АЭТЕРНА
НАУЧНО-ИЗДАТЕЛЬСКИЙ ЦЕНТР

ISSN 2541-8076

№ 7-2/2022

НАУЧНЫЙ

ЭЛЕКТРОННЫЙ

ЖУРНАЛ

Академическая Публицистика

ПЕДАГОГИЧЕСКИЕ НАУКИ

ВЕТЕРИНАРНЫЕ НАУКИ

ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ

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CALCULATING PERIODIC OSCILLATION OF A SINGLE-LINK FLEXIBLE MANIPULATOR

ABSTRACT

In the robot manipulators operating at high speeds, the elastic vibration of links is inevitable. The present paper deals with problem of calculation periodic oscillation of a single-link flexible manipulator. First, linearize the motion equations of flexible robot around the basic motion based on a Taylor expansion. Then, calculate the periodic oscillation. The proposed procedure is demonstrated and verified by the model of a flexible single-link manipulator.

KEYWORDS

Flexible manipulator, linearization, Taylor expansion, periodic oscillation

1. Introduction

Recently, flexible robots have been used in space technology, nuclear reactors, medical engineering, and many other fields. Flexibility, small volume, high speed, and low power consumption are advantages over rigid robots. However, the elastic displacements created by flexible links are the main cause of questions about position accuracy, structure stability and vibration. Some scientists have done research to solve those problems. However, the research results obtained are still relatively small and need to be studied further.

Bayo et al. [1] and Asada et al. [2] have proposed two different algorithms for calculating the torques required to move the end effector of flexible manipulators. A brief description about the development of stabile and vibration analysis of flexible manipulators has been depicted here.

The assumed mode method has been used to study the stability and vibration of flexible manipulators. Chiou and Shahinpoor [3] analyzed the stability limitations for force-controlled two-link flexible manipulator and compared it with the model considering rigid body dynamics. Poppelwell and Chang [4] determined the natural frequencies of single link flexible manipulator when the center of the payload does not coincide with manipulator end. Coleman [5] analyzed the vibration eigen-frequency of a flexible slewing beam with a payload attached at one end using wave propagation method. The results showed that the large frequencies are asymptotically identical to those for the clamped free beam independent of the payload.

Using the singular perturbation approach, X. Yang et al. [6] investigated the tracking control of a two-link flexible manipulator by adaptive sliding mode control scheme and linear quadratic regulator control method. With the proposed control, the closed-loop stability under unknown disturbances has been proven. Using the numerical method. Kumar and Pratiher [7] investigated the free vibration of a two-link flexible manipulator.

In this study, the linearization problem of the non-linear equations governing the motion of flexible manipulators in the vicinity of periodic fundamental motion is addressed. Then, calculates the periodic oscillation of a single-link flexible manipulator.

2. Dynamics of a single-link flexible manipulator

2.1. *Fundamental motion of the flexible manipulator*

The fundamental motion of the manipulator is the virtual rigid link motion of the link OE [2] such as fig 1.

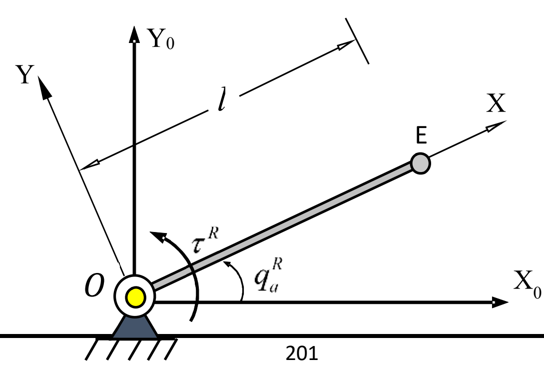

Fig.1. Single-link rigid manipulator

From the virtual rigid link motion, the position of the point *E* on the link is given as

$$x_E^R = l\cos q_a^R(t),\quad y_E^R = l\sin q_a^R(t) \tag{1}$$

The mass moment of inertia of the virtual rigid link with respect to point O takes the form

$$J_O = \frac{1}{3}\rho A l^3 + m_E l^2 + J_1\,, \tag{2}$$

where J_1 is the mass moment of inertia of link 1 (including the motor) with respect to point *O*, $\rho$is the density of beam and A is the sectional area of beam, $m_E$is the mass of the payload. Using the momentum theorem, it follows that

$$(\frac{1}{3}\rho A l^3 + m_E l^2 + J_1)\ddot{q}_a^R(t) = -m_{OE}g\frac{l}{2}\cos q_a^R(t) - m_E g l\cos q_a^R(t) + \tau^R(t) \tag{3}$$

