Journal of Mining and Earth Sciences

Website: http://tapchi.humg.edu.vn

An approach to improve the resolution of vertical electrical sounding data

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ARTICLE INFO

Article history:
Received 17 Feb. 2018
Accepted 19 Apr. 2018
Available online 29 Jun. 2018

Keywords:
Resistivity
Vertical electrical sounding
Resolution
Mean sensitivity depth

ABSTRACT

Vertical electrical sounding has proved its efficiency in mineral exploration, geotechnical prospecting, environment and hydrogeology. With the widespread application of this method, there have been a lot of researches of data processing and interpretation. In this paper, we describe our approach to improve the resolution of vertical electrical sounding data, followed by comparing it to the so-called "N-transformation" method. The mean sensitivity depth was applied to evaluate the investigation depth, and several simple mathematical formulas to transform apparent resistivity was used to get the transformed curve which is better match to geological models. To illustrate the effectiveness of this approach, we carried out research on six synthetic models that had been used in other research. The results were pretty good and could be used for training students, as well as additional information to interpret vertical electrical sounding data. We also proved that the interpolation method is inadequate to approximate apparent resistivity data, and the wrong anisotropy predicted may create several distortions to investigation depth. This causes the fact that the interpolation method may not have any geological sense, and it can be done only when the authors consider the geological models and resistivity parameters before performing their method, which is mainly impossible in field data.

1. Introduction

Vertical electrical sounding (VES) has been considered as an important method in mineral exploration, geotechnical engineering, as well as hydrogeology. 1D and 2D investigation are now routine while 3D and even 4D investigations are becoming more common (Butler, 2016). However, as other geophysical methods, the problem of ambiguity always makes it difficult to interpret VES data and delineate subsurface targets. There have been several researches about VES data processing. While some authors proposed the function (functions) to calculate the depth of the electrical resistivity method (Apparao, 1992; Banerjee, 1986; Reynolds, 2011; Ward, 1988; Koefoed, 1979; Barker, 1979; Edwards, 1977), others created and/or improved inversion procedures for interpretation purpose that included approaches to determine the depth

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(Loke, 2013; Zohdy, 1949; Zohdy, 1965; Zohdy, 1989; Butler, 2016) proposed a simple formula to calculate the mean sensitivity depth, which is useful for geophysical education and for the aim of aiding the understanding of electrical resistivity method.

In Vietnam, (Nguyen, 2016) introduced N transformation method with the perspective that the subsurface is a system of continuous thin layers. (Truong, 2017) used a high-order polynomial function as an interpolation algorithm for approximating apparent resistivity curves to calculate the so-called "N transformation method" formulas. However, the square of derivative of apparent resistivity versus distance used to calculate anisotropy coefficient, depth and resistivity, is very unstable, leading to bad results.

In this paper, we will propose new formulas to transform plot of apparent resistivity curves versus distance into resistivity curves versus depth with the point of view similar to (Nguyen, 2016). Our simple formulas could be used for the same purposes as that of Butler (2016). After that, we will compare our results, which is carried out with synthetic models, with the mean sensitivity depth calculated from Butler's formula. We will then show the effectiveness of this new way to determine the resistivity versus depth as opposed to that of (Nguyen, 2016; Truong, 2017). In addition to this, we will also prove the unsuitability of the interpolation algorithm (Truong, 2017) to calculate above mentioned derivative.

2. Materials and methodology

2.1. Materials

In this research, we used the same six synthetic models as (Nguyen, 2016; Truong, 2017; Butler, 2016), to calculate synthetic data. The parameters of the models are showed in Table 1.

We solved forward problem for these models with Wenner-Schlumberger array, using IPI2win software for the first four models and Res2Dmod software for the last two ones in order to match to the results from references. In addition to this, we used the smallest electrode spacing (a) is 10 meters for model 5 and 6; separation factors (n) are integers and range from one to 16. Figure 1 show the Wenner - Schlumberger array in resistivity surveys. In this figure, while C1 and C2 are current electrodes, P1 and P2 are potential ones. The geometric factor (k) can be calculated from the formula in figure 1, which vary related to smallest electrode spacing and separation factor.

