A COMBINED SOLUTION FOR DETERMINATION OF MULTI-BRANCHED REFRACTIVE INDEX IN 1D METAMATERIALS

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Abstract: *Negative-refractive metamaterials have attracted a great attention in recent years due to their unique physical properties. It is possible to obtain a negative refractive index material by periodically combining a split-ring resonator and a wire structure. In this work, we first numerically investigated frequency-dependent refractive index of a metamaterial by the retrieval method, figured out and solved multi-branch problem occuring regularly in this method, especially in high frequency regime. Importantly, we co-realized the true refractive index by the phase shift method and 2D refraction method respectively. Finally we compared the results obtained from each method to converge an eventual one.*

Keywords: Metamaterials, Negative refraction, Split-ring resonator, Retrieval method, Phase shift method, 2D refraction method, Multi-branch problem.

1. INTRODUCTION

MMs are simply defined as artificially structured media in a size scale smaller than the wavelength of external stimuli. They are composed by periodically arranged unit cells which play a role of “meta-atoms” in metarials. Therefore, MMs can possess novel properties which cannot be easily attainable from natural metarials [1].

Since every material is featured by their optical parameters, identifying these values is very important in studying materials. Thanks to Nicolson and Ross [2], an appropriate method was proposes, allowed us to determine complex optical parameters of linear materials in the frequency domain by a single time-domain measurement. Twenty years later, Baker-Jarvis *et. al*. [3] improved the method by examining special cases of permittivity measurement and estimating the errors incurred due to the uncertainty in scattering parameters, length measurement and reference plane position plane position. However, many issues had still remained until 2004, when Chen *et. al*. proposed “a robust method to retrieve the constitutive effective parameters of metamaterials” which provided advanced performance than the two mentioned methods including the determination of the first boundary and the thickness of the effective slab, the selection of the correct sign of effective impedance, and especially, a mathematical method to choose the correct branch of the real part of the refractive index [4]. Then, this method has been extensively used to extract effective parameters of MMs.

In Vietnam, many publication results employed this retrieval method to extract optical parameters of MMs [5, 6]. However, by far, none of these investigations was carried out to clarify the multi-branch problem in determination refractive index of MMs and also the validity of the obtained value. Therefore, in this work, We solve the multi-branch problem and compare the acquired results to those obtained by two other independent methods for ensuring the accuracy of the retrieval one.

2. SIMULATION SETUP

In this work, a simulation system was designed to obtain S parameters of the MM structure (Fig. 1). The system included two ports playing a role of transmitter and also receiver antenna, a MM unit cell stays in between. The incident EM wave in z-direction is S-polarized with electric field in y-direction and magnetic field in x-direction. A periodic boundary condition is set in x and y direction while boundary condition in z direction is open. After simulation, S parameters data including magnitude and phase of transmittance and reflectance are excuted by matlab to retrieve optical paramaters of the MM.

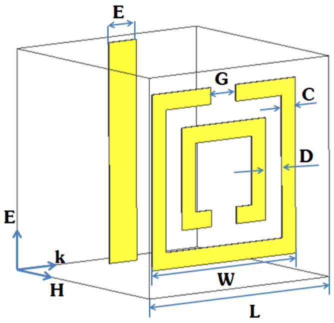


Figure 1: *Unit cell of the combined metamaterial used in the simulation. C=0.25 mm, D=0.3 mm, G=0.46 mm, W=2.63 mm, E=0.5 mm, and L=3.3 mm. The metallic elements are copper with a thickness of 0.017 mm. The SRR is square and the unit cell is cubic.*

Using the idea of conventional prism we constructed a simulation setup which is shown in Fig. 2. Three MM prisms with different wedge angles were assembled by MM unit cells. From an initiator, unit cells are expanded in electric field and wave propagation directions. A plane wave playing a role of transmitter antenna was utilized to emit S-polarized incident wave to the prism (EM constituents distribute in directions as previous simulations). Since the incident angle is identified by wedge angle of the prism, one only has to measure the refracted angle to calculate value of refractive index [9].