Assuming the motion rule of the drive has the following form

$$q_a^R(t) = \frac{p}{2} + \frac{p}{2}\sin(2pt) \tag{4}$$

By differentiating Eq (4) and then substituting the obtained result into Eq. (3) we have

$$\begin{aligned}\tau^R(t) = &-2p^3(\frac{1}{3}rAl^3 + m_E l^2 + J_1)\sin(2pt)\\ &+m_{OE}g\frac{l}{2}\cos(\frac{p}{2}+\frac{p}{2}\sin(2pt)) + m_E g l\cos(\frac{p}{2}+\frac{p}{2}\sin(2pt))\end{aligned} \tag{5}$$

From Eq. (4) the position of the point *E* on the link is given as

$$x_E^R = l\cos q_a^R(t) = l\cos(\frac{p}{2}+\frac{p}{2}\sin(2pt))\,;y_E^R = l\sin q_a^R(t) = l\sin(\frac{p}{2}+\frac{p}{2}\sin(2pt)) \tag{6}$$

2.2 Equations of motion of a single – link flexible manipulator.

Using the floating frame of reference approach [8], in this subsection we set up the motion equations for a single-link flexible manipulator. Consider a single-link flexible manipulator OE of length *l* with a rotor located at the hut and a payload at the free end. As shown in Fig.2, the end of the link is attached to the O point (including the motor) revolving around the O axis, at the E of the link carries mass m_E. The link is considered as homogeneous beam with the area A.

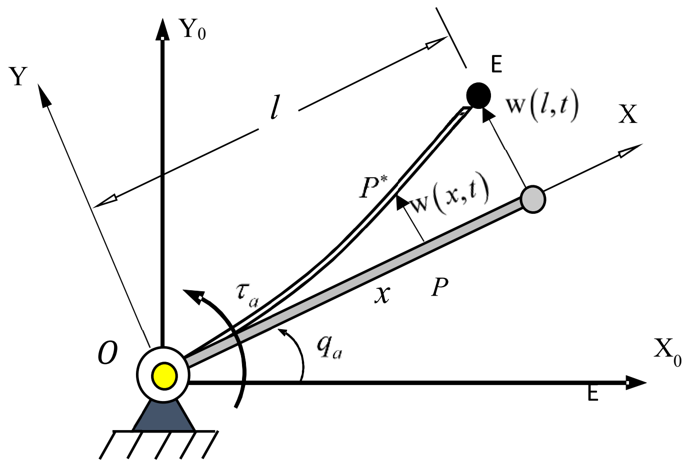

Fig.2. Single-link flexible manipulator

To describe the kinematics, the position of point *P* on the flexible beam is given as

$$\begin{aligned} x_P &= x\cos q_a - w(x,t)\sin q_a \\ y_P &= x\sin q_a + w(x,t)\cos q_a \end{aligned} \qquad (7)$$

Differentiation of Eq. (7) yields

$$v_P^2 = \dot{x}_P^2 + \dot{y}_P^2 = (w^2 + x^2)(\dot{q}_a)^2 + \dot{w}^2 + 2x\dot{w}\dot{q}_a \qquad (8)$$

It follows that

$$v_E^2 = (w_E^2 + l^2)(\dot{q}_a)^2 + \dot{w}_E{}^2 + 2l\dot{w}_E\dot{q}_a \qquad (9)$$

