A Robust Density-Based Approach to Production Data Analysis of Oil/Water Multiphase Flow System

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This paper presents a novel approach to estimate reserves of oil and water reservoirs undergoing boundary-dominated flow conditions in a simplified yet accurate manner. The methodology incorporates rescaled density-based exponential models and is based on the coupling of two-phase oil and water material balances with multiphase well deliverability equations. Current multiphase production data analysis methods employed for reserve calculations, including density-based approach, are subjected to the determination of saturation–pressure relationship, multiphase pseudo-pressure, and pseudo-time, as well as the iterative nature of its own algorithm. The herein proposed approach circumvents the need for pseudo-variables calculations, thus precluding the determination of saturation–pressure relationship and removing the iterative nature often present in state-of-the-art approaches. The proposed model is validated by comparing its predictions to numerical models with both constant and variable bottomhole pressure constrains, and has been found to match closely. For all cases, relative errors are found to be less than 1%.
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1 Introduction

Production data analysis (PDA) is one of the most widely used tools in reservoir engineering to forecast future production and estimate reserve/properties of hydrocarbon fields by analyzing available production data. The development of PDA techniques started with the work of Arps [1], where a set of empirical equations was identified to characterize the decline behavior of oil and gas wells as exponential, hyperbolic, or harmonic declination. Ayeni [2] developed the estimate equations for non-linear parameters in hyperbolic decline curve analysis. Fetkovich [3] introduced his well performance analysis type curves, pioneered modern PDA by combining analytical solution for the well deliverability equations and Arps' relations. For the past five decades, considerable effort has been made to develop analytical solutions for the flow equations of greater challenging systems such as gas reservoir and multiphase environment.

For the analysis of gas reservoir, the strongly pressure-dependent fluid properties render substantial nonlinearities for the governing fluid flow equation. To account for these nonlinearities effect in a boundary-dominated flow (BDF) reservoir under constant bottomhole pressure (BHP), several mathematical transformation concepts were proposed: pseudo-pressure [4], pseudo-time [5,6], and viscosity compressibility product [7]. In extension to a more practical application of variable BHP, the concepts of normalized pseudo-pressure and material balance pseudo-time were introduced by Blasingame and Lee [8] and later derived by Palacio and Blasingame [9]. These types of reservoir predictions are within the analytical category, while there are others that can use artificial neural networks [10,11].

Rescaled density-based exponential models have been extensively employed as efficient analytical techniques for decline curve analysis and reserve estimation in complex reservoir systems. This methodology was first introduced by Ayala and Ye [12] and demonstrated the ability to describe the decline of liquid exponential production data using a dimensionless term, $\bar{\lambda}$, and its time-averaged evolution, $\bar{\beta}$, to account for the effects of depletion on single-phase gas reservoirs producing under constant BHP conditions. Zhang and Ayala [13] refined the methodology by proposing an updated rescaled exponential solution that incorporated a more comprehensive and rigorously defined parameter, $\bar{\lambda}$. This solution was able to effectively analyze the boundary-dominated production data of single-phase dry gas reservoirs, demonstrating the versatility and robustness of the rescaled exponential models in characterizing complex hydrocarbon production systems. It has also been adapted to analyze production data under variable bottomhole pressure conditions. Additionally, straight line analysis methods were developed using the rescaled exponential and
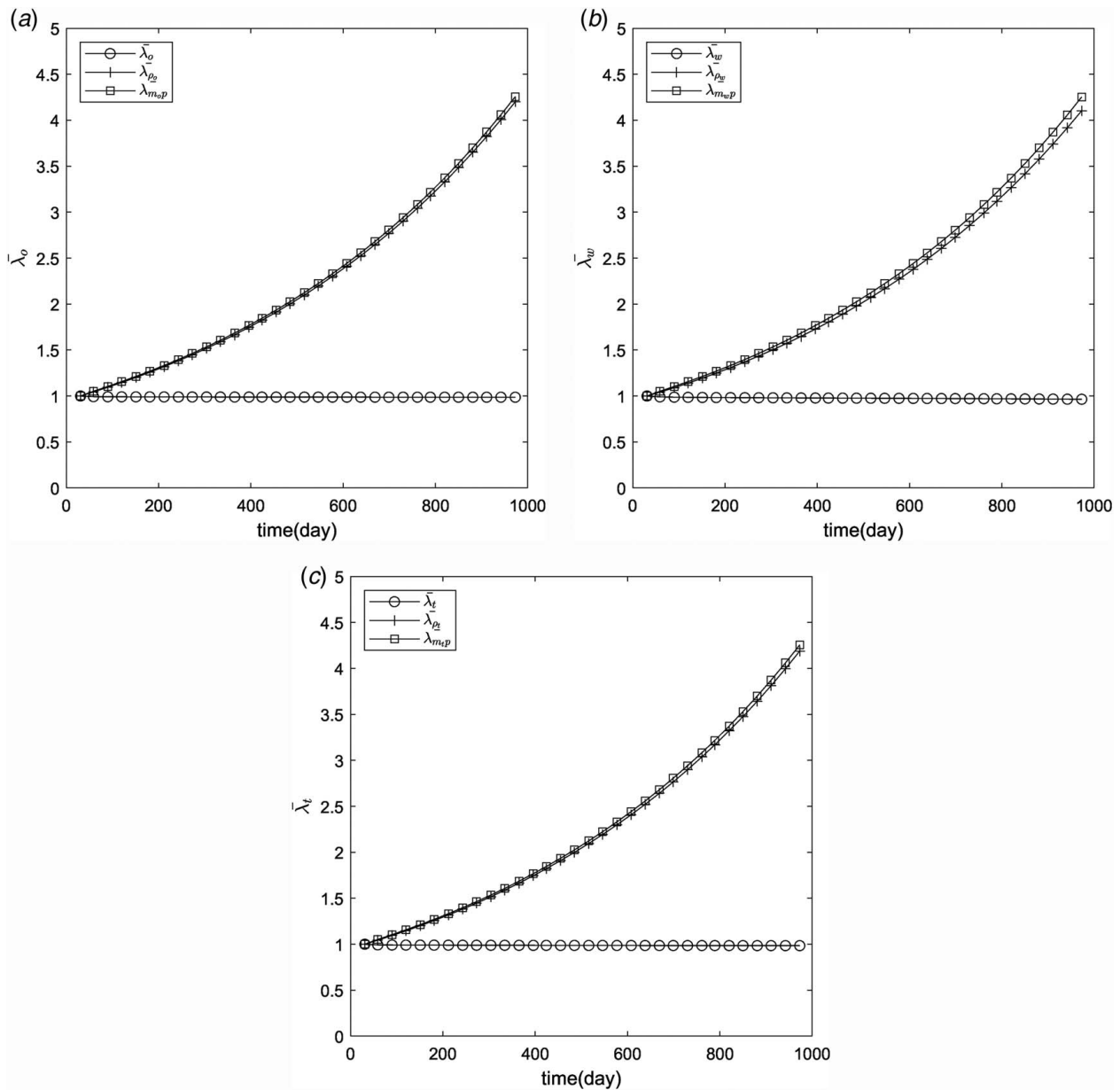
Fig. 1 $\bar{\lambda}$ behavior of (*a*) oil component, (*b*) water component, and (*c*) total component—constant BHP case

density-based material balance equations to estimate reserves from field production data. These studies support the use of rescaling parameters $\bar{\lambda}$ and $\bar{\beta}$ to incorporate gas flow nonlinearities into exponential decline models and demonstrate the potential for application in complex multiphase systems.