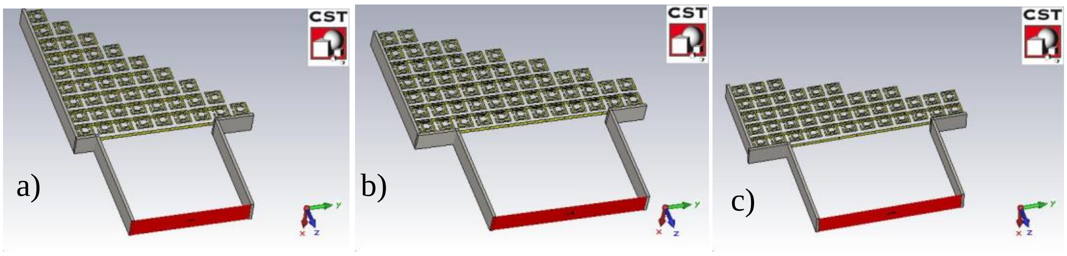


Figure 2. *Three MM prisms with different wedge angle of (a) 45^o, (b) 26.6^o and (c) 18.4^o assembled by MM unit cells.*

3. RESULTS AND DISCUSSION

We would start at the origin of MMs with one of the primary structures – Split-ring resonators (SRR) and continuous wires. The unit cell of combined MM is cube-shaped with lattice parameters shown in Fig. 1 similar to those used by Shelby [10]. However here we put all magnetic and electric resonators in one unit cell and separated from each other in a distance of $t_s = 2.5$ mm. In fact, this distance exerts a minor influence on S-parameters of the structure when $0.5 \text{ mm} < t_s < 3.0$ mm (it means these two resonators should not be too close as well as too far from each other that gets over lattice constant of the unit cell) The value of 2.5 mm is chosen to avoid coupling effects between the two resonators.

At the beginning, unit cell structure was simulated to extract S parameters including reflection and transmission coefficients and phase. Hereafter, these values were utilized as input for a matlab code based on X. Chen's equations as mentioned above to retrieve optical parameters of the MM. The same procedure were applied while increasing number of layers of MM, frequency-dependent refractive index values of MM were procured corresponding to one, two and three layers MM (Fig. 3). Theoretically, all of three values should be the same. Unfortunately, Fig. 3 presents three different lines with fluctuations escalating proportional to the number of layers. Multi-branch problem is now formed and strongly influences the result of retrieval method. The result shown in Fig. 3 is obviously unreasonable especially in cases of two and three layers.

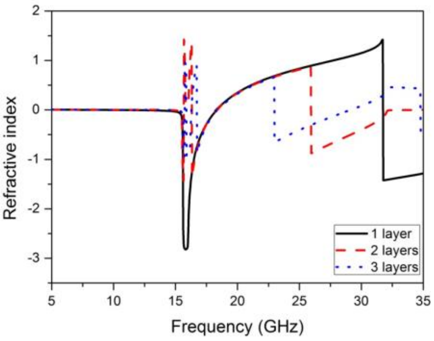


Figure 3: *Frequency-dependent refractive index of one layer (black curve), two layers (red curve) and three layers (blue curve) SRR structure with multi-branch problem.*

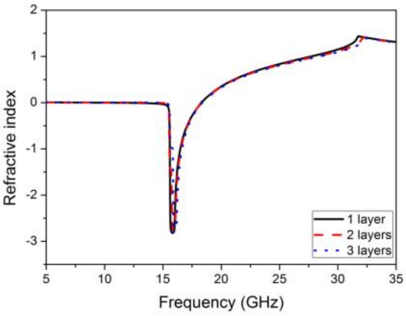


Figure 4: *Frequency-dependent refractive index of one layer (black curve), two layers (red curve) and three layers (blue curve) SRR structure with appropriate values of m.*

Multi-branch problem was nominated as the reason behind this inaccuracy. In an effort to find out the true result, we tuned the value of "m" (the branch index as mentioned above) in each fluctuation section from -2 to +2. A reasonable result was achieved as shown in Fig. 4 by allocating each fluctuation frequency range for an appropriate value of m. The continuity of frequency-dependent refractive index result is set out to be the object of this adjustment.