The Euler-Bernoulli beam theory and Ritz-Galerkin method are applied to the flexible manipulator with assuming that the deformation in the longitudinal direction is negligibly small. Let the transverse deformation of the beam be written as

$$w(x,t) = \sum_{i=1}^{N} X_i(x)q_{ei}(t)\,, w_E = \sum_{i=1}^{N} X_i(l)q_{ei}(t), \qquad (10)$$

where $q_{ei}(t)$ are unknown generalized coordinates of transverse deformation, $X_i(x)$ are a set of mode shapes of transverse deformation of a clamped- free beam and N is the number of modes used to describe the defection of the flexible link. The mode shapes are given as [9]

$$X_i(x) = \cos(b_i x) - \cosh(b_i x)$$

$$+ \frac{\cos b_i l + \cosh b_i l}{\sin b_i l + \sinh b_i l}\left(-\sin(b_i x) + \sinh(b_i x)\right) \quad (11)$$

The kinetic energy of the flexible manipulator shown in Fig. 1 is given by

$$T = T_1 + T_E + T_{OE} = \frac{1}{2}J_1(\dot{q}_a)^2 + \frac{1}{2}m_E v_E^2 + \frac{1}{2}\int_0^l \rho A v_P^2 dx, \quad (12)$$

where J_1 is the mass moment of inertia of link 1 (including the motor) with respect to the point O, m_E is the mass of the point E, ρA is the mass per unit length of the beam.

By substitution of Eqs. (7), (8), (9) and (10) into Eq. (11), we obtain the kinetic energy of system

$$T = \left(\frac{1}{2}J_1 + \frac{1}{2}m_E l^2 + \frac{1}{6}\rho A l^3\right)(\dot{q}_a)^2 + \frac{1}{2}m_E[w_E^2(\dot{q}_a)^2 + \dot{w}_E^2 + 2l\dot{w}_E\dot{q}_a] +$$

$$\frac{1}{2}\rho A\int_0^l \dot{w}^2 dx + \frac{1}{2}\rho A(\dot{q}_a)^2\int_0^l w^2 dx + \rho A\dot{q}_a\int_0^l x\dot{w}dx \quad (13)$$

The strain energy of the beam OE according to Reddy [9] is given by

$$\Pi_{dh} = \frac{1}{2}EI\int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx, \quad (14)$$

where E and I is the modulus of elasticity, area moment of inertia of the beam, respectively.

By substituting Eqs. (7), (10) and (11) into Eq. (14), we obtain

$$\Pi = m_E g[l\sin q_a + \sum_{i=1}^{N} X_i(l) q_{ei}(t)\cos q_a] + \frac{m_{OE} g l \sin q_a}{2}$$

$$+ mg\cos q_a \sum_{i=1}^{N} C_i q_{ei} + \frac{1}{2}EI\sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij} q_{ei} q_{ej}\ , \quad (15)$$

where

$$C_i = \int_0^{l_2} X_i dx;\ k_{ij}^* = \int_0^{l_2} X_i'' X_j'' dx \quad (16)$$

The Lagrange equations have the following form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = -\frac{\partial \Pi}{\partial q_j} + Q_j^*,\ j=1,2,\ldots,n\ , \quad (17)$$

where q_j are the generalized coordinates which include rigid body coordinete q_a as well elastic modal q_{ei}, and Q_j^* are generalized forces.

By substituting Eqs. (13) and (15) into Eq.(17), we obtain the equations of motion of the system as

$$[J_1 + m_E l^2 + \frac{1}{3}\rho A l^3 + \rho A \sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\, q_{ei} q_{ej} + m_E \sum_{i=1}^{N}\sum_{j=1}^{N} X_i(l) X_j\,(l) q_{ei} q_{ej}]\, \ddot{q}_a$$

$$+[2m_E \sum_{i=1}^{N}\sum_{j=1}^{N} X_i(l) X_j\,(l) + 2\rho A \sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}]\dot{q}_a \dot{q}_{ei} q_{ej} + [\rho A \sum_{i=1}^{N} D_i + m_E l \sum_{i=1}^{N} X_i(l)]\ddot{q}_{ei}$$

$$= - m_E g[l \cos q_a - \sum_{i=1}^{N} X_i(l) q_{ei} \sin q_a] - \frac{m_{OE} g l \cos q_a}{2} + mg \sin q_a \sum_{i=1}^{N} C_i q_{ei} + \tau \qquad (18)$$