The development of effective tools for multiphase BDF analysis has been more challenging than for single-phase gas reservoirs. This is due to the more complex nonlinearities that arise in the governing equations as a result of factors such as relative permeability, mass transfer between phases, and changes in phase composition and properties. Traditionally, solving the multiphase problem requires the use of a saturation–pressure correlation, which is derived from laboratory data such as constant volume depletion and constant composition expansion experiments or wellbore conditions from producing well stream compositions [14–16]. Some recent PDA models have used the well deliverability equation proposed by Fevang and Whitson [17] and have analyzed gas-condensate production data using two-phase pseudo-variables [18,19]. Rescaled density-based modeling approach has been also extended to multiphase gas-condensate reservoirs [20,21]. Like other approaches, it

also requires additional closure relationships such as prior knowledge of the saturation–pressure relationship.

Evaluation of current research suggests that it can be challenging to determine a reliable relationship between saturation and pressure without utilizing data from production wells and laboratory experiments. In fact, multiphase PDA models may be not applicable if data on well stream composition and laboratory results are not available. Raghavan [22] argues that in the case of saturated oil–gas systems, the saturation–pressure relationship developed by Muskat [23] can be used in multiphase systems when saturation gradients are not significant. This assumption is based on the idea that capillary effects in the multiphase system are negligible and there is no fluid intrusion or injection. The relationship connects saturation and pressure changes using fluid compressibility, mobility, and average saturation. Recently, Sun and Ayala [24] adopted Muskat's saturation–pressure method and proposed an extended density-based model that can estimate reserves for two-phase oil/water systems without prior knowledge of producing well composition or laboratory pressure-volume-temperature (PVT) data. The model is based on multiphase pressure
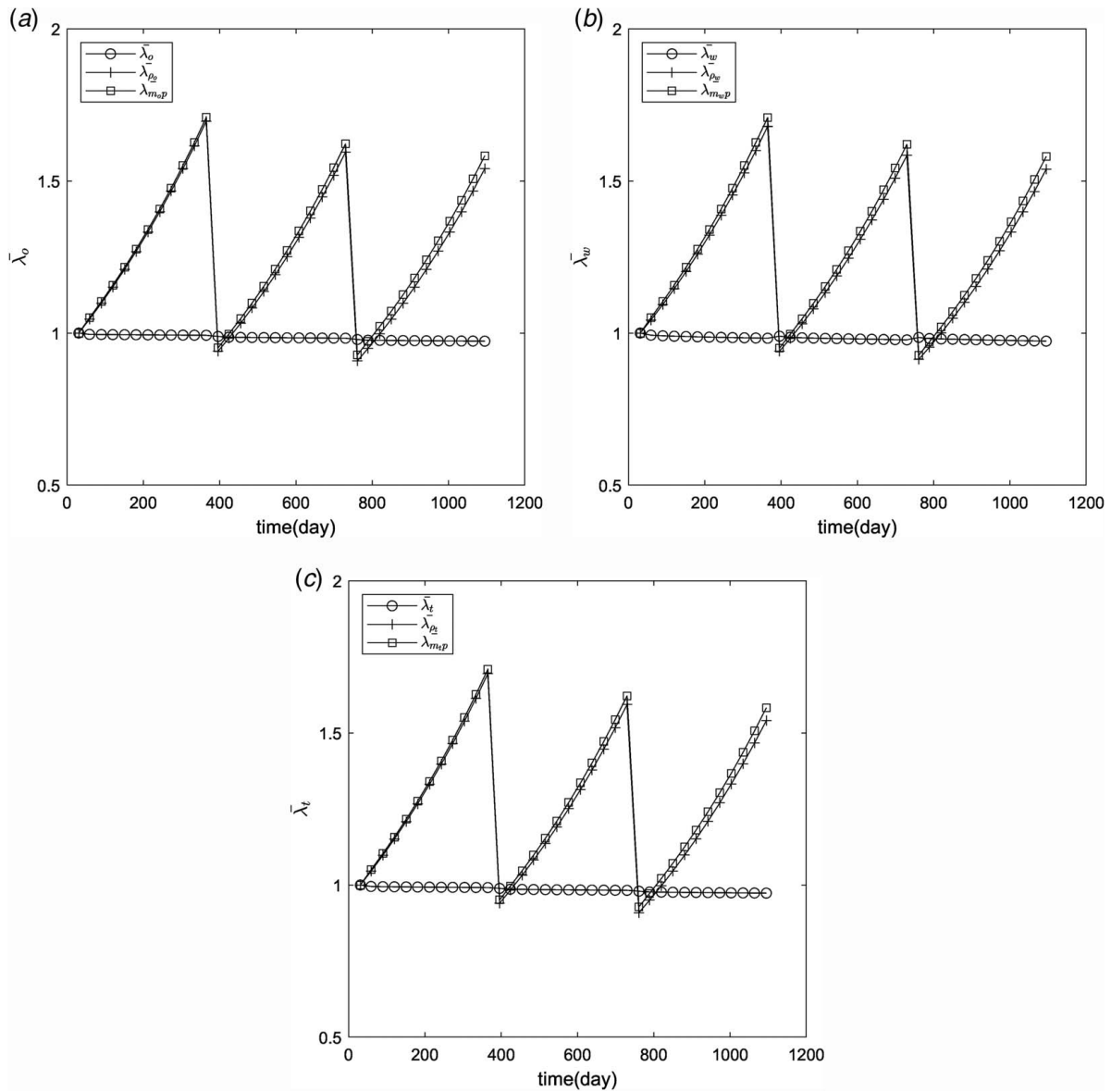
Fig. 2 $\bar{\lambda}$ behavior of (*a*) oil component, (*b*) water component, and (*c*) total component—variable BHP case

calculations, thus requiring not only the saturation–pressure path but also average reservoir pressure, which results in a iterative-based approach. In this study, we propose a new and improved density-based approach independent of pseudo-pressure calculation. It is introduced for the first time a simple yet accurate approach that does not require saturation–pressure relationship nor average reservoir pressure, and iterationless.

