL7 tunnel has collapsed and was converted into a cavity due to the tunnel excavation. It also has caused the creation of a sinkhole at the ground surface.

Fig. 26 shows the stages that the qanat of Molavi has probably passed to completely convert into the cavity during the TBM passage schematically. Based on this figure, the conversion of qanat into cavity has taken place through four stages. According to these stages and the numerical results, it is found that the geological characteristics of the studying zone, such as soil type and its geotechnical parameters, groundwater condition, and other factors are important in the conversion process. The TBM excavation process has also had an impact. Finally, regarding all of these factors, it can be said that a combination of natural and artificial factors has caused the collapse of the ground surface and the creation of the hole of Molavi Street.

Conclusion

A 3D numerical modeling was used to investigate the impact of pre-existing qanats on ground settlements due to tunneling. The case of the Molavi Street collapse is considered in terms of geological and geotechnical properties and the settlements of the ground surface, occurred due to the excavation of EWL7 tunnel, are considered as the basis of in-situ investigations. According to the monitoring results, the real settlements can be separated into normal and abnormal settlements. In this study, normal measured settlements were monitored in the range from 5 mm to 20 mm whereas abnormal ones are reached more than 155 mm.

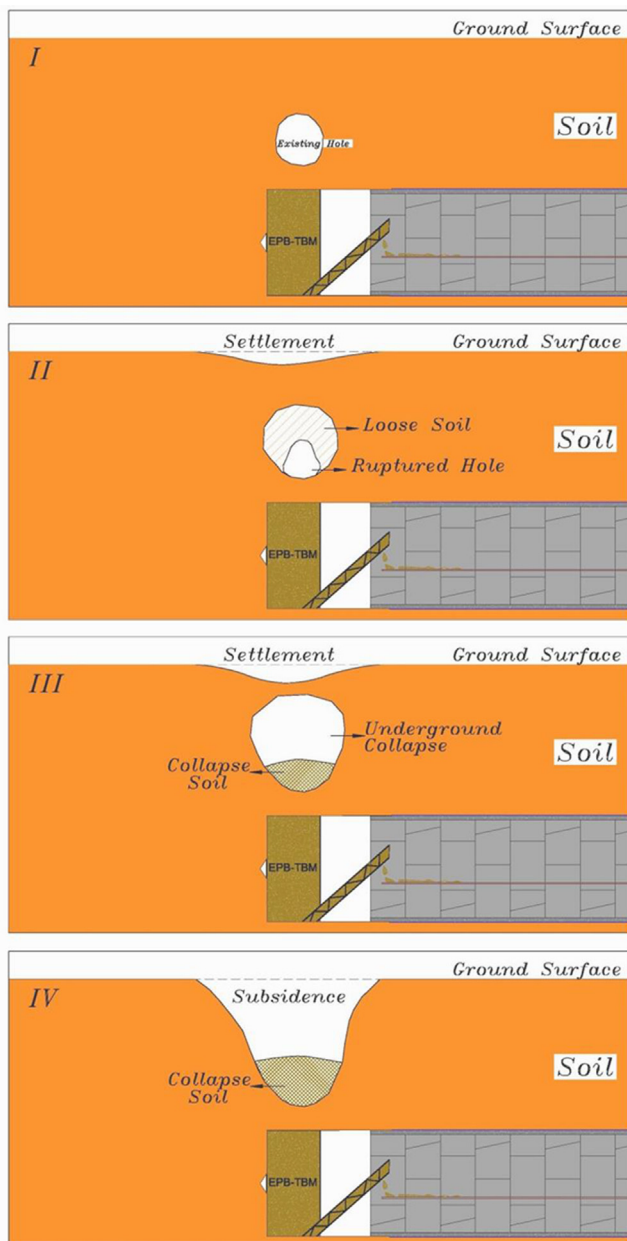


Fig. 26. The probable process of the collapse of Molavi Street.