$$[m_E l X_i(l) + \rho A D_i]\ddot{q}_a + [m_E X_i(l) \sum_{j=1}^{N} X_j\,(l) + \rho A \sum_{j=1}^{N} m_{ij}]\ddot{q}_{ej} + EI \sum_{j=1}^{N} k_{ij}^* q_{ej}$$

$$-[m_E X_i(l) \sum_{j=1}^{N} X_j\,(l) q_{ej} + \rho A \sum_{j=1}^{N} m_{ij} q_{ej}]\dot{q}_a^2 = - m_E g X_i(l) \cos q_a - mg C_i \cos q_a, \quad i = 1,2,\ldots,N. \qquad (19)$$

where

$$D_i = \int_0^{l_2} x X_i dx;\; m_{ij} = \int_0^{l_2} X_i X_j dx \qquad (20)$$

If we choose N = 1 and use of symbols $q_{e1} = q_e$, the differential equations of the single-link flexible manipulator give the following form

$$[J_1 + m_E l^2 + \frac{1}{3}\rho A l^3 + (\rho A m_{11} q_{e1}^2 + m_E X_1^2(l) q_{e_1}^2)]\ddot{q}_a + [\rho A D_1 + m_E l X_1(l)]\ddot{q}_{e1} +$$

$$[2 m_E X_1^2(l) + 2\rho A m_{11}]\dot{q}_a \dot{q}_{e1} q_{e1} + \frac{m_{OE} g l \cos q_a}{2} - mg \sin q_a C_1 q_{e1}$$

$$= - m_E g[l \cos q_a - X_1(l) q_{e1} \sin q_a] + \tau \qquad (21)$$

$$m_E X_1^2(l)\ddot{q}_{e1} + m_E l X_1(l)\ddot{q}_a + \rho A D_1 \ddot{q}_a + \rho A m_{11}\ddot{q}_{e1} - m_E \dot{q}_a^2 X_1^2(l) q_{e1} - \rho A \dot{q}_a^2 m_{11} q_{e1}$$

$$+EI k_{11}^* q_{e1} = - m_E g X_1(l) \cos q_a - mg \cos q_a C_1 \qquad (22)$$

3. Linearization of the motion equations of flexible manipulator about the fundamental motion

Now consider the problem of linearizing motion equations of the single-link flexible manipulator, which consists of a single flexible beam with a link at one end and the hub as an example. A rigorous model for the dynamics of a flexible slewing beam, with a rotor located at the hut and a payload at the free end is shown in Fig.2.

The fundamental motion of the manipulator is described by $\boldsymbol{q}^R(t)$ and $\tau^R(t)$, where $\boldsymbol{q}^R(t)$ is the generalized coordinate of the manipulator

$$\boldsymbol{q}^R(t) = [q_a^R(t) \quad q_e^R(t)]^T = [q_a^R(t) \quad 0]^T. \tag{23}$$

and $\tau^R(t)$ is the torque

$$\tau^R(t) = [\tau_a^R \quad \tau_e^R]^T = [\tau_a^R \quad 0]^T \tag{24}$$

In Eqs. (21) and (22) $q_e^R(t)$ is the elastic generalized coordinate, and $\tau_e^R(t)$ is the elastic torque of the virtual rigid link.

The differential equations of the single-link flexible manipulator (21) and (22) can be expressed in the following matrix form

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \tau(t) \tag{25}$$

where $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are vectors of generalized position, velocity and acceleration variables, respectively

$$\boldsymbol{q} = [q_a, q_e]^T,\ \tau(t) = [\tau_a(t), \tau_e(t)]^T = [\tau_a(t), 0]^T. \tag{26}$$