Table 1: *Branch index m corresponds for each frequency range and number of layer. (blank cells and unmentioned frequency ranges set n = 0 as default)*

m	1 layer	2 layers	3 layers
-1	31.746 – 35.000 GHz	15.654 – 16.300 GHz	15.722 – 17.708 GHz
+1		25.922 – 35.000 GHz	22.964 – 34.762 GHz
+2			34.762 – 35.000 GHz

Details are presented in Table 1. It is important to note that the higher number layers of MM, the denser branches of refractive index. Nevertheless, one should be still confused if these identified values of m are correct.

Throughout simulating SRR-MMs, transmission phases of one and two layers were attained and compared (Fig. 5). As expected, in the frequency regime of negative refractive index, the phase shift is negative from one to two layers indicates that the phase velocity is negative. Meanwhile, it is clearly seen that the phenomena happens inversely in the positive refractive index regime with phase shift growing up by layers. Continuously, frequency-dependent values of n were calculated by $n = \frac{\Delta\emptyset.c}{\omega\Delta L}$ [11]. Fig. 6 performs a comparision between results obtained by the two methods (retrieval method and phase shift method) in a good agreement with small difference at high frequencies.

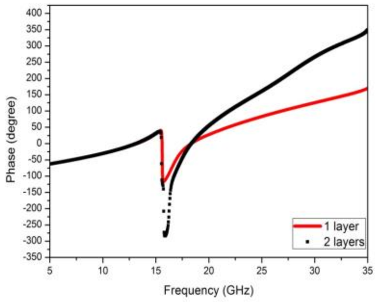


***Figure 5**: Transmission phase of one layer (red curve) and two layers (black squares) SRR structure.*

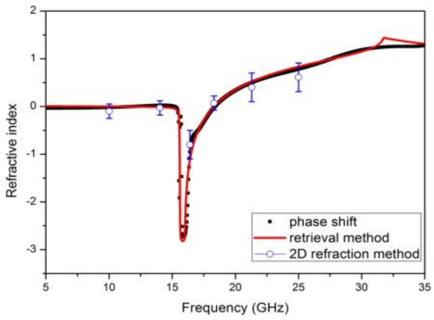


***Figure 6**: Frequency-dependent refractive index obtained by phase shift (black squares), retrieval method (red curve) and 2D refraction method (blue circles).*

Last but not least, 2D refraction method was employed to verify the correction of collected values of refractive index above. In order to demonstrate negative refraction of MM, Fig. 7 is simulated phenomena of how EM wave propagates through a MM prism at negative refractive index frequency. There is a good agreement between these three prisms with varied wedge angle of 18.4^o, 26.6^o and 45^o at 16.4 GHz, the value of n attained from the three prisms are -1.02, -0.99 and -1.00 respectively according to Snell's law $\frac{\sin\emptyset_1}{\sin\emptyset_2} = \frac{n_2}{n_1}$. This is a quite good agreement with n equals to -1.2 from retrieval method.

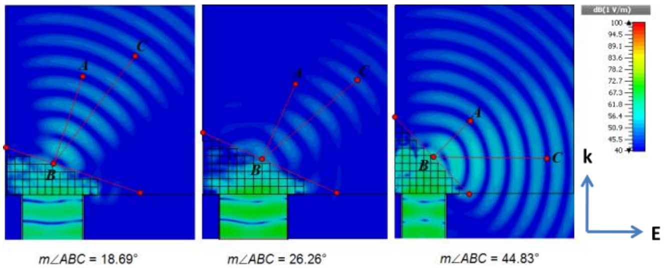
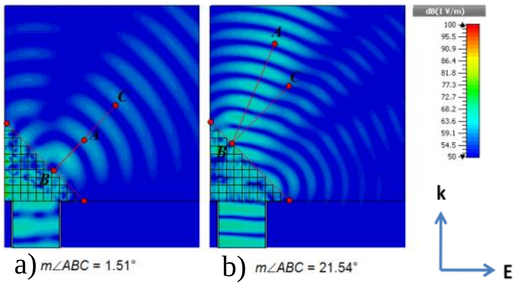