Table 1 Reservoir input parameters

	Constant BHP	Variable BHP	
T	120	150	°F
p_i	3000	5000	psi
r_e	1500	2500	ft
r_w	0.2	0.2	ft
h	50	50	ft
K	10	30	md
Φ	0.2	0.2	Fraction
S_{wi}	0.3	0.3	Fraction
μ_o	1	1	cp
μ_w	2	2	cp
p_{wf}	1000	3000/2000/1000	psi

2 Mathematical Formulation

This section introduces the development of density-based formulation for an oil and water reservoir system. Following the blackoil modeling approach, the generalized governing flow equation for a surface component "I" (i.e., surface water and surface oil) can be written as

$$\nabla \cdot (\gamma_I(p)\nabla p) = \frac{1}{V_b k} \frac{\partial}{\partial t}(M_I) \qquad (1)$$

where the reservoir is assumed incompressible with homogenous and isotropic property distribuiton. In this study, oil and water are assumed immiscible (slightly compressible) fluids, therefore, each phase composition is entirely formulated by the original component. Capillary pressure is not considered, since our focus is on analyzing primary depletion drawdown effects caused by the multiphase flow nature of the system. Here, γ_I is defined as the mobility of surface component "I"; V_b is the bulk volume; k is the absolute permeability; and M_I is the mass-in-place of component "I." The mobility and mass-in-place of oil and water component are defined as $\gamma_o = \rho_o(k_{ro}/\mu_o)$, $M_o = V_b\phi(\rho_o S_o)$ and $\gamma_w = \rho_w(k_{rw}/\mu_w)$, $M_w = V_b\phi(\rho_w S_w)$, respectively. Surface oil and surface water
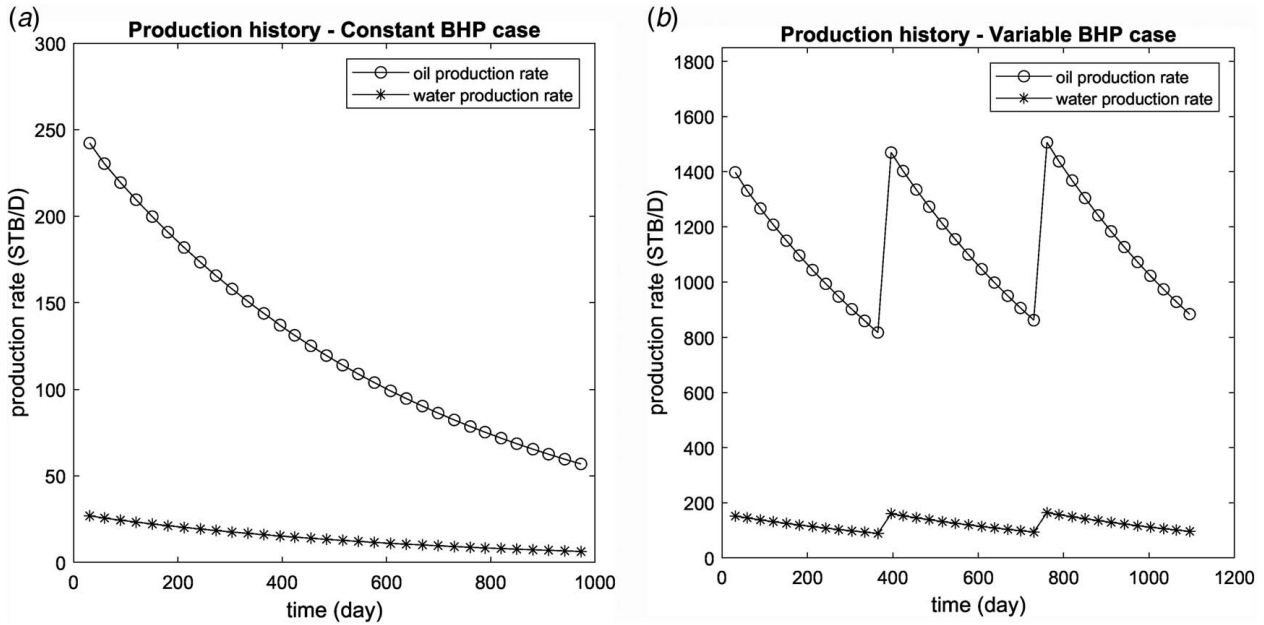
Fig. 3 Production history: (*a*) constant BHP case and (*b*) variable BHP case

components can be combined to express the total mobility and mass-in-place of all components in the system: $\gamma_t = \rho_o(k_{ro}/\mu_o) + \rho_w(k_{rw}/\mu_w)$ and $M_t = V_b \phi(\rho_o S_o + \rho_w S_w)$.

Based on the abovementioned assumptions and definitions, the governing flow equations for oil, water, and total components are

$$\nabla \cdot \left(\rho_o \frac{k_{ro}}{\mu_o} \nabla p \right) = \frac{\phi}{k} \frac{\partial}{\partial t} (\rho_o S_o) \tag{2}$$

$$\nabla \cdot \left(\rho_w \frac{k_{rw}}{\mu_w} \nabla p \right) = \frac{\phi}{k} \frac{\partial}{\partial t} (\rho_w S_w) \tag{3}$$

$$\nabla \cdot \left(\left(\rho_o \frac{k_{ro}}{\mu_o} + \rho_w \frac{k_{rw}}{\mu_w} \right) \nabla p \right) = \frac{\phi}{k} \frac{\partial}{\partial t} (\rho_o S_o + \rho_w S_w) \tag{4}$$

The corresponding well deliverability equations for such system described above could be expressed in terms of mass rate using multiphase pseudo-pressure:

$$\dot{m}_o = \frac{2\pi k h \alpha_c}{b_{D,\text{PSS}}} [m_o(\overline{p}) - m_o(p_{wf})] \tag{5}$$

$$\dot{m}_w = \frac{2\pi k h \alpha_c}{b_{D,\text{PSS}}} [m_w(\overline{p}) - m_w(p_{wf})] \tag{6}$$

$$\dot{m}_t = \frac{2\pi k h \alpha_c}{b_{D,\text{PSS}}} [m_t(\overline{p}) - m_t(p_{wf})] \tag{7}$$

where the multiphase pseudo-pressure for oil, water, and total component is defined as $m_o(\overline{p}) - m_o(p_{wf}) = \int_{p_{wf}}^{\overline{p}} \gamma_o \, dp$,
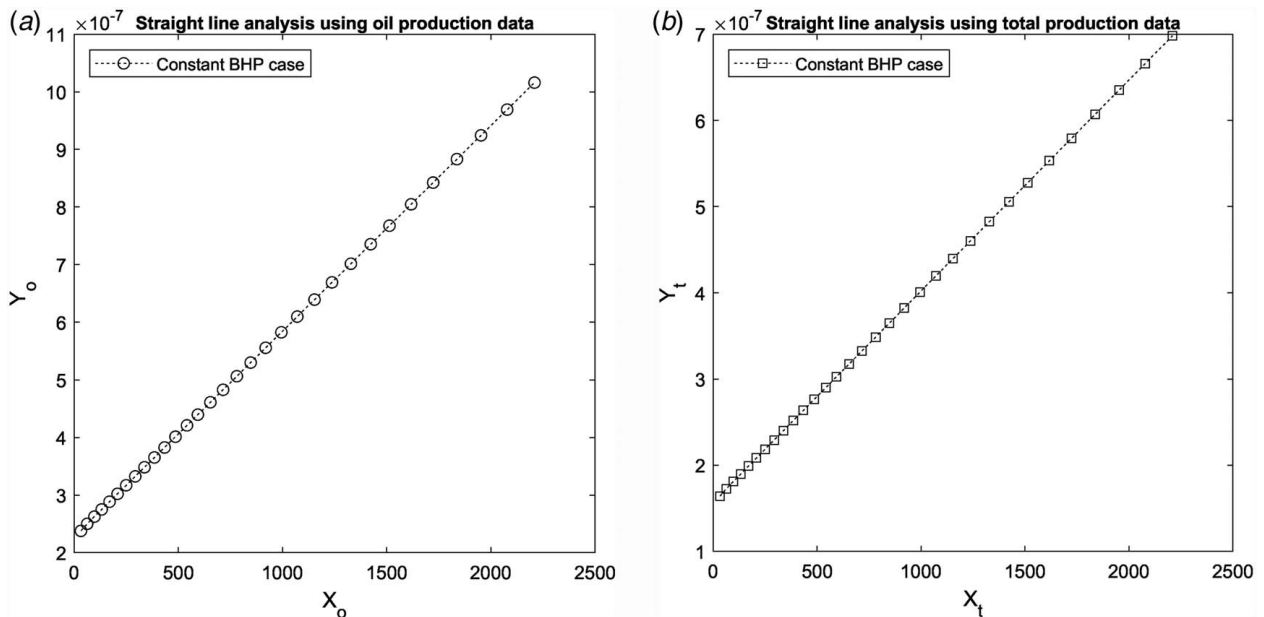