Field observations showed that the ancient and obsoleted qanat chains in this region, as well as their minor branches and conduits, are the main reasons which cause the collapse of Molavi Street. Numerical models were then used to better understand the position effect of the qanat, cavity, and tunnel on a possible collapse. The following results were obtained:

- If the qanat did not become a cavity, the presence of the qanat did not have a considerable impact on the ground settlements.
- In general, when the qanats become a cavity, the ground settlement due to tunneling increase. In the case of a shallower cavity, the cavity collapsed.
- A numerical model was made in accordance with the Molavi Street conditions. The numerical model cannot reach an equilibrium, which confirms the collapse occurrence in reality.

Acknowledgements

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References

- [1] Atapour H, Aftabi A. Geomorphological, geochemical and geo-environmental aspects of Karstification in the Urban Areas of Kerman City, Southeastern, Iran. *Environ Geol* 2002;42(2002):783–92.
- [2] Beaumont P, B.A. Qanat systems in IRAN. *International Association of Scientific Hydrology Bulletin* 1971;16:1 39–50, DOI: <http://doi.org/10.1080/02626667109493031>.
- [3] Bi J, Zhou XP, Qian QH. The 3D numerical simulation for the propagation process of multiple pre-existing flaws in rock-like materials subjected to biaxial compressive loads. *Rock Mech Rock Eng* 2016;49(5):1611–27. <https://doi.org/10.1007/s00603-015-0867-y>.
- [4] Bi J, Zhou XP, Xu XM. Numerical simulation of failure process of rock-like materials subjected to impact loads. *Int J Geomech* 2016;17(3):04016073.
- [5] Bianchi Fasani G, Bozzano F, Cardarelli E, Cercato M. Underground cavity investigation within the city of Rome (Italy): a multi-disciplinary approach combining geological and geophysical data. *Eng Geol* 2013;152:109–21. <https://doi.org/10.1016/j.enggeo.2012.10.006>.
- [6] Cheshomi A. Study of mechanical properties of Tehran coarse-grained alluvium based on combination of geology and mechanical tests PhD. thesis Tarbiat Modares University; 2006. p. 238.
- [7] Castellanza R, Lollino P, Ciantia M. A methodological approach to assess the hazard of underground cavities subjected to environmental weathering. *Tunn Undergr Sp Technol* 2018;82:278–92.
- [8] Cundall PA, Strack ODL. A discrete numerical model for granular assemblies. *Geotechnique* 1979;29:47–65.
- [9] DAUB (Deutscher Ausschuss für unterirdisches Bauen). Recommendations for selecting and evaluating tunnel boring machines. *Tunnel* 1997;5(97):20–35.
- [10] Dias D, Kastner R. Movements caused by the excavation of tunnels using face pressurized shields – analysis of monitoring and numerical modeling results. *Eng Geol* 2013;152:17–25. <https://doi.org/10.1016/j.enggeo.2012.10.002>.
- [11] Do NA, Dias D, Oreste PP, Djeran-Maigre I. Three-dimensional numerical simulation of a mechanized twin tunnels in soft ground. *Tunn Undergr Sp Technol* 2014;42:40–51.