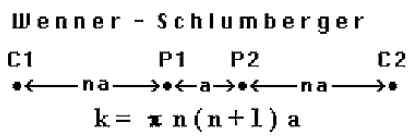

Figure 1. Wenner-Schlumberger array with its geometric factor k (Loke, 2013).

Table 1. Parameters of six synthetic models.

	Model 1 (Nguyen, 2016)		Model 2 (Nguyen, 2016)	
Layer	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	120	10	120	10
2	60	60	60	35
3	1500	∞	200	55
4	-	-	40	∞
Layer	Model 3		Model 4 (Truong, 2017)	
1	600	5	250	5
2	100	10	76	11
3	1000	50	21	100
4	50	∞	10000	∞
Layer	Model 5 (Butler, 2016)		Model 6 (Butler, 2016)	
1	10	10	5	10
2	2	20	10	20
3	20	∞	1	∞

We then used the results of forward problem as synthetic data for our resistivity curves transformation that will be showed in the following sections.

2.2. Methodology

The aim of our research is to derive resistivity versus depth curves from apparent resistivity ones. In this section, we propose several formulas to do the transformation from electrode spacing and apparent resistivity values to mean sensitivity depths and transformed resistivities for Wenner-Schlumberger array.

First, the mean sensitivity depth was calculated by a very simple mathematical formula for the general four electrode configuration (Butler, 2016), which can be rewritten as:

$$z_{mean} = \frac{k}{4\pi} ln \left[\frac{r_{C_1P_2} . r_{C_2P_1}}{r_{C_1P_1} . r_{C_2P_2}} \right] \quad (1)$$

Where k is the geometric factor, and $r_{C_1P_2}, r_{C_2P_1}, r_{C_1P_1}, r_{C_2P_2}$ are the distance between each pair of electrodes showed in Figure 1.

We then try to transform resistivity values from apparent resistivity. Because apparent resistivity curves are always plotted in dual-logarithmic coordinates system, the apparent resistivity can be seen as a function versus r which is the distance between either C_1 or C_2 and the center of the Wenner-Schlumberger array.

$$Lg(\rho_a) = f\,[lg(r)],\ r = r_{C_1C_2}/2 \quad (2)$$

At each apparent resistivity value, we can plot two asymptotic lines (45 degree and -45 degree) to determine the rates of increase or decrease in longitudinal conductance and transverse resistance between that point and the previous ones, which are noted as ΔS and ΔT, respectively. It is noted that those values for the first point of the curve are equal to zero. Figure 2 shows the