Fig. 4 Straight line analysis for constant BHP case using (*a*) oil production data and (*b*) total production data

$m_w(\overline{p}) - m_w(p_{wf}) = \int_{p_{wf}}^{\overline{p}} \gamma_w dp$, and $m_t(\overline{p}) - m_t(p_{wf}) = \int_{p_{wf}}^{\overline{p}} \gamma_t dp$,

respectively; $\overline{p}$ and p_{wf} are the average reservoir and bottomhole flowing pressure, respectively; h is the reservoir thickness; $\alpha_c = 0.00633$ is the unit conversion in field unit system ($\alpha_c = 1$ for SI unit system); $b_{D,\text{PSS}} = \ln(r_e/r_w) - (3/4)$ is the pseudo steady-state component for radial reservoir.

Following the procedure from Ref. [13], the development for the presenting density-based model starts with defining the dimensionless depletion-dependent parameters $\overline{\lambda}_o$, $\overline{\lambda}_w$, and $\overline{\lambda}_t$ for oil, water, and total component, respectively:

$$\overline{\lambda}_o = \frac{\mu_o^* c_o^*|_{\text{reference}}}{\mu_o^* c_o^*|_{\overline{p}}}$$
$$= \frac{(\rho_{o,\overline{p}}^* - \rho_{o,p_{wf}}^*|_{\text{reference}})/(\rho_{o,\overline{p}}^* - \rho_{o,p_{wf}}^*)}{(m_o(\overline{p}) - m_o(p_{wf})|_{\text{reference}})/(m_o(\overline{p}) - m_o(p_{wf}))} = \frac{\overline{\lambda}_{\rho_o}}{\overline{\lambda}_{m_o(p)}} \quad (8)$$

$$\overline{\lambda}_w = \frac{\mu_w^* c_w^*|_{\text{reference}}}{\mu_w^* c_w^*|_{\overline{p}}}$$
$$= \frac{(\rho_{w,\overline{p}}^* - \rho_{w,p_{wf}}^*|_{\text{reference}})/(\rho_{w,\overline{p}}^* - \rho_{w,p_{wf}}^*)}{(m_w(\overline{p}) - m_w(p_{wf})|_{\text{reference}})/(m_w(\overline{p}) - m_w(p_{wf}))} = \frac{\overline{\lambda}_{\rho_w}}{\overline{\lambda}_{m_w(p)}} \quad (9)$$

$$\overline{\lambda}_t = \frac{\mu_t^* c_t^*|_{\text{reference}}}{\mu_t^* c_t^*|_{\overline{p}}}$$
$$= \frac{(\rho_{t,\overline{p}}^* - \rho_{t,p_{wf}}^*|_{\text{reference}})/(\rho_{t,\overline{p}}^* - \rho_{t,p_{wf}}^*)}{(m_t(\overline{p}) - m_t(p_{wf})|_{\text{reference}})/(m_t(\overline{p}) - m_t(p_{wf}))} = \frac{\overline{\lambda}_{\rho_t}}{\overline{\lambda}_{m_t(p)}} \quad (10)$$

where c^*, μ^*, and ρ^* are the equivalent compressibility, viscosity, and density functions defined for each surface component present in a two-phase system, with the subscripts "o," "w," and "t" represent oil, water, and total component: $c_o^* = (1/\rho_o^*)((\rho_{o,\overline{p}}^* - \rho_{o,p_{wf}}^*)/(\overline{p} - p_{wf}))$, $c_w^* = (1/\rho_w^*)((\rho_{w,\overline{p}}^* - \rho_{w,p_{wf}}^*)/(\overline{p} - p_{wf}))$, $c_t^* = (1/\rho_t^*)((\rho_{t,\overline{p}}^* - \rho_{t,p_{wf}}^*)/(\overline{p} - p_{wf}))$, $\mu_o^*/\rho_o^* = (\overline{p} - p_{wf})/(m_o(\overline{p}) - m_o(p_{wf}))$, $\mu_w^*/\rho_w^* = (\overline{p} - p_{wf})/(m_w(\overline{p}) - m_w(p_{wf}))$, $\mu_t^*/\rho_t^* = (\overline{p} - p_{wf})/(m_t(\overline{p}) - m_t(p_{wf}))$, $\rho_o^* = S_o\rho_o$, $\rho_w^* = S_w\rho_w$, and $\rho_t^* = S_o\rho_o + S_w\rho_w$. In these equations, the $|_{\text{reference}}$ label is designated to the onset of the BDF period. Most importantly, the definition of $\overline{\lambda}$ in this work is rearranged into the ratio of $\overline{\lambda}_\rho/\overline{\lambda}_{m(p)}$. This newly proposed arrangement enables rigorous assessment of $\overline{\lambda}$ behavior by separately tracking the influences of density and pseudopressure drawdown effects: $\overline{\lambda}_\rho = (\rho_{\overline{p}}^* - \rho_{p_{wf}}^*|_{\text{reference}})/(\rho_{\overline{p}}^* - \rho_{p_{wf}}^*)$ and $\overline{\lambda}_{m(p)} = (m(\overline{p}) - m(p_{wf})|_{\text{reference}})/(m(\overline{p}) - m(p_{wf}))$.

Upon substituting the definition of $\overline{\lambda}$ in Eqs. (8)–(10) into Eqs. (5)–(7), the mass flowrate of surface oil, water, and total component in a two-phase oil/water system can be obtained from the following density-based rate equations:

$$\dot{m}_o = \frac{2\pi kh}{b_{D,\text{PSS}}\mu_o^* c_o^*|_{\text{reference}}} \overline{\lambda}_o(\rho_{o,\overline{p}}^* - \rho_{o,p_{wf}}^*) \quad (11)$$

$$\dot{m}_w = \frac{2\pi kh}{b_{D,\text{PSS}}\mu_w^* c_w^*|_{\text{reference}}} \overline{\lambda}_w(\rho_{w,\overline{p}}^* - \rho_{w,p_{wf}}^*) \quad (12)$$

$$\dot{m}_t = \frac{2\pi kh}{b_{D,\text{PSS}}\mu_t^* c_t^*|_{\text{reference}}} \overline{\lambda}_t(\rho_{t,\overline{p}}^* - \rho_{t,p_{wf}}^*) \quad (13)$$

This study invokes the density-based material balance formulation, originally proposed by Zhang et al. [20] for liquid-rich gas systems, which has been validated for oil/water system by Sun and Ayala [24]:

$$\rho_{o,\overline{p}}^* = \rho_{o,\text{initial}}^* \left(1 - \frac{M_{p,o}}{\text{OMIP}_o}\right) \quad (14)$$

$$\rho_{w,\overline{p}}^* = \rho_{w,\text{initial}}^* \left(1 - \frac{M_{p,w}}{\text{OMIP}_w}\right) \quad (15)$$

$$\rho_{t,\overline{p}}^* = \rho_{t,\text{initial}}^* \left(1 - \frac{M_{p,t}}{\text{OMIP}_t}\right) \quad (16)$$

where $M_{p,o}$, $M_{p,w}$, and $M_{p,t}$ are the cumulative mass production; OMIP_o, OMIP_w, and OMIP_t are original mass in-place of oil, water, and total component at standard condition, respectively.