- [12] Do NA, Dias D, Oreste PP, Djeran-Maigre I. Numerical investigation of segmental tunnel linings-comparison between the hyperstatic reaction method and a 3D numerical model. *Geomech Eng* 2018;14(3):293–9.
- [13] Dunnyclif J, Green GE. *Geotechnical instrumentation for monitoring field performance*. United State: John Wiley and Sons Inc.; 1993. p. 577.
- [14] English PW. The origin and spread of qanats in the old world. *Proc Am Philosoph Soc* 1968;112(3):170–81.
- [15] English P. Qanats and life worlds on Iranian Plateau villages. In: Albert J, Bernhardsson M, Kenna R, editors. *Transformation of Middle Eastern Natural Environment, Bulletin Series 103, Yale School of Forestry and Environmental Studies*. Yale University Press; 1998. p. 187e205.
- [16] Fookes PG, Knill JL. The application of engineering geology in the regional development of northern and central Iran. *Eng Geol* 1969;3(2):81–120. [https://doi.org/10.1016/0013-7952\(69\)90001-5](https://doi.org/10.1016/0013-7952(69)90001-5).
- [17] Foroughnia F, Nemati S, Maghsoudi Y, Perissin D. An iterative PS-InSAR method for the analysis of large spatio-temporal baseline data stacks for land subsidence estimation. *Int J Appl Earth Obs Geoinformation* 2018;74:248–58. <https://doi.org/10.1016/j.jag.2018.09.018>.
- [18] Fuenkajorn K, Archeeploha S. Prediction of cavern configurations from subsidence data. *Eng Geol* 2009;110:21–9. <https://doi.org/10.1016/j.enggeo.2009.10.003>.
- [19] Geranmayeh VR, Foroughi M, Tarigh Azali S. Feasibility of Ground Improvement Techniques for Safe Tunneling in Urban Environments, Tehran metro line 7. ITA-AITES World Tunnel Congress (WTC) Bangkok, Thailand; 2012.
- [20] Golpasand MRB, Nikudel MR, Uromeihy A. Effect of engineering geological characteristics of Tehran's recent alluvia on ground settlement due to tunneling. *Geopersia* 2014;4(2):185–99.
- [21] Golpasand MRB. Evaluation of the effect of Engineering Geological characteristics of soil on ground settlement induced by shallow Tunneling in urban area PhD. Dissertation Tehran, Iran: Tarbiat Modares University; 2015.
- [22] Golpasand MRB, Nikudel MR, Uromeihy A. Specifying the real value of volume loss (VL) and its effect on ground settlement due to excavation of Abuzar tunnel, Tehran. *Bull Eng Geol Environ* 2016;75(2):485–501. <https://doi.org/10.1007/s10064-015-0788-8>.
- [23] Golpasand MRB, Do NA, Dias D, Nikudel MR. Effect of the lateral earth pressure coefficient on settlements during mechanized tunneling. *Geomech Eng* 2018;16(6):643–54. <https://doi.org/10.12989/gae.2018.16.6.643>.
- [24] Hamidian A, Ghorbani M, Abdolshahnejad M, Abdolshahnejad A. Qanat, traditional eco-technology for irrigation and water management. *Agric Agric Sci Procedia* 2015;4(2015):119–25.
- [25] Hajian AR, Ardestani EV, Lucas C, Saghaiannejad SM. Detection of subsurface qanats by artificial neural network via microgravity data. *J Earth Space Phys* 2009;35(1):9–15.
- [26] Helm PR, Davie S, Glendinning S. Numerical modelling of shallow abandoned mine