Let $\Delta q_a$and $\Delta q_e$are the difference between the real motion$\boldsymbol{q}(t)$ and the fundamental motion $\boldsymbol{q}^R(t)$, we have

$$q_a(t) = q_a^R(t) + \Delta q_a(t) = q_a^R(t) + y_1(t) \tag{27}$$

$$q_e(t) = q_e^R(t) + \Delta q_e(t) = y_2(t) \tag{28}$$

Where y_1 and y_2 are called the additional motion or the perturbed motion. Similarly, we have

$$\tau(t) = [\tau_a(t), \tau_e(t)]^T = [\tau_a(t), 0]^T \tag{29}$$

By substituting Eqs. (25), (26) into Eq. (25) and using Taylor series expansion around fundamental motion, then neglecting nonlinear terms, we obtain the system of linear differential equations with time-varying coefficients for the single-link flexible manipulator as follows [10]

$$\boldsymbol{M}_L(t)\ddot{\boldsymbol{y}} + \boldsymbol{C}_L(t)\dot{\boldsymbol{y}} + \boldsymbol{K}_L(t)\boldsymbol{y} = \boldsymbol{h}_L(t)\ . \tag{30}$$

The matrices $\boldsymbol{M}_L(t), \boldsymbol{C}_L(t), \boldsymbol{K}_L(t)$ and vector $\boldsymbol{h}_L(t)$ of the linear differential equations (30) have the following forms

$$\boldsymbol{M}_L(t) = \begin{bmatrix} J_1 + m_E l^2 + \frac{1}{3} m_{OE} l^2 & \rho A D_1 + m_E l X_1(l) \\ m_E l X_1 + \rho A D_1 & m_E X_1^2(l) + \rho A m_{11} \end{bmatrix} \quad (31)$$

$$\boldsymbol{C}_L(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (32)$$

$$\boldsymbol{K}_L(t) = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad (33)$$

where

$$\begin{aligned} k_{11} &= -l \sin q_a^R(t) m_E g - \frac{m_{OE} g l \sin q_a^R(t)}{2}, \\ k_{12} &= k_{21} = -m_E g X_1(l) \sin q_a^R(t) - m g \sin q_a^R(t) C_1, \\ k_{22} &= -m_E [\dot{q}_a^R(t)]^2 X_1^2(l) - \rho A [\dot{q}_a^R(t)]^2 m_{11} + E I k_{11}^*. \end{aligned} \quad (34)$$

and

$$\mathbf{h}_L(t) = \begin{bmatrix} 0 \\ m_E g X_1(l) \cos q_a^R(t) - m g \cos q_a^R(t) C_1 - m_E l X_1 \ddot{q}_a^R(t) - \rho A D_1 \ddot{q}_a^R(t) \end{bmatrix} \quad (35)$$

where fundamental motion $q_a^R(t)$ is given by Eq. (4) and constants $C_1, D_1, X_1, m_{11}, k_{11}^*$ are determined by Eqs. (11), (16) and (20). It should be noted that the matrices $\boldsymbol{M}_L(t), \boldsymbol{C}_L(t), \boldsymbol{K}_L(t)$ and vector $\boldsymbol{h}_L(t)$ are time-periodic with least period *T*.

The calculating parameters of the considered manipulator are listed in Tab. 1.

Table 1. Parameters of the manipulator

Parameters of the model	Variable and Unit	Value
Length of link	*l* (m)	0.9
Sectional area of beam	*A* (m^2)	4×10^{-4}
Density of beam	ρ (kg/ m^3)	2700
Inertial moment of sectional area of beam	*I* (m^4) = $bh^3/12$	1.33334×10^{-8}
Modulus	*E* (N/ m^2)	7.11×10^{10}
Mass moment of inertia of link 1 (including the motor)	$J_1 \left(\mathrm{kg.m}^2\right)$	5.86×10^{-5}
Mass of payload	m_E (kg)	0.1

It follows from the parameters in Tab. 1 that

$$C_1 = -0.7046317896,\ D_1 = -0.4607100845,$$

$$m_{11} = 0.8998501520,\ k_{11}^* = 16.95515100,\ X_1 = -2 \tag{36}$$

4. Calculating the periodic oscillation of a single – link flexible manipulator.

4.1. Periodic oscillation

we go to find the periodic solution for the equations.