For reserve estimation purpose, Eqs. (14)–(16) can be substituted into Eqs. (11)–(13) to take the form of the straight line equations below:

$$\overline{\lambda}_o \frac{r_{\rho_o^*}}{\dot{m}_o} = \frac{1}{\text{OMIP}_o}\left(\overline{\lambda}_o \frac{M_{p,o}}{\dot{m}_o}\right) + \frac{1}{J_o^*} \quad (17)$$

$$\overline{\lambda}_w \frac{r_{\rho_w^*}}{\dot{m}_w} = \frac{1}{\text{OMIP}_w}\left(\overline{\lambda}_w \frac{M_{p,w}}{\dot{m}_w}\right) + \frac{1}{J_w^*} \quad (18)$$

$$\overline{\lambda}_t \frac{r_{\rho_t^*}}{\dot{m}_t} = \frac{1}{\text{OMIP}_t}\left(\overline{\lambda}_t \frac{M_{p,t}}{\dot{m}_t}\right) + \frac{1}{J_t^*} \quad (19)$$

where $r_{\rho_o^*}$, $r_{\rho_w^*}$, and $r_{\rho_t^*}$ are the equivalent density drawdown ratio of oil, water, and total component, respectively: $r_{\rho_o^*} = 1 - (\rho_{o,p_{wf}}^*/\rho_{o,\text{initial}}^*)$, $r_{\rho_w^*} = 1 - (\rho_{w,p_{wf}}^*/\rho_{w,\text{initial}}^*)$, $r_{\rho_t^*} = 1 - (\rho_{t,p_{wf}}^*/\rho_{t,\text{initial}}^*)$; J_o^*, J_w^*, and J_t^* are the density-based well productivity indexes: $J_o^* = 2\pi kh\rho_{o,\text{initial}}^*/b_{D,\text{PSS}}\mu_o^* c_o^*|_{\text{reference}}$, $J_w^* = 2\pi kh\rho_{w,\text{initial}}^*/b_{D,\text{PSS}}\mu_w^* c_w^*|_{\text{reference}}$, and $J_t^* = 2\pi kh\rho_{t,\text{initial}}^*/b_{D,\text{PSS}}\mu_t^* c_t^*|_{\text{reference}}$.

As dictated by Eq. (17), plotting $\overline{\lambda}_o(r_{\rho_o^*}/\dot{m}_o)$ versus $\overline{\lambda}_o(M_{p,o}/\dot{m}_o)$ on the Cartesian coordinate system renders a straight line with the slope of $1/\text{OMIP}_o$ and allows obtaining reserve-in-place from the reciprocal of the slope. Similar analysis could be performed using Eq. (18) to calculate OMIP_w or Eq. (19) to calculate OMIP_t.

3 Proposed Approach

The model presented in Sec. 2 is derived in a specific manner such that the predictive capabilities of the straight line equations (Eqs. (17)–(19)) are revolved around the determination of $\overline{\lambda}$. As a result, the calculation of $\overline{\lambda}_\rho$ and $\overline{\lambda}_{m(p)}$ (Eqs. (8)–(10)) requires both average reservoir pressure and a saturation–pressure relationship. Conventionally, the average pressure calculation is performed by solving one of the tank material balance equations (Eqs. (14)–(16)) using the appropriate values of reserve (OMIP$_o$, OMIP$_w$, and OMIP$_t$). Thus, the inverse analysis to estimate the original fluid in place must employ an iterative protocol to simultaneously solve the tank material balance while updating the reserve values at every iteration upon convergence. The iterative procedure was discussed by Sun and Ayala [24]. Furthermore, multiple assumptions (such as constant liquid viscosity compressibility product, negligible variation of relative permeability) need to be taken into account in order to establish a saturation–pressure relationship for the pseudo-pressure calculation. We detailed the iterative protocol for a rigorous inverse analysis in Appendix C.

In this study, we propose a simplified yet accurate implementation of Eqs. (17)–(19) based on careful examination pertaining the behavior of the dimensionless depletion-dependent parameter $\overline{\lambda}$. It is observed that in oil and water systems, $\overline{\lambda}$ follows a predictable pattern for both constant and variable BHP constraints. A set of numerical simulation studies have been conducted with the aim of illustrating the behavior of $\overline{\lambda}$. Details about the numerical simulation inputs/settings are given in the next section. The results from these numerical studies, such as average reservoir presure and saturation–pressure profile, are used to compute the value of $\overline{\lambda}$ as the ratio of $\overline{\lambda}_\rho/\overline{\lambda}_{m(p)}$. Figures 1 and 2 showcase the transient profiles of $\overline{\lambda}$, $\overline{\lambda}_\rho$, and $\overline{\lambda}_{m(p)}$ for constant BHP case and variable BHP cases, respectively. Inspection of both figures reveals that while the values of
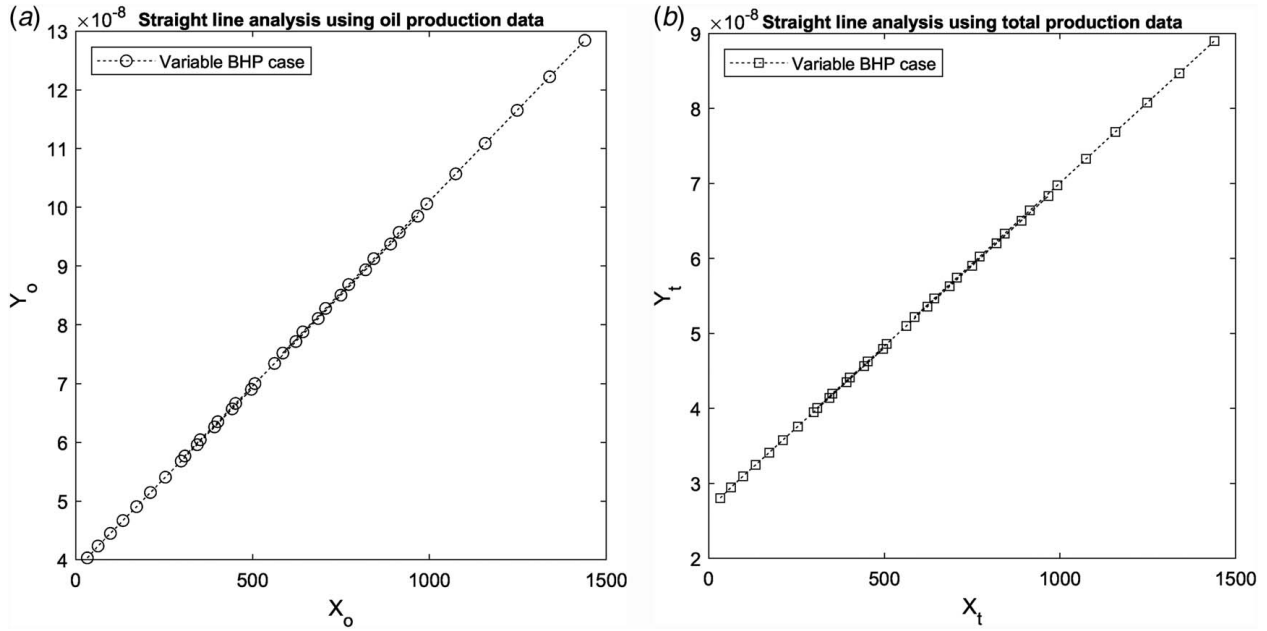
Fig. 5 Straight line analysis for variable BHP case using (*a*) oil production data and (*b*) total production data