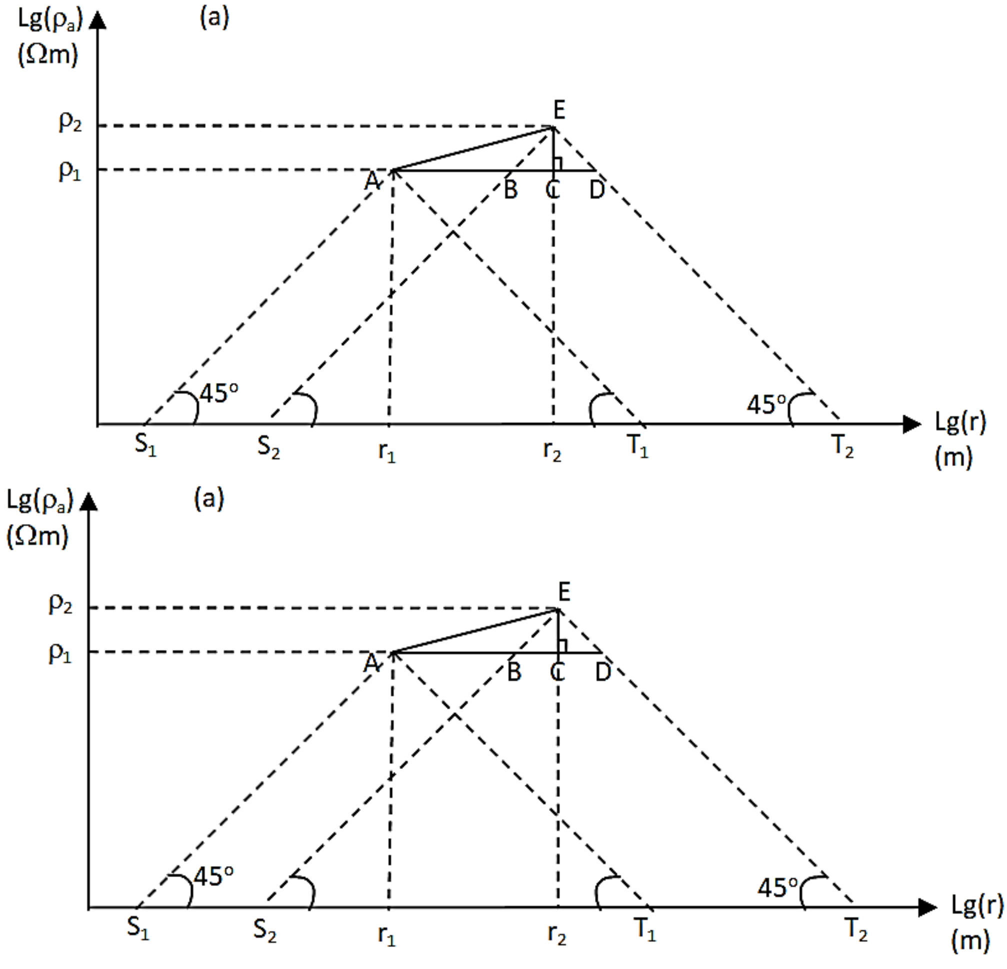

Figure 2. Apparent resistivity curve (a) in case of going up, (b) in case of going down.

way to calculate these values in cases of increasing and decreasing curves.

When the curve goes up (Figure 2a), $\Delta S = S_2 - S_1$, which has the same length with AB, and can be calculated as:

$$\Delta S = AB = AC - BC = [lg(r_2) - lg(r_1)] - [lg(\rho_2) - lg(\rho_1)] \quad (3)$$

Meanwhile, the formula for $\Delta T = T_2 - T_1$ has the same length with AD is:

$$\Delta T = AD = AC + CD = [lg(r_2) - lg(r_1)] + [lg(\rho_2) - lg(\rho_1)] \quad (4)$$

In contrast, we can also define ΔS and ΔT values in case of going down curve as: (Figure 2b)

$$\Delta S = AD = AC + CD = [lg(r_2) - lg(r_1)] + [lg(\rho_2) - lg(\rho_1)] \quad (5)$$

$$\Delta T = AB = AC - BC = [lg(r_2) - lg(r_1)] - [lg(\rho_2) - lg(\rho_1)] \quad (6)$$

It is clear that while $\Delta S < \Delta T$ when the curve goes up, the opposite is true in the rest case, except $\Delta S = \Delta T$ in case of apparent resistivities remain unchanged. This means that the main direction of current flow is parallel to the strata if $\Delta S > \Delta T$, and perpendicular if $\Delta S < \Delta T$ (Zohdy, 1965), which are related to conductive and resistive layer, respectively. We introduce our formulas based on the right asymptote conditions of the apparent resistivity curves, which makes the derivatives of the curve at each datum point never larger than 1 or smaller than -3.721 in dual-logarithmic coordinates system. With the first apparent resistivity value, whose distance (r) is the smallest, it can be considered as a real resistivity of a homogeneous layer, reaching the left asymptote of the curve. This is consistent with many research of the vertical electrical sounding method. For the rest values, we propose the following formulas to transform the apparent resistivity.