$$\boldsymbol{M}_L^{(1)}(t)\ddot{\boldsymbol{y}} + \boldsymbol{C}_L^{(1)}(t)\dot{\boldsymbol{y}} + \boldsymbol{K}_L^{(1)}(t)\boldsymbol{y} = \boldsymbol{h}_L^{(1)}(t) \tag{37}$$

Using the periodic solution algorithm of the system of linear differential equations [11] we find the periodic oscillation of the system of equations (37) in the form:

$$\boldsymbol{y}^* = [y_1^* \quad y_2^*]^T \tag{38}$$

When the system is stable then

$$\boldsymbol{y}^* \approx \boldsymbol{y} \tag{39}$$

Using PD control method with k_p =120 and k_d =80 we get

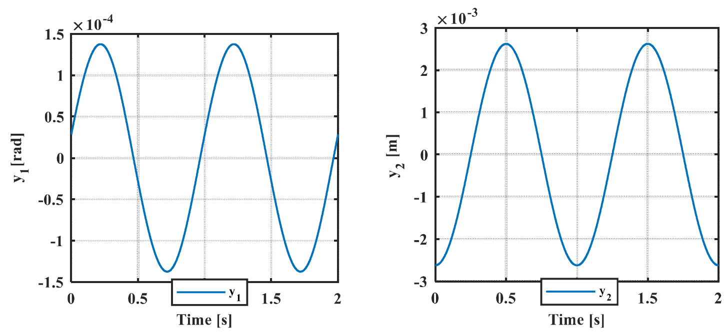

Fig. 3. Periodic solution

From Figure 3, we see that the periodic elastic oscillation is small.

4.2. Approximate motion of a single – link flexible manipulator.

After determining the elastic oscillation of the robot as in section 4.1, then we have the driving link coordinates such as:

$$q_a(t) = q_a^R(t) + y_1(t)$$

$$q_e(t) = y_2(t) \tag{40}$$

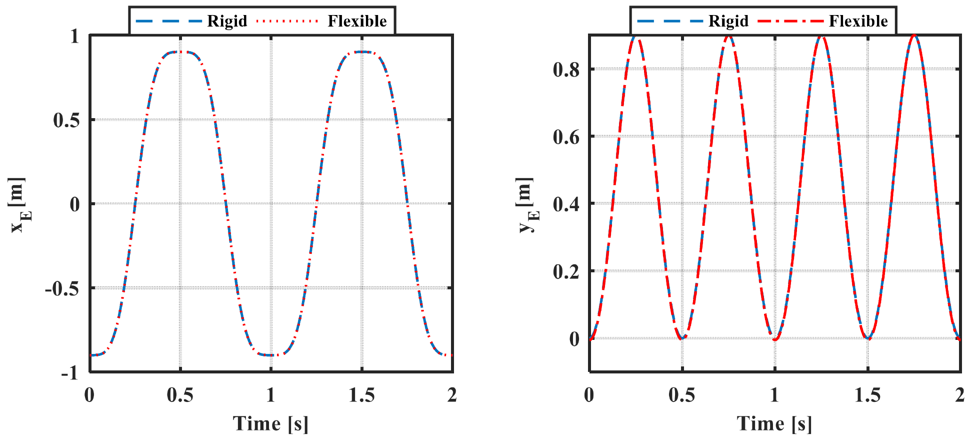

Fig. 4. Motion of end point

Now, the elastic displacement at the end point such as:

$$w(l,t) = X_1(l)y_2(t) \tag{41}$$

Then, the motion of end point E to given as

$$x_E = l\cos q_a(t) - w(l,t)\sin q_a(t) \tag{42}$$

$$y_E = l\sin q_a(t) + w(l,t)\cos q_a(t) \tag{43}$$

Cucalating by Matlab we obtain the motion of end point as shown in Figure 4, we can see that the motion deviation of end point when flexibe and rigid is very small.

5. Conclusions

In the present paper, the linearization problem of the equation of motion of flexible manipulators in the vicinity of a fundamental motion is addressed. Determine the approximate periodic oscillation for flexible manipulaters which are described by linear differential equations with time-periodic coefficients

Through numerical simulation, the efficiency and usefulness of the proposed algorithm were demonstrated as well as the problem and further issues.

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