- working subsidence affecting transport infrastructure. *Eng Geol* 2013;154:6–19. <https://doi.org/10.1016/j.enggeo.2012.12.003>.
- [27] ITA (International Tunnelling Association) Working Group No. 2. Guidelines for the design of shield tunnel lining. *Tunn Undergr Sp Technol* 2000;15(3):303–31.
- [28] Itasca. FLAC3D Fast Lagrangian analysis of continua in 3D dimensions. Users and Theory Manuals. Minneapolis, Minn; 2006.
- [29] Lightfoot D. Traditional wells as phreatic barometers: a view from qanats and tube wells in developing arid lands. Proceedings of the UCOWR conference: water security in the 21st Century, Washington, DC. 2003.
- [30] Mahmoudpour M, Khamhechyan M, Nikudel MR, Ghassemi MR. Numerical simulation and prediction of regional land subsidence Caused by Groundwater Exploitation in the Southwest Plain of Tehran, Iran. *Eng Geol* 2016;201:6–28. <https://doi.org/10.1016/j.enggeo.2015.12.004>.
- [31] Noel E. Qanats. *J R Central Asian Soc* 1944;31(1944):191–202.
- [32] Peck RB. Deep excavation and tunneling in soft ground. In: Proc. Of the 7th int. conference on soil mechanics and foundation engineering. State of the art Volume. Sociedad Mexican de Mecanica de Suelos, A. C; 1969.
- [33] Pellet F, Amini Hosseini K, Jafari MK, Zohra Zerfa F, Mahdavi MR, Keshavarz Bakhshayesh M. Geotechnical performance of Qanats during the 2003 Bam, Iran, earthquake. *Earthquake Spectra* 2005;21:137–64.
- [34] Potyondy DO, Cundall PA. A bonded-particle model for rock. *Int J Rock Mech Min Sci* 2004;41(8):1329–64.
- [35] Rayhani MHT, El Naggar MH. Collapse hazard zonation of qanats in greater Tehran area. *Geotech Geol Eng* 2007;25(3):327–38.
- [36] Rezaei F, Dadsetan A. Evaluation of gradual ground linear subsidence and geotechnical parameters assessment in the Taleghani Site of Eshtehard. *J Geosci* 2012;21(83):3–12. <https://doi.org/10.22071/gsj.2012.54428>.
- [37] Rieben EH. The geology of the Tehran plain. *Am J Sci* 1955;253:617–39. <https://doi.org/10.2475/ajs.253.11.617>.
- [38] Salehi F, Hafezi moghadas N, Ghafoori M, Lashkaripour G. Hazard evaluation of qanat in the west of mashhad by plaxis software. *jeg* 2014; 8(3)::2277–300, URL: <http://jeg.khu.ac.ir/article-1-1610-en.html>.
- [39] Silling SA. Reformulation of elasticity theory for discontinuities and long-range forces. *J Mech Phys Solids* 2000;48(1):175–209. [https://doi.org/10.1016/S0022-5096\(99\)00029-0](https://doi.org/10.1016/S0022-5096(99)00029-0).
- [40] Stiros SC. Accurate Measurements with primitive instruments: the “Paradox” in the Qanat Design. *J Archaeol Sci* 2006;33:1058–64.
- [41] Tarigh Azali S, Ghafoori M, Lashkaripour GR, Hassanpour J. Engineering geological investigations of mechanized tunneling in soft ground: a case study, East-West lot of line 7, Tehran Metro, Iran. *Eng Geol* 2013;166:170–85. <https://doi.org/10.1016/j.enggeo.2012.12.003>.
- [42] Wang T, Yang C, Chen J, Daemen JJK. Geomechanical investigation of roof failure of China's first gas storage salt cavern. *Eng Geol* 2018;243:59–69. <https://doi.org/10.1016/j.enggeo.2018.06.013>.
- [43] Wang Y, Zhou X, Shou Y. The modeling of crack propagation and coalescence in rocks under uniaxial compression using the novel conjugated bond-based peridynamics. *Int J Mech Sci* 2017;128:614–43. <https://doi.org/10.1016/j.ijmecsci.2017.05.019>.
- [44] Wang Y, Zhou X, Xu X. Numerical simulation of propagation and coalescence of flaws in rock materials under compressive loads using the extended non-ordinary state-based peridynamics. *Eng Fract Mech* 2016;163:248–73. <https://doi.org/10.1016/j.engfracmech.2016.06.013>.
- [45] Wang Y, Zhou X, Wang Y, Shou Y. A 3-D conjugated bond-pair-based peridynamic formulation for initiation and propagation of cracks in brittle solids. *Int J Solids Struct* 2018;134:89–115. <https://doi.org/10.1016/j.ijsolstr.2017.10.022>.
- [46] Yu HS. Cavity expansion methods in geomechanics. School of civil engineering. Dordrechd/ Boston/ London: University of Nottingham, U. K. Kluwer Academic Publisher; 2000. ISBN: 978-0-412-79990-7.
- [47] Zhou XP, Bi J, Qian QH. Numerical Simulation of crack growth and coalescence in rock-like materials containing multiple pre-existing flaws. *Rock Mech Rock Eng* 2015;48(3):1097–114. <https://doi.org/10.1007/s00603-014-0627-4>.