In case of the curve decreases and remain constant:

$$lg(\rho_{z,i}) = lg\left\{lg(\rho_{a,i-1}) + \left[\frac{1}{2.721}\left(\frac{3.721+m_i}{1-m_i} - 3.721\right) + m_i - 2\Delta T_i\right].[lg(r_i) - lg(r_{i-1})]\right\} \quad (7)$$

And in case of the curve increases:

$$lg(\rho_{z,i}) = lg\left\{lg(\rho_{a,i-1}) + \left[2.721\left(1 - \frac{1-m_i}{3.721+m_i}\right) + m_i - 2\Delta S_i\right].[lg(r_i) - lg(r_{i-1})]\right\} \quad (8)$$

Where $\rho_{z,i}, \rho_{a,i-1}, r_i, r_{i-1}$ are transformed resistivity, apparent resistivity, distances of i^{th} and $i\text{-}1^{th}$ datum points. $\Delta S_i, \Delta T_i$ are determined between i^{th} and $i\text{-}1^{th}$ datum points, using the way that has been mention above. m_i is the derivative of the function (2) at i^{th} value. It is simple to calculate formulas (7) and (8) from the first point to the last point of the curve with i range from 2 to the number of total datum points.

$$m_i = \frac{\partial lg(\rho_a)}{\partial lg(r)} \quad (9)$$

It is noted that we set $m_i = \left.\frac{\partial lg(\rho_a)}{\partial lg(r)}\right|_{i=1} = 0$; $\rho_{z,i=1} = \rho_{a,i=1}$, according to the term of the left asymptote of apparent resistivity curve. Besides, we also set $\rho_{z,i} = \rho_{a,i}$ for the i^{th} datum points, at which the derivative values (m_i) change its sign. This is because we want to remain the shape of transformed curve as similar to the apparent curve as possible, and mitigate the abrupt change at these data. At peaks and troughs of apparent resistivity curve, where the derivatives (m_i) are equal to zero, formulas (7) and (8) sometimes make $\rho_{z,i}$ smaller and larger than the previous values $\rho_{z,i-1}$, respectively, due to the first term in square brackets. Therefore, at those point, we calculated $\rho_{z,i}$ using the following formulas:

$$\rho_{z,i} = \rho_{z,i-1} + 10^{[lg(\rho_{a,i}) - lg(\rho_{a,i-1})]} \quad \text{for peaks} \quad (10)$$

$$\rho_{z,i} = \rho_{z,i-1} - 10^{[lg(\rho_{a,i-1}) - lg(\rho_{a,i})]} \quad \text{for troughs} \quad (11)$$

This keeps the curves from distortions owing to the derivative changes in (7), (8), so that the positions of maximum and minimum points of transformed curve are the same as apparent curve.

Because the first derivative $m_{i=1}$ is set to be zero, and the apparent resistivity curve is a series of discrete datum points, we calculate derivative of i^{th} point related to its previous point as:

$$m_i = \frac{lg(\rho_{a,i}) - lg(\rho_{a,i-1})}{lg(r_i) - lg(r_{i-1})} \quad (12)$$

In which, i varies from 2 to the end point of the apparent curve. Formulas (12) can be easily defined via Matlab with the function diff.

In next section, we will compare our method with that of (Nguyen, 2016) and (Truong, 2017) who consider the anisotropy of the media at each datum point as:

$$\lambda_i = \begin{cases} \left(1 - \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i\right)^2 when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i \le 0 \\ \left(1 + \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i\right)^2 when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i > 0 \end{cases} \quad (13)$$

(Truong, 2017) used the highest degree (n-1) polynomial as approximating function for a set of n apparent resistivity values in order to calculate the derivative of lg(ρ_a) versus lg(r) in formula (13).