Table 2 Estimated reserves for constant BHP and variable BHP case

	Mass-in-place (lb)	Numerical model	Estimated	Error
Constant BHP case	$OMIP_o$	2.787×10^9	2.802×10^9	0.54%
	$OMIP_w$	1.323×10^9	1.330×10^9	0.50%
	$OMIP_t$	4.110×10^9	4.084×10^9	0.63%
Variable BHP case	$OMIP_o$	1.580×10^{10}	1.595×10^{10}	0.97%
	$OMIP_w$	7.355×10^9	7.426×10^9	0.96%
	$OMIP_t$	2.315×10^{10}	2.307×10^{10}	0.33%

$\bar{\lambda}_\rho$ and $\bar{\lambda}_{m(p)}$ vary significantly with time, both variables shift in a synchronized manner. Consequently, $\bar{\lambda}$ maintains a constant value versus time (or relatively constant in variable BHP case) as the undeviating ratio of $\bar{\lambda}_\rho / \bar{\lambda}_{m(p)}$. The insight of a constant $\bar{\lambda}$ behavior in oil and water system leads to a beneficial realization for the reserve estimation practice: without calculating or iterating on $\bar{\lambda}$ (or by simply supposing $\bar{\lambda} = 1$), the straight line analysis could be performed in a straightforward and robust manner by plotting $r_{\rho^*}/\dot{m}$ versus $M_p/\dot{m}$.

4 Benchmarking Study

To showcase and validate the proposed approach, a benchmarking study against numerical simulation is performed. The numerical modeling for a two-phase oil and water system following a 1D radial geometry is implemented in IMEX-CMG, a commercial blackoil simulator. The first case study represents a constant BHP application while the second case renders a variable BHP application. Reservoir input parameters are described in Table 1. Fluid properties and relative permeability data are showed in Appendix A, as well as the BHP schedules for the variable BHP application. The predicted rate behavior for both cases are shown in Fig. 2.

The proposed approach was implemented to analyze the production history of constant and variable BHP systems, displayed in Fig. 3. Figures 4 and 5 represent the dataset linearized based on the straight line approach previously discussed in Sec. 2. The analysis is performed based on oil and total production rates. Following the density-based methodology, the original mass in-place of oil component $OMIP_o$ and total component $OMIP_t$ are estimated independently by plotting $Y_o = r_{\rho_o^*}/\dot{m}_o$ versus $X_o = M_{p,o}/\dot{m}_o$ using oil production data and $Y_t = r_{\rho_t^*}/\dot{m}_t$ versus $X_t = M_{p,t}/\dot{m}_t$ using total production data, respectively. The original mass in-place of water component $OMIP_w$ is calculated using the volumetric relationship (Eq. (B5)). Table 2 summarizes the comparative study results for both constant and variable bottomhole pressure constraints. The estimated mass-in-place (or reserves) is compared against the true values obtained from the simulator. An excellent agreement against numerical simulation data is observed, with errors kept at less than 1%. The satisfactory matching indicates that the analysis of oil and water systems can be greatly simplified through the realization that $\bar{\lambda}$ is nearly constant. The advantages associated with such a finding are the independence of the proposed inverse analysis technique on information regarding saturation–pressure relationship and average reservoir pressure, and the avoidance of an iterative scheme.

5 Concluding Remarks

This study introduces a revised version of the density-based formulation, originally developed for the analysis of dry gas and gas-condensate reservoirs, to model oil and water systems. By identifying a predictable pattern in the drawdown behavior of oil and water systems, we propose an effective approach for inverse analysis that eliminates the need for iterative procedures. Our method is validated through numerical simulations with both constant and variable bottomhole pressure applications, demonstrating reliable reserve estimation without the need for traditional computations of average reservoir pressure, saturation–pressure relationships, and multiphase pseudo-pressure. The results indicate a straightforward and robust approach to reserve estimation for oil and water systems.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The authors attest that all data for this study are included in the paper.

Nomenclature

h = formation thickness (ft)
k = absolute permeability (md)
p = pressure (psi)
$\overline{p}$ = average reservoir pressure (psi)
t = time (days)
A = reservoir drainage area (ft^2)
$b_{D,\text{PSS}}$ = pseudo-steady-state component
k_{ro} = relative permeability of oleic phase
k_{rw} = relative permeability of aqueous phase
$\dot{m}_o$ = mass production rate of oil component (lb/day)
$\dot{m}_t$ = mass production rate of total component (lb/day)
$\dot{m}_w$ = mass production rate of water component (lb/day)
p_{wf} = bottomhole pressure (psi)
q_{osc} = oil production rate at standard condition (14.7 psi, 60 °F) = STB/day
q_{wsc} = water production rate at standard condition (14.7 psi, 60 °F) = STB/day
r_e = radius of the outer boundary of the system (ft)
r_w = wellbore radius (ft)
$r_{\rho_o^*}$ = equivalent density drawdown ratio of oil component
$r_{\rho_t^*}$ = equivalent density drawdown ratio of total component
$r_{\rho_w^*}$ = equivalent density drawdown ratio of water component
S_o = oil saturation
$S_{o,\overline{p}}$ = average oil saturation of the reservoir
$S_{o,\text{initial}}$ = initial oil saturation
S_w = water saturation
$S_{w,\overline{p}}$ = average water saturation of the reservoir
$S_{w,\text{initial}}$ = initial water saturation
V_b = bulk volume (ft^3)
c_o^* = equivalent compressibility of oil component (psi^{-1})
c_t^* = equivalent compressibility of total component (psi^{-1})
c_w^* = equivalent compressibility of water component (psi^{-1})
J_o^* = density-based well productivity index oil component
J_t^* = density-based well productivity index total component
J_w^* = density-based well productivity index water component
M_o = mass-in-place of oil component (lb)
M_t = mass-in-place of total component (lb)
M_w = mass-in-place of water component (lb)
$M_{p,o}$ = cumulative mass production of oil component (lb)
$M_{p,t}$ = cumulative mass production of total component (lb)
$M_{p,w}$ = cumulative mass production of water component (lb)
$m_o(p)$ = multiphase pseudo-pressure of oil component (lbm psi/ft^3 cp)
$m_t(p)$ = multiphase pseudo-pressure of total component (lbm psi/ft^3 cp)
$m_w(p)$ = multiphase pseudo-pressure of water component (lbm psi/ft^3 cp)