***Figure 7**: Probed emergent angle of each prism with incident wave of a) 18.4^o, b) 26.6^o and c) 45^o at 16.4 GHz.*

Values of refractive index obtained via retrieval method were continuously verified by to 0.01 in retrieval method at 14 GHz, the emergent EM wave sticks closely to normal of the emergent plane. Indeed, the measured emergent angle in this case is 1.51^o (Fig. 8a) which means refractive index value is about -0.03. This is the region so called single negative index regime (SNG) with only $\varepsilon < 0$. Theoretically, the loss in this regime is significant. Therefore quite a few studies have been done due to lack of applications in comparison to ones of double negative index regime (DNG). However, recently, the interest of refractive index absolute zero has attracted MMs researchers since phase velocity and wavelength of EM wave are infinite [9, 10].



a) $m\angle ABC = 1.51°$ b) $m\angle ABC = 21.54°$

Figure 8: *Probed emergent angles of 45^o MM prism with incident angles at a) 14 GHz and b) 21.3 GHz.*

Besides, in positive refractive index frequency 21.3 GHz, the measured emergent angle equals to 21.54^o (Fig. 8b) referred refractive index valued around 0.52. The difference to n equals to 0.53 obtained at the same frequency by retrieval method is negligible. In summary, the refractive index of SRR structure in microwave regime from 5 GHz – 35 GHz have been calculated by three methods: retrieval method, phase shift method and 2D refraction method and converged in a good agreement (Fig. 6). The result after solving multi-branch problem in retrieval method can be now affirmed to be true.

Interestingly, we would remind you about the total reflection effect happening when the incident angle comes over the critical one. Here one can see the same phenomena in Fig. 9 that at 15.8 GHz, refractive index of prism equals to -2.8 refers to a critical angle of 20^o. However the incident angle to emergent plane is 45^o, much larger than the critical one. Thus, incoming EM waves are totally reflected inside the prism.

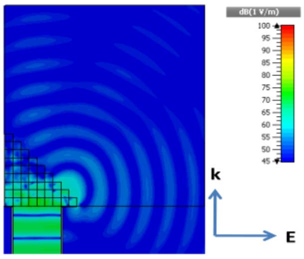


Figure 9: *Total reflection of 45^o MM prism with incident EM wave at 15.8 GHz.*

According to the results discussed above, the retrieval method in refractive index determination is evaluated as the most favored and trustworthy. Therefore, in this study, the values of refractive index procured by retrieval method are chosen to be the conclusive result of this study. Nevertheless, in Fig 6, one can see that there is a small difference between the results of retrieval method and phase shift method at high frequencies. Due to the fact that, at high frequencies, the wavelength of EM wave is much smaller than one at low frequencies (wavelength of EM wave at 30 GHz is shorter twice than one at 15 GHz). At a certain boundary, the condition of "effective medium theory" would be broken which leads to untrusted simulated results at high frequencies. Therefore, we would narrow the trusted frequency range to 5 ↔ 30 GHz.

4. CONCLUSION

We applied numerical methods used in simulations to calculate the value of refractive index by three independent methods and compared these results with each other. An

excellent agreement between the three results was obtained to release a conclusive value of frequency-dependent refractive index in the frequency range from 5 to 30 GHz.

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TÓM TẮT

BÀI TOÁN ĐA NHÁNH TRONG XÁC ĐỊNH CHIẾT XUẤT CỦA VẬT LIỆU MỘT CHIỀU METAMATERIALS

Vật liệu chiết suất âm gần đây đã thu hút được sự chú ý lớn của giới khoa học trong thời gian gần đây bởi những tính chất vật lý hết sức đặc biệt. Chiết suất âm có thể được tạo ra bằng sự kết hợp của cấu trúc vòng cộng hưởng và cấu trúc dây kim loại tuần hoàn. Trong bài viết này, đầu tiên chúng tôi xác định chiết suất vật liệu theo sự thay đổi tần số bằng phương pháp truy hồi, chỉ ra và giải quyết vấn đề chia nhánh xuất hiện trong phương pháp này. Sau đó, chiết suất của vật liệu lần lượt được tính toán lại bằng phương pháp dịch chuyển pha và phương pháp khúc xạ 2D. Cuối cùng, chúng tôi so sánh những kết quả thu được với nhau để thu được vùng kết quả tin cậy.

Từ khóa: Vật liệu meta, Chiết suất âm, Vòng cộng hưởng, phương pháp truy hồi, Phương pháp dịch chuyển pha, Phương pháp khúc xạ 2D, Vấn đề chia nhánh.

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