They then used that anisotropy value for the “so-called” N transformation method calculation by the following formulas:

$$z_N|_i = \frac{r}{\lambda}\Big|_i ; \rho_N(z_N)\Big|_i = \begin{cases} \frac{\rho_a}{\lambda}\Big|_i when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i \le 0 \\ \rho_a \lambda|_i when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i > 0 \end{cases} \quad (14)$$

However, the authors had changed the formula for ρ_N in (14), without any mention, to fit the transformed curve with their models as:

$$\rho_N(z_N)|_i = \begin{cases} \frac{\rho_a}{\left(1-\alpha\frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i\right)^2}\Bigg|_i when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i \le 0 \\ \rho_a \left(1 + \alpha \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i\right)^2\Bigg|_i when \ \frac{\partial lg(\rho_a)}{\partial lg(r)}\Big|_i > 0 \end{cases} \quad (15)$$

Here, α is a coefficient which varies from 0.2 to 0.3, depending on the models.

This makes unreasonable results when other applies their transformation method, also requires prior information about resistivity model which is difficult to find down in practical application.

3. Results

We first compared derived results by our method to those of Nguyen (Nguyen, 2016) and Truong (Truong, 2017) in Figure 3. Here, the coefficient α in formula (15) was set equal to 0.2. In figure 3, the blue line is synthetic model, the red line with square markers is synthetic curve calculated by 1D forward problem, the yellow line with triangle markers (Rho_N curve) is the result of Nguyen (Nguyen, 2016) and Truong (Truong, 2017), and the magenta line with circle markers (transformed curve) is our curve transformation result.

It is clearly seen that transformed curves which are calculated by our method are better fit with synthetic models than both Rho_N curves and synthetic curves. In general, most of the transformed curves reach all the peaks and troughs of synthetic models, except the left trough in figure 3c, and remain the similar shape to synthetic curves. In contrast, although Rho_N curves increase the amplitude of anomalies as opposed to synthetic ones, it is undoubted that Rho_N curves have very strange shapes compared to synthetic curves. This distortion is due to the wrong anisotropy (λ) values in equation (13) and the unreasonable interpolation by highest degree polynomial in (Truong, 2017), which made several datum points had smaller depths than their previous points. We will discuss about the interpolation algorithm of (Truong, 2017) in the next part of this section. In order to fix these problems, (Nguyen, 2016; Truong, 2017) might fix an equal depth for those points to confirm the growing values for their transformed depths. However, this was not mentioned in their publications so that no one can use their method and get the same results theirs. Strikingly, the mean sensitivity depth proposed by (Butler, 2016) helped us to solve this problem, and the combination of our proposed formulas and the mean sensitivity depth showed pretty good results for synthetic models in Figure 3. Therefore, we recommend using our method instead of that of (Nguyen, 2016; Truong, 2017) to transform apparent resistivity curve so as to define the depth values and improve the resolution for vertical electrical sounding data.

Figure 4 is the results from model 5 and model 6 in table 1. In this figure, the blue line is synthetic model; the red line with square markers is synthetic curve calculated by 1D forward problem; the yellow line with triangle markers, rhoa (z mean) curve, is apparent resistivity versus mean sensitivity depth; and the magenta line with circle markers is our curve transformation result. It can be seen that the rhoa (z mean) curves

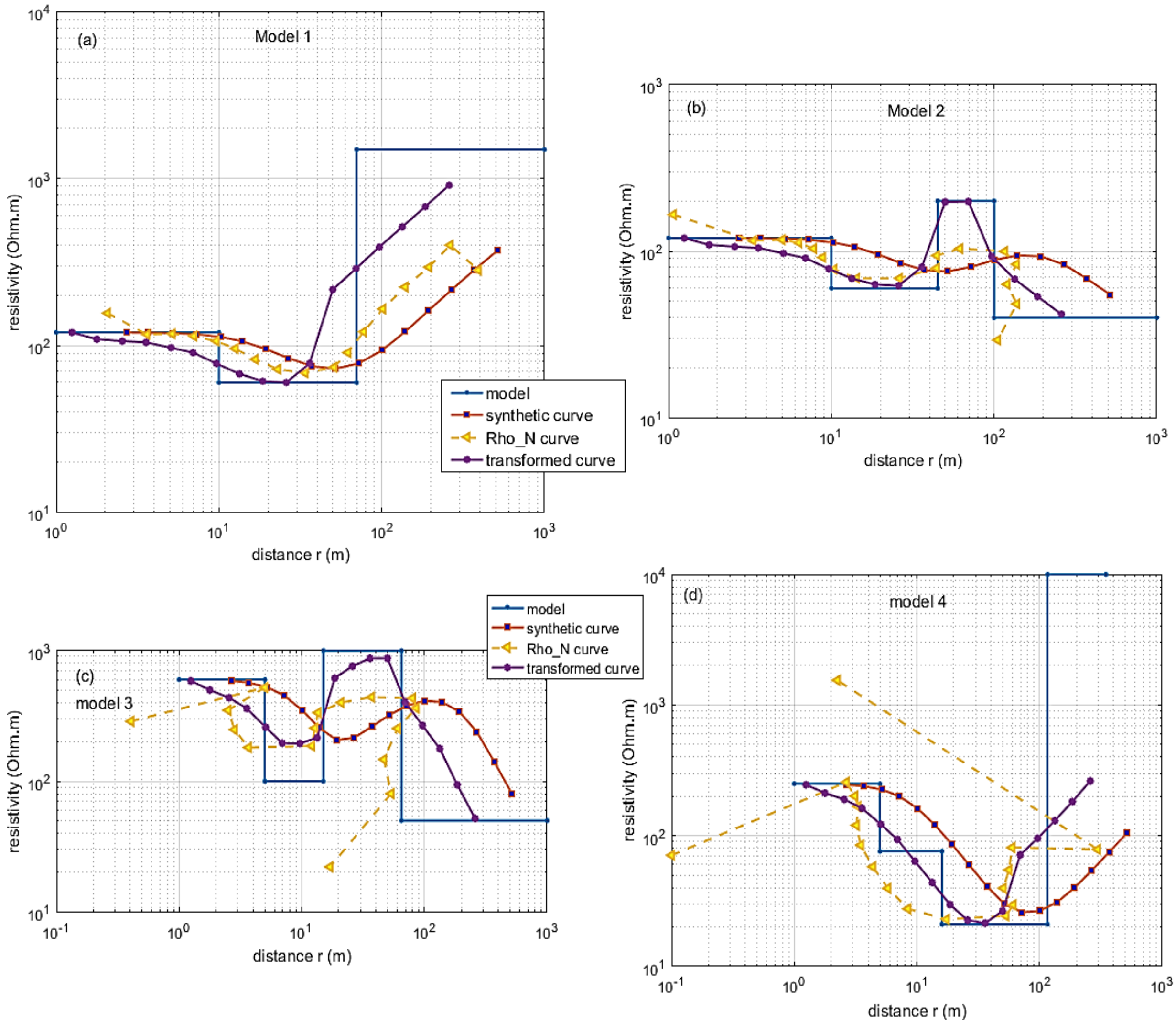

Figure 3. Results for the first four model in table 1. Synthetic curve is calculated from forward problem, Rho_N curve is derived from Nguyen (2016) and Truong (2017) method, transformed curve is the result of our proposed method; (a), (b), (c), (d) are the results for model 1 to model 4, respectively.

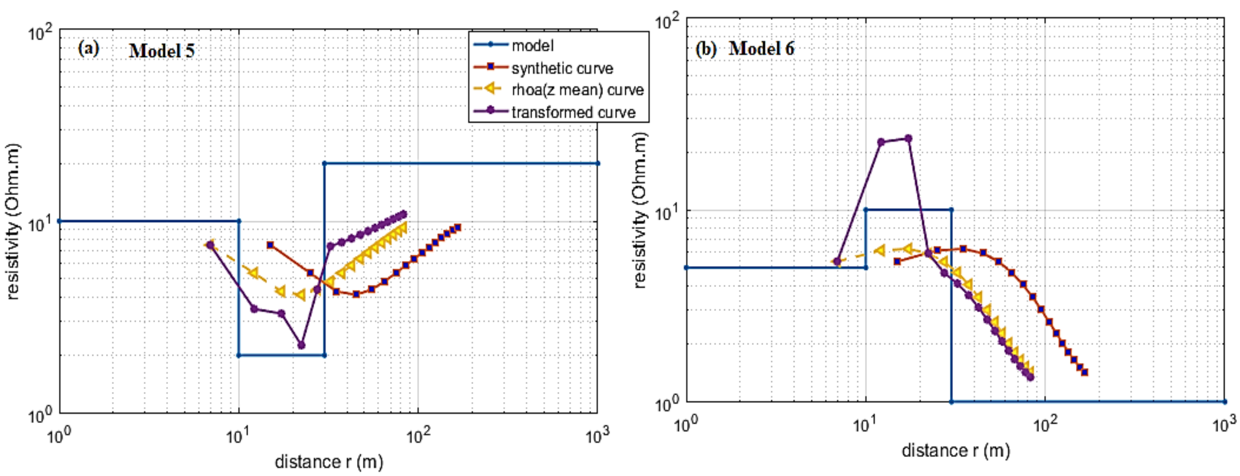

Figure 4. Results for (a) - model 5 and (b) - model 6 in Table 1.

showed better fit with models than synthetic curves. Thus, the mean sensitivity depth gave adequate determination for electrical resistivity method, which had been proved in (Butler, 2016). Besides, transformed curves had resistivity values those are more consistent with synthetic model than apparent resistivity values, except an over-estimation for the peak in model 6. This may be caused by the small resistivity contrast of the upper-most two thin layers, leading to the unreasonable high value for the first term in square brackets in formula (8).

Figure 5 presents synthetic curve from model 4 and its approximation by interpolation algorithm proposed by (Truong, 2017). Because our data had 17 datum points, we conducted the approximation with a 16-degree polynomial function, which is like what the author showed in her paper. It is clearly seen that in spite of fitting with well in each datum point of synthetic curve, approximating curve illustrates two troughes at the beginning and the end points, leading to the wrong derivatives when we use the approximating result to carry out derivation.

To specify, in Table 2, we presented results of derivative values and depths with synthetic data from model 4. In the table, r is the half of the distance between two current electrodes, ρ_a is apparent resistivity, discrete derivative and Z mean are the derivatives calculated by our proposed method and the mean sensitivity depth (Butler, 2016), 16-degree polynomial derivative and Z_N are the derivatives and depths calculated by the method of (Nguyen, 2016; Truong, 2017). It is possible to see that discrete derivatives were consistent with the increase and decrease in apparent resistivity versus distance, while 16-degree polynomial derivative values showed at least four wrong values at the distances of 2.7 m, 51.8 m, 72 m, and 518m, which were bold characters in Table 2. This means that the interpolation algorithm in (Truong, 2017) caused errors when using it to carry out the derivation of synthetic data. Thus, if we still want to fit observed data as a function of high-degree polynomial, it should be done carefully. Moreover, there were four wrong depths Z_N, in Table 2, which were larger than the maximum depths ($r/2$). We presented those values as bold characters. Meanwhile, Z mean values were always smaller than the corresponding maximum depths, increasing gradually versus distance (r). Theoretically speaking, the investigation depth is smaller than or equal to $r/2$ depending on the anisotropy of the earth (Nguyen, 1996). Therefore, the Z_N values that have been mentioned above do not make any geological sense. In addition to this, we can easily find out several Z_n values which are smaller than their previous values in the table, it is certainly wrong because the larger distance r the bigger investigation depth. However, (Nguyen, 2016; Truong, 2017) has fixed these values by their conditions which had not been appeared in the publications.

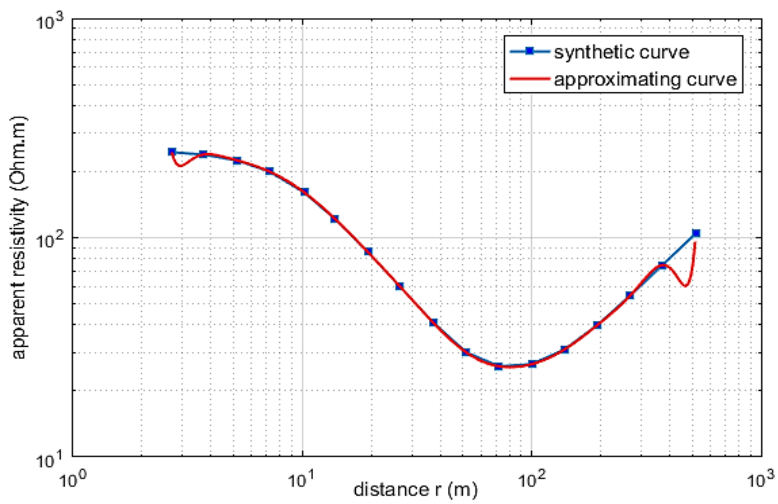

Figure 5. Synthetic curve approximation as 16-degree polynomial function for the data from model 4.

Table 2. Results of derivative values and depths with synthetic data from model 4.

r (m)	ρ_a (Ω.m)	Discrete derivative	16-Degree polynomial derivative	Z_N (m)	$r/2$ (m)	Z mean (m)
2.7	245.49	0	0.7158	0.1	1.35	1.248
3.7	239.39	-0.0799	-0.0617	2.66	1.85	1.776
5.2	225.24	-0.179	-0.0229	3.117	2.6	2.548
7.2	200.2	-0.3621	-0.0675	3.308	3.6	3.562
10.2	160.74	-0.6303	-0.1476	3.213	5.1	5.073
13.9	121.76	-0.8974	-0.258	3.485	6.95	6.931
19.3	86.023	-1.0586	-0.3738	4.375	9.65	9.636
26.7	59.737	-1.1236	-0.3934	5.805	13.35	13.34
37.2	40.951	-1.1385	-0.2407	8.47	18.6	18.593
51.8	30	-0.9399	0.0496	17.423	25.9	25.895
72	25.928	0	0.3885	53.189	36	35.996
100	26.532	0.0701	0.6874	61.014	50	49.997
139	30.99	0.4716	0.853	50.704	69.5	69.498
193	39.984	0.7764	0.8934	56.375	96.5	96.499
267	54.281	0.9419	0.8573	59.735	133.5	133.499
370	75.03	0.9922	0.932	297.786	185	184.999
518	105.03	0.9997	-0.7435	2.244	259	258.999

4. Conclusion and discussion

The so-called "N transformation method" of (Nguyen, 2016) is a mathematical data processing procedure. Because of the inexact anisotropy values (Nguyen, 2016) and inadequate interpolation algorithm (Truong, 2017), the derived results from that data processing method showed a lot of errors, distortions and sometimes do not make any geological sense.

Therefore, we proposed another approach to calculate the derivatives for vertical electrical sounding data and the transformed resistivity values by some simple mathematical formulas. With the help of mean sensitivity depth (Butler, 2016), our method showed much more adequate results than that of (Nguyen, 2016; Truong, 2017). These formulas can be used for education and give additional information for interpretation. Although, the results derived from our method were pretty good, there were some transformed resistivity values did not match well with the model. Therefore, we still need to carry out further research to solve this problem.

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