Greek Symbols

α_c = field unit conversion factor
γ_o = mobility of oil component (lbm/ft^3 md)
γ_t = mobility of total component (lbm/ft^3 md)
γ_w = mobility of water component (lbm/ft^3 md)
$\overline{\lambda}_o$ = depletion-dependent parameter for oil component
$\overline{\lambda}_t$ = depletion-dependent parameter for total component
$\overline{\lambda}_w$ = depletion-dependent parameter for water component
$\overline{\lambda}_{\rho_o}$ = density drawdown parameter for oil component

$\overline{\lambda}_{\rho_t}$ = density drawdown parameter for total component
$\overline{\lambda}_{\rho_w}$ = density drawdown parameter for water component
$\overline{\lambda}_{m_o(p)}$ = pseudo-pressure drawdown parameter for oil component
$\overline{\lambda}_{m_t(p)}$ = pseudo-pressure drawdown parameter for total component
$\overline{\lambda}_{m_w(p)}$ = pseudo-pressure drawdown parameter for water component
μ_o = oil viscosity (md)
μ_w = water viscosity (md)
μ_o^* = equivalent viscosity of oil component (md)
μ_t^* = equivalent viscosity of total component (md)
μ_w^* = equivalent viscosity of water component (md)
π = constant ≈ 3.1415927
ρ_o = oil density (lbm/ft^3)
ρ_{osc} = oil density at standard condition (14.7 psi, 60 °F) = lbm/ft^3
ρ_w = water density (lbm/ft^3)
ρ_{wsc} = water density at standard condition (14.7 psi, 60 °F) = lbm/ft^3
ρ_w = water density (lbm/ft^3)
ρ_o^* = equivalent density of oil component (lbm/ft^3)
ρ_t^* = equivalent density of total component (lbm/ft^3)
ρ_w^* = equivalent density of water component (lbm/ft^3)
ϕ = porosity

Abbreviations

BDF = boundary-dominated flow
BHP = bottomhole pressure
CMG = Computer Modeling Group Ltd
OMIP$_o$ = original mass in-place of oil component
OMIP$_t$ = original mass in-place of total component
OMIP$_w$ = original mass in-place of water component
PDA = production data analysis
WOR = water–oil ratio

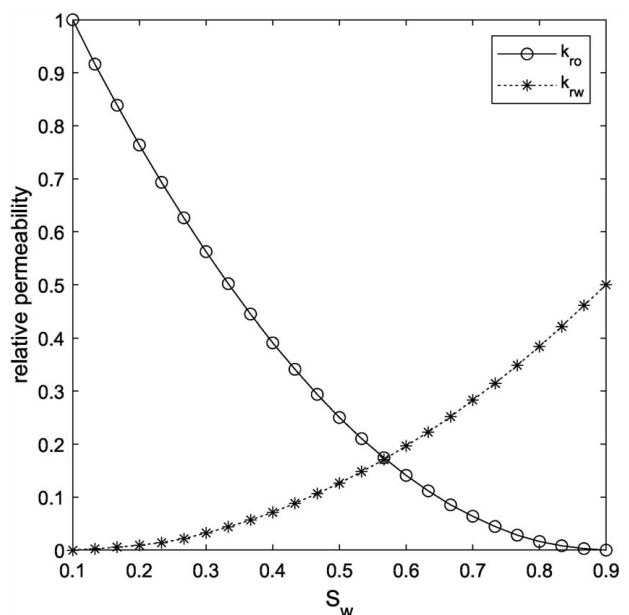## Appendix A: Additional Input Parameters for Numerical Simulation

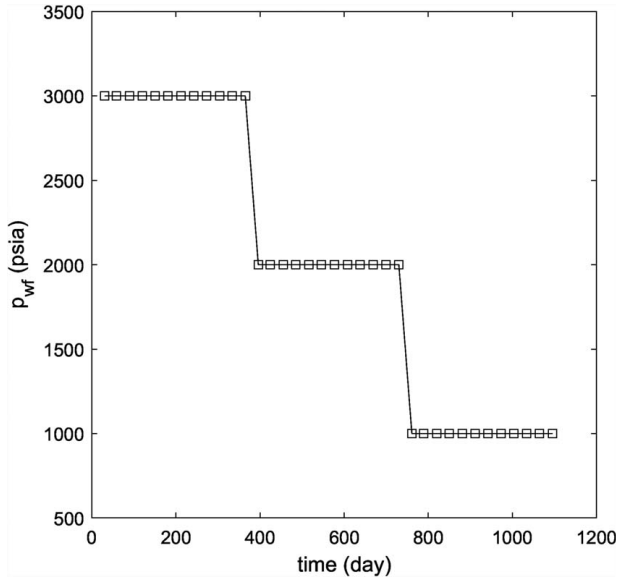**Fig. 6 Relative permeability data for both cases study**

Fig. 7 BHP schedule for variable BHP case

Table 3 Fluid properties

Pressure (psi)	ρ_o (lbm/ft³)	ρ_w (lbm/ft³)	B_o (RB/STB)	B_w (RB/STB)
14.7	54.672	62.4	1	1
200	54.774	62.401	0.9981	0.99998
500	54.938	62.403	0.9952	0.99995
1000	55.214	62.406	0.9902	0.9999
1500	55.491	62.409	0.9853	0.99985
2000	55.769	62.412	0.9803	0.9998
2500	56.048	62.416	0.9755	0.99975
3000	56.329	62.419	0.9706	0.9997
4000	56.895	62.425	0.9609	0.9996
5000	57.467	62.431	0.9514	0.9995

Appendix B: Average Pressure Calculation and S_o–p Behavior

The next subsection is dedicated to present a straightforward protocol to compute average pressure using tank material balance equations and in turn, a saturation–pressure relationship using constant in situ composition (in situ water–oil ratio (WOR)) assumption.

By expanding Eqs. (14)–(16), the phase saturation at average pressure condition can be expressed as Eqs. (B1) and (B2), where the left-hand side solely depend on average pressure $\overline{p}$:

$$S_{o,\overline{p}} = \frac{\rho_{o,\text{initial}}S_{o,\text{initial}}}{\rho_{o,\overline{p}}}\left(1 - \frac{M_{p,o}}{\text{OMIP}_o}\right) \tag{B1}$$

$$S_{w,\overline{p}} = \frac{\rho_{w,\text{initial}}S_{w,\text{initial}}}{\rho_{w,\overline{p}}}\left(1 - \frac{M_{p,w}}{\text{OMIP}_w}\right) \tag{B2}$$

In a two-phase oil/water system, the summation of phases saturation always equal to 1: $S_o + S_w = 1$. Thus, the density of oil and water at average reservoir pressure condition ($\rho_{o,\overline{p}}$ and $\rho_{w,\overline{p}}$) must honor Eq. (B3) and average pressure $\overline{p}$ can be interpolated

from previously prepared table of p versus ρ_o and ρ_w.

$$\frac{\rho_{o,\text{initial}}S_{o,\text{initial}}}{\rho_{o,\overline{p}}}\left(1 - \frac{M_{p,o}}{\text{OMIP}_o}\right) + \frac{\rho_{w,\text{initial}}S_{w,\text{initial}}}{\rho_{w,\overline{p}}}\left(1 - \frac{M_{p,w}}{\text{OMIP}_w}\right) = 1 \tag{B3}$$

Noticeably, the original mass in-place of oil and water component, OMIP_o and OMIP_w, must satisfy the volumetric relationship: $V_b\phi = \text{OMIP}_o/\rho_{o,\text{initial}}S_{o,\text{initial}} = \text{OMIP}_w/\rho_{w,\text{initial}}S_{w,\text{initial}}$. Therefore, OMIP_o can be correlated to OMIP_w and vice versa:

$$\text{OMIP}_o = \text{OMIP}_w\frac{\rho_{o,\text{initial}}S_{o,\text{initial}}}{\rho_{w,\text{initial}}S_{w,\text{initial}}} \tag{B4}$$

$$\text{OMIP}_w = \text{OMIP}_o\frac{\rho_{w,\text{initial}}S_{w,\text{initial}}}{\rho_{o,\text{initial}}S_{o,\text{initial}}} \tag{B5}$$

We rely on the premise that the in situ fluid ratio in an oil/water reservoir follows the behavior of region I in gas-condensate reservoir, where both fluids simultaneously flow together. A constant in situ WOR across the whole reservoir domain is clearly observed in simulation, meaning that the producing WOR recorded at surface could be assigned for any pressure value in between p_{wf} and $\overline{p}$ when tubing effect and temperature gradient are negligible [16]:

$$\text{WOR} = \frac{q_{wsc}}{q_{osc}} = \frac{(kA/\rho_{wsc})(\gamma_W(\partial p/\partial r))|_{r=0}}{(kA/\rho_{osc})(\gamma_o(\partial p/\partial r))|_{r=0}} = \frac{\rho_{osc}}{\rho_{wsc}}\left.\left(\frac{\rho_w(k_{rw}/\mu_w)}{\rho_o(k_{ro}/\mu_o)}\right)\right|_{r=0} \tag{B6}$$

Thus, the pressure–saturation relationship at any given pressure value can be obtained by rearranging Eq. (B7):

$$\left.\frac{k_{rw}}{k_{ro}}\right|_{S_w} = \text{WOR}\frac{\rho_{wsc}}{\rho_{osc}}\frac{\rho_o\mu_w}{\rho_w\mu_o}\bigg|_p \tag{B7}$$

Appendix C: Iterative Protocol for Reserve Estimation

In this part, we present a step-by-step iterative procedure to estimate the original fluid in place of a two-phase oil/water system under boundary-dominated flow condition. It should be noted that the following procedure is to be performed with the straight line relationships written in Eq. (17) or Eq. (19), using surface oil production data or total production data, respectively. The required inputs for this straight line analysis are fluid properties (density and viscosity data), relative permeability data, initial reservoir condition (pressure and saturation), and production history (flowrate and cumulative production of oil and water).

The following steps (illustrated in Fig. 8) described the iterative protocol to obtain original mass in-place of oil component (OMIP_o), using the straight line relationships in Eq. (17):

(1) Calculate an initial guess for $\text{OMIP}_{o,\text{guess}}$, by plotting $r_{\rho_o^*}/\dot{m}_o$ versus $M_{p,o}/\dot{m}_o$ and fitting the boundary-dominated data to a straight line. The initial guess for $\text{OMIP}_{o,\text{guess}}$ can be obtained from the reciprocal of the slope.

(2) Calculate $\text{OMIP}_{w,\text{guess}}$ using Eq. (B5) and initial values of density and saturation.

(3) Solve the material balance equaion (B3) to obtain average reservoir pressure $\overline{p}$.

(4) Based on the constant WOR assumption, compute a saturation–pressure profile for the whole pressure domain ($p_{wf} - \overline{p}$) at each boundary-dominated time-step.

(5) Use Eq. (B7) to calculate the values of $\overline{\lambda}_{\rho_o}$, $\overline{\lambda}_{m_o(p)}$, and in turn, $\overline{\lambda}_o$.

(6) Obtain the new value of $\text{OMIP}_{o,\text{guess}}$ using Eq. (17), then calculate the difference between the new value and the previous guess of OMIP_o.

(7) Repeat steps 2–6 until ΔOMIP_o satisfies a prescribed tolerance (set at 10^3 lb).
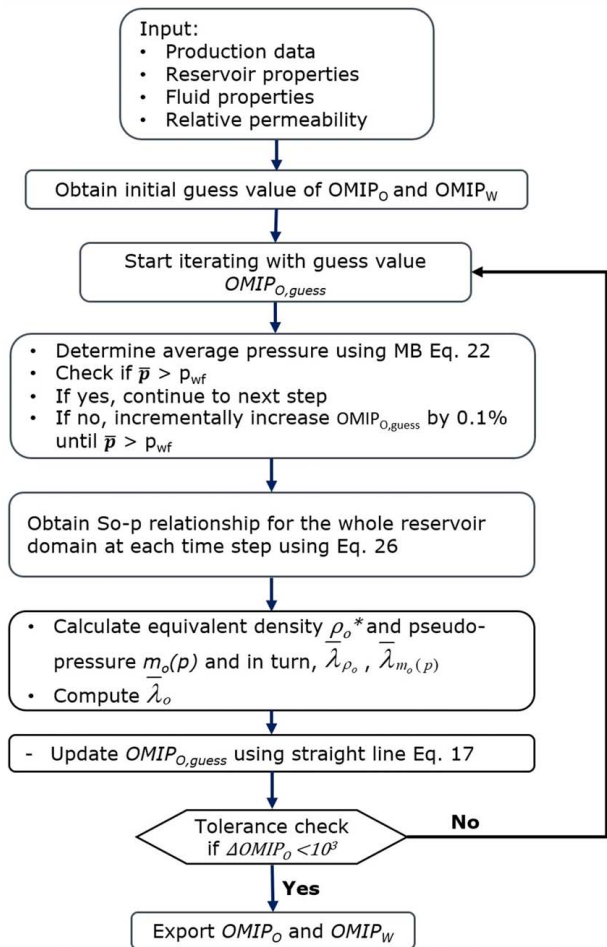
Fig. 8 Flowchart of iterative protocol for reserve estimation

The flowchart contains the following text boxes:

Input:
- Production data
- Reservoir properties
- Fluid properties
- Relative permeability

Obtain initial guess value of $OMIP_O$ and $OMIP_W$

Start iterating with guess value $OMIP_{O,guess}$

- Determine average pressure using MB Eq. 22
- Check if $\bar{p} > p_{wf}$
- If yes, continue to next step
- If no, incrementally increase $OMIP_{O,guess}$ by 0.1% until $\bar{p} > p_{wf}$

Obtain So-p relationship for the whole reservoir domain at each time step using Eq. 26

- Calculate equivalent density $\rho_o{}^*$ and pseudo-pressure $m_o(p)$ and in turn, $\bar{\lambda}_{\rho_o}$, $\bar{\lambda}_{m_o(p)}$
- Compute $\bar{\lambda}_o$

- Update $OMIP_{O,guess}$ using straight line Eq. 17

Tolerance check if $\Delta OMIP_O < 10^3$ — No / Yes

Export $OMIP_O$ and $OMIP_